Lecture

on

Relativistic Mechanics

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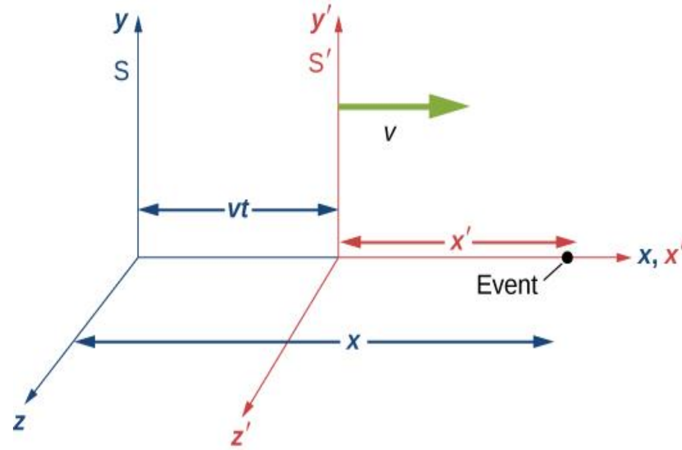
Frame of Reference: A system of coordinate axes which defines the position of a particle or specifies the location of an event is called a frame of reference.

Inertial Frames of Reference: Two frames are said to be inertial with respect to each other if there is no acceleration between them i.e. un-accelerated. In such frames all the laws of Newtonian Mechanics hold good. For example – The person and vehicle are inertial frames as shown below.

Non-inertial Frames of Reference: Two frames are said to non-inertial with respect to each other if there is an acceleration between them i.e. accelerated. In such frames, the laws of Newtonian Mechanics do not hold good. For example - car2 is accelerated along the direction of motion as shown below.

Galilean Transformation Equations: The equations which provide the relationship between the coordinates of two reference systems are called transformation equations. Galilean transformations are used to transform the coordinates of position and time from one inertial frame to another.

In order to obtain the Galilean transformation equations, consider two frames of reference S and S' with axes (x,y,z) and (x',y',z') respectively. The frame S' is moving with a uniform velocity v along the x-axis. At t=0, the two frames coincided which means that the axis of S and S' overlapped. At any time t, the x-coordinate of event in S exceeds that in S' by vt, the distance covered by S' in time t in the positive x direction as shown in figure below. Therefore, the observed coordinates in the two frames are given by the following transformation equations



$X' = x - vt$, $y' = y$, $z' = z$, $t' = t$ ------------- (1)

The set of equations (1) are known as Galilean transformations. We can consider that frame S is moving with velocity –v along the negative x-axis with respect to frame S'. Then the transformation equations from S' to S are as follows

$X = x' + vt'$, $y = y'$, $z = z'$, $t = t'$ ------------- (2)

The set of equations (2) are known as inverse Galilean transformation equations. Differentiating the transformation eq. (1), we get velocity transformation equations from s to S'. These equations are

$U_x' = u_x - v$, $u_y' = u_y$, $u_z' = u_z$ and $dt' = dt$ ----------- (3) because $dx/dt = u_x$ & $dx'/dt' = u_x'$

Thus, velocity is not invariant under Galilean transformations.

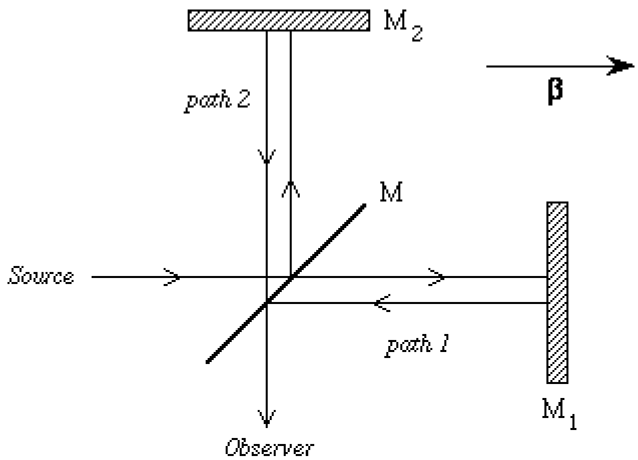
The acceleration transformation equations are obtained by differentiating equation 3, we have

$a_x' = a_x$, $a_y' = a_y$, $a_z' = a_z$ ---------- (4)

Thus, the acceleration is invariant under Galilean transformations.

Michelson – Morley Experiment: In 1887, Albert Michelson and Edward Morley carried out an experiment to detect the motion of the earth relative to ether medium at rest using Michelson interferometer. In 19th century, scientists had assumed that a hypothetical medium called luminiferrous ether is required for the propagation of the light. It was considered that the ether exists uniformly in the space and it is at rest relative to the earth and other planets. Ether which is transparent, invisible, massless, perfectly non-resistive, high elastic and negligible density. Thus, ether provides a fixed frame of reference which was called ether frame or rest frame of reference.

Figure A



The arrangement for Michelson-Morley experiment is shown in Figure A above. A beam of light from the source is incident upon a 45^0 inclined glass plate M. It splits into two components one is reflected and other is refracted. These beams travel at right angles to each other and are normally incident on mirror M1 and M2 placed at equal distances MM1 = MM2 = l = L from the glass plate M. After reflections from the mirrors, the two beams interfere at point M. The interference fringes are observed in the telescope as observer. If the apparatus were at rest, the two beams would take the same time to return to M.

Let us consider that earth along with the apparatus moves with a velocity v in ether. Suppose c is the velocity of light through the ether. Here we consider $\beta = v$. When light goes from M to M1, the relative velocity of light is $c - v$. From M1 to M the relative velocity is $c + v$. Finally, from either M to M2 or M2 to M, the relative velocity of light is $(c^2 - v^2)^{1/2}$. Thus the time required by light to go along the parallel path from M to M1 and back to M, as measured by the observer is

$$t1 = L/(c - v) + L/(c + v) = 2Lc/(c^2 - v^2) = (2L/c)/(1 - v^2/c^2)$$

However, the time required to go along the perpendicular path from M to M2 and back to M, as measured by observer is

$$t2 = 2L/(c^2 - v^2)^{1/2} = (2L/c)/(1 - v^2/c^2)^{1/2}$$

Hence, the time difference between the times of the travel of the two beams is

$$\Delta t = t1 - t2 = (2L/c)/(1 - v^2/c^2) - (2L/c)/(1 - v^2/c^2)^{1/2}$$

$= (2L/c)[\{1 – v^2/c^2\}^{-1} – \{1 – v^2/c^2\}^{-1/2}]$

Using binomial theorem and neglecting higher terms, we get

$\Delta t = (2L/c)[(1 + v^2/c^2 + \ldots\ldots\ldots) – (1 + v^2/2c^2 + \ldots\ldots\ldots)]$

$= (2L/c)[v^2/2c^2] = Lv^2/c^3$

Now the corresponding path difference is

$\Delta = c\Delta t = c(Lv^2/c^3) = Lv^2/c^2$

Finally, the whole apparatus is turned through 90^o so that the path MM1 becomes longer than the path MM2 by an amount Lv^2/c^2. As a result, a path difference of same amount in opposite direction is introduced so that the total path difference between the two rays becomes $2Lv^2/c^2$. Thus, the fringe shift is

$\Delta n = \text{Path diff.}/\lambda = 2Lv^2/c^2\lambda$

If L =11 m, λ = 6000Å and v = 3 x 10^4 m/s then $\Delta n = 0.4$

No shift in the fringe was observed even when the interferometer was rotated through 90^o. This indicates that the relative velocity between the earth and the ether is zero.

Negative Results of Michelson-Morley Experiment

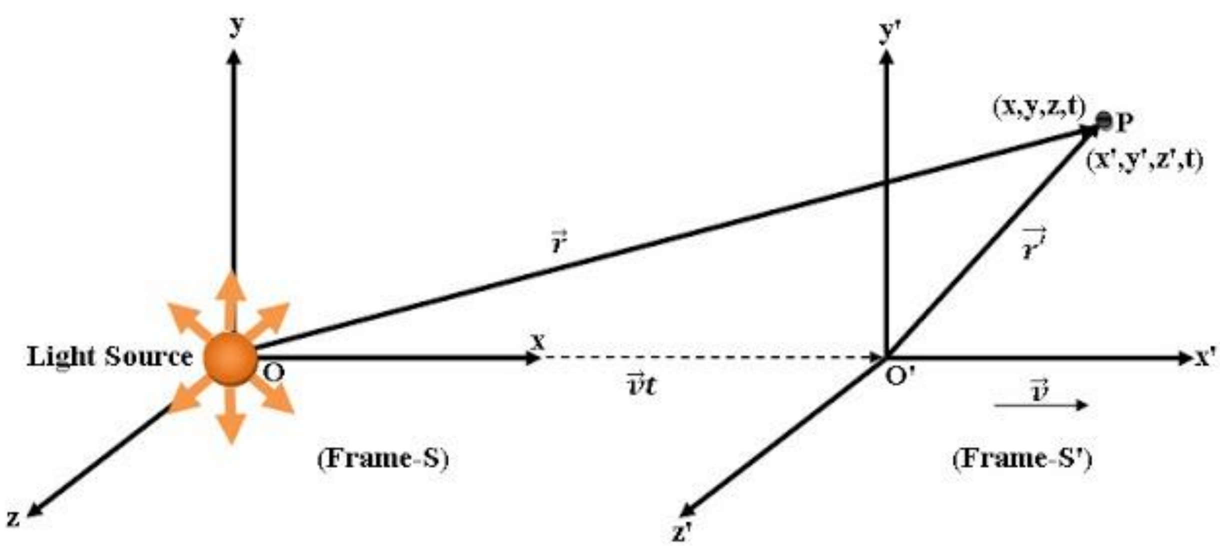
1. ***Ether-Drag Hypothesis:*** Michelson & Morley assumed that the earth drags the "ether", with the same velocity as that of its. This implies that there is no relative motion between earth and ether.
2. ***Fitzgerald-Lorentz Contraction Hypothesis:*** According to length contraction theory, the length of an object appears to be contracted by a relative observer along the direction of motion as $L = L_O(1 – v^2/c^2)^{1/2}$
   Where LO is the length of an object in stationary condition & L is the observed or relative length when object moves with velocity v with respect to observer. If L is replaced by $L_O(1 – v^2/c^2)^{1/2}$, then t1 = t2 i.e. $\Delta t = 0$ i.e. $\Delta n = 0$
3. ***Constancy of the speed of light:*** The idea was formulated by Einstein. According to constancy of speed of light is an absolute physical quantity i.e. it does not depend upon the relative motion between light source and observer.

Einstein's postulates: Einstein proposed a new theory of relativity known as Einstein's special theory of relativity. This theory is based on the following postulates.

1. Principle of equivalence: All the fundamental laws of physics are the same for all the systems that move uniformly (inertial frames) relative to one another.
2. Principle of Constancy of the speed of light: The speed of light in free space (vacuum) is constant in all inertial frames and is independent of the relative motion of source, observer and inertial frame.

Lorentz Transformation equations: A transformation that changes space-time coordinates (x,y,z,t) into (x',y',z',t) in such a way that the speed of light is constant in all inertial frames was first obtained by Lorentz and is hence called Lorentz transformation.

Consider two inertial frames S and S', S' is moving with a velocity v relative to S. Both frames coincide at time t = t'. Now consider an event that occurs at the point P (x,y,z,t) as measured in S. The same event occurs at (x',y',z',t') in S' as shown in Fig. below.



In new transformation, the measurement in the x-direction made in frame S must be linearly proportional to that made in S'. That is

$x' = K(x - vt)$ ……(1), Where K is the proportionality constant, which does not depend upon either x or t but may be a function of v. As the laws of physics are same in both frames S and S' (first postulate); therefore the corresponding equation of x in terms of x' and t' will have similar nature except that –v replaces v, so that

$x = K(x' + vt')$ ………. (2), where $t \neq t'$. Now substituting the value of x' from equation 1 into 2, we have

$x = K[K(x-vt) + vt']$

$x = K^2(x - vt) + Kvt'$ or $(1 - K^2)x + K^2vt = Kvt'$

$t' = Kt + x(1 - K^2)/Kv = Kt - Kx(1 - 1/K^2)/v$ ---------- (3)

The value of K can be evaluated with the help of second postulate. Let a light signal be given at the origin O at time t = 0, t' = 0; this means O and O' coincide. The signal travels with a speed c which is same in both the frames

$x = ct$ and $x' = ct'$ (position in S and S' frame, respectively). Substituting these values of x and x' in equations 1 and 2, we get

$ct' = Kt(c - v)$ and $ct = Kt'(c+v)$

After multiplying both these equations with each other, we get

$c^2 = K^2(c^2 - v^2)$ or $K^2 = c^2/(c^2 - v^2)$ or $K = 1/(1 - v^2/c^2)^{1/2}$ ………………(4)

Now, substituting the value of K from equation (4) in equation (1), we get

$x' = (x - vt)/(1 - v^2/c^2)^{1/2}$ …………… (5)

From equation (3) $t' = Kt - Kx(1 - 1/K^2)/v$ From equation (4)

$1/K^2 = 1 - v^2/c^2$ …………… (6), Putting equation (6) in equation (3)

$t' = Kt - Kx(1-1 + v^2/c^2)/v$ or $t' = Kt - Kxv^2/vc^2 = Kt - Kxv/c^2$

$t' = K(t - xv/c^2)$ or $t' = (t - xv/c^2)/(1 - v^2/c^2)^{1/2}$ …………. (7)

Due to the relative motion of the two reference frames, is in the x-direction. Therefore

$y' = y$ and $z' = z$ …………. (8)

Hence the Lorentz transformation equation for space and time are

$x' = (x - vt)/(1 - v^2/c^2)^{1/2}$, $y' = y$, $z' = z$ and $t' = (t - vx/c^2)/(1 - v^2/c^2)^{1/2}$

Inverse Lorentz Transformation Equation:

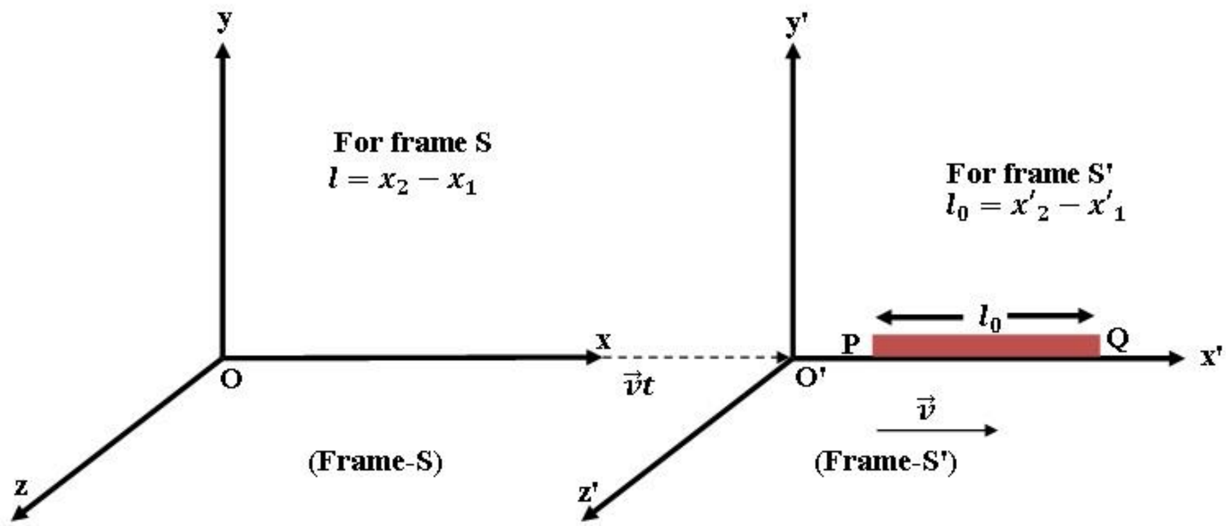
From Lorentz transformation equations, the following equations may be derived

$x = (x' + vt')/(1 - v2/c2)1/2$, $y = y'$ and $z = z'$, $t = (t' + x'v/c^2)/(1 - v^2/c^2)^{1/2}$

Above equations are known as inverse Lorentz transformation equations. If $v << c$, then these equations will be $x = x' + vt'$, $y = y'$, $z = z'$ and $t = t'$ because $1/(1 - v^2/c^2)^{1/2} = 1$ and $t + x'v/c^2 = t$

Length Contraction: Lorentz-Fitzgerald, for the first time, proposed that the length of moving body (comparable with the velocity of light) measured by a stationary observer appears to be contracted in the direction of motion. The appeared decrease in the length of the body in the direction of motion is called length contraction.

Derivation of length contraction: Consider two frames S and S', S' is moving with uniform velocity v relative to frame S in the direction of x-axis. Let a rod be placed in S' along



x-axis as shown in Fig. above. If x_1' and x_2' be the coordinates of the ends of the rod, then its length l_O in frame S' is given by $l_O = L_O = x_2' – x_1'$ ------------- (1)

Now let an observer O in S frame measures the length of the same rod at the same time t, then the length l of the rod is given by

$l = L = x_2 – x_1$ -------------- (2)

According to Lorentz transformation equation, we have

$x_2' = (x_2 – vt)/\sqrt{(1 – v^2/c^2)}$ and $x_1' = (x_1 – vt)/\sqrt{(1 – v^2/c^2)}$

Substituting these values in equation(1), we have

$L_O = (x_2 – vt)/\sqrt{(1 – v^2/c^2)} - (x_1 – vt)/\sqrt{(1 – v^2/c^2)}$

$L_O = (x_2 – x_1)/\sqrt{(1 – v^2/c^2)}$ or $L_O = L/\sqrt{(1 – v^2/c^2)}$ as $L = x_2 – x_1$

$L = L_O\sqrt{(1 - v^2/c^2)}$ ------------- (3)

Thus the rod is contracted by a factor $\sqrt{(1 - v^2/c^2)}$ in the direction of motion.

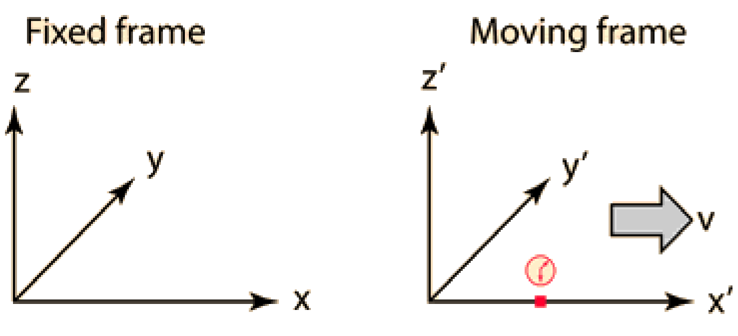
Important points: If $v << c$ then v^2/c^2 is negligible so $L = L_O$

If v = c, then L =0, This is impossible. Nobody can attain the velocity of light.

Time Dilation: The time interval between two events that occurs at the same place in an observer's frame of references, is called the proper time of the interval between the events.

A clock moving with a uniform velocity v relative to an observer at rest appears to him to go slow by a factor $\sqrt{(1 - v^2/c^2)}$ than when at rest. This effect is called time dilation.

Derivation of Time Dilation: Let a clock be placed at a point in the frame S (fixed frame) and another at a point in the frame S' moving with velocity v with respect to frame S along the positive x –axis as shown in Fig. below.



According to inverse Lorentz transformation for time

$t = (t' + vx'/c2)/ \sqrt{(1 - v^2/c^2)}$ --------- (1)

Again consider a light signal is emitted at point x' at time t1' and another at the same location at time t2' in the frame S' as measured by an observer in frame S'. Therefore time measured in frame S by observer O for the same will be

$t_1 = (t_1' + vx'/c^2)/ \sqrt{(1 - v^2/c^2)}$ and $t_2 = (t_2' + vx'/c^2)/ \sqrt{(1 - v^2/c^2)}$ ----------- (2)

Therefore $\Delta t = t_2 - t_1$

$= (t_2' + vx'/c^2)/ \sqrt{(1 - v^2/c^2)} - (t_1' + vx'/c^2)/ \sqrt{(1 - v^2/c^2)}$

Or $\Delta t = (t_2' - t_1')/ \sqrt{(1 - v^2/c^2)}$ ----------- (3)

Or $\Delta t = \Delta t' / \sqrt{(1 - v^2/c^2)}$ ------------ (4)

Equation (4) shows that to the stationary observer O in S, the time interval Δt appears to be lengthened by a factor $1/\sqrt{(1 - v^2/c^2)}$. Thus a moving clock appears to be slowed down to a stationary observer. This effect is known as time dilation (to dilate is to become larger)

If $v = c$, then $v^2/c^2 = 1$ this means Δt = infinity i.e. the clock moving with the speed of light will appear to be completely stopped to a stationary observer. If $v << c$, then $1/\sqrt{(1 - v^2/c^2)}$

$= (1 - v^2/c^2)^{-1/2} = 1 + v^2/2c^2$. In this v2/c2 is very small and can be neglected. So $\Delta t = \Delta t'$ i.e. if the clock is moving with the speed very-very smaller than speed of light, then time interval will remain same for moving observer ($\Delta t = \Delta t'$).

Time dilation is real effect: Take an example of cosmic ray particles called mesons, μ-mesons are created at high altitudes in the earth atmosphere (= 10 Km) by the fast cosmic ray photons and are projected towards the earth surface with a very high speed of about 2.994×10^8 m/s which is 0.998c. μ – mesons are unstable and decay into electrons or positrons with an average life time of about 2.0×10^{-6} sec. Hence, in its life time a μ – meson can travel a distance $d = vt = 2.994 \times 10^8 \text{ m/s} \times 2.0 \times 10^{-6} \text{ s} = 600$ m. But the question is that how μ- mesons have an average life time i.e. $\Delta t' = t' = 2.0 \times 10^{-6}$ s.

In observer's frame of reference

$\Delta t = \Delta t' / \sqrt{(1 - v^2/c^2)}$ or $t = t'/\sqrt{(1 - v^2/c^2)} = 2.0 \times 10^{-6}/\sqrt{\{1 - (0.998c/c)^2\}}$

$= 2.0 \times 10^{-6}/\sqrt{\{1 - (0.998)^2\}} = 3.17 \times 10^{-5}$ sec.

In this dilated life time the presence of μ – mesons on the earth surface i.e. $d = 2.994 \times 10^8 \times 3.17 \times 10^{-5} = 9500$ m $= 9.5$ km. Hence time dilation is real effect.

Relativistic addition of velocities: The Lorentz transformation equations enable us to transform velocity from one frame of reference to another, in relative motion with respect to it. This leads to a relativistic formula for the addition of velocities. This formula is known as Einstein's velocity addition theorem.

Let S and S' be the two inertial frames in relative motion, so that S' moves with a uniform velocity v to the right, along the x-axis, relative to S.

Let u and u' be the velocities of a particle measured in the inertial frame S and S' respectively. The components of these velocities are (u_x, u_y, u_z) and (u_x', u_y', u_z')

$u_x = dx/dt$, $u_y = dy/dt$, $u_z = dz/dt$ ------------ (1)

$u_x' = dx'/dt'$, $u_y' = dy'/dt'$, $u_z' = dz'/dt'$ --------- (2)

We know Lorentz transformation equations

$x' = (x - vt)/\sqrt{(1 - v^2/c^2)}$, $y' = y$, $z' = z$ and $t' = (t - vx/c^2)/\sqrt{(1 - v^2/c^2)}$ --------- (3)

Taking the differentials of the equation (3), we get

$dx' = (dx - vdt)/\sqrt{(1 - v^2/c^2)}$, $dy' = dy$, $dz' = dz$ and $dt' = (dt - vdx/c^2)/\sqrt{(1 - v^2/c^2)}$ ------(4)

Now we can write by using equation (4)

$dx'/dt' = (dx - vdt)/(dt - vdx/c^2) = (dx/dt - v)/(dt/dt - vdx/c^2dt)$

or $dx'/dt' = (dx/dt - v)/(1 - vdx/c^2dt)$

or $u_x' = (u_x - v)/(1 - vu_x/c^2)$ ----------- (5)

Similarly

$dy'/dt' = dy\sqrt{(1 - v^2/c^2)}/(dt - vdx/c^2)$

$dy'/dt' = (dy/dt)\sqrt{(1 - v^2/c^2)}/(1 - vdx/c^2dt)$

or $u_y' = u_y\sqrt{(1 - v^2/c^2)}/(1 - vu_x/c^2)$ ------------- (6)

Similarly

$u_z' = u_z\sqrt{(1 - v^2/c^2)}/(1 - vu_x/c^2)$ ------------ (7)

Similarly by using inverse Lorentz transformation equations, we get

$u_x = (u_x' + v)/(1 + vu_x'/c^2)$, $u_y = u_y'\sqrt{(1 - v^2/c^2)}/(1 + vu_x'/c^2)$ and $u_z = u_z'\sqrt{(1 - v^2/c^2)}/(1 + vu_x'/c^2)$

If the velocity of the particle is along the x-axis, then $u_x' = u'$, $u_y' = 0$, $u_z' = 0$

And $u_x = u$, $u_y = 0$, $u_z = 0$

Then equations (5), (6) & (7) may be written as

$u' = (u - v)/(1 - vu/c^2)$ and $u = (u' + v)/(1 + vu'/c^2)$

Consistency with Einstein's second postulate:

We know that $u = (u' + v)/(1 + vu'/c^2)$

1. When u' and v are smaller as compared to c, $u'v/c^2$ can be neglected. Therefore $u = u' + v$ which is classical formula.
2. When $v = c$, then
   $u = (u' + v)/(1 + vu'/c^2) = c(u' + c)/(u' + c) = c$
   i.e. if one object moves with velocity c with respect to other then their relative velocity is always c, whatever may be the velocity of the other.

3. when $u' = c = v$ then

   $u = (c + c)/(1 + c^2/c^2) = 2c/2 = c$

3. If $u' = c$ then
   $u = (v + c)/(1 + vc/c^2) = c$

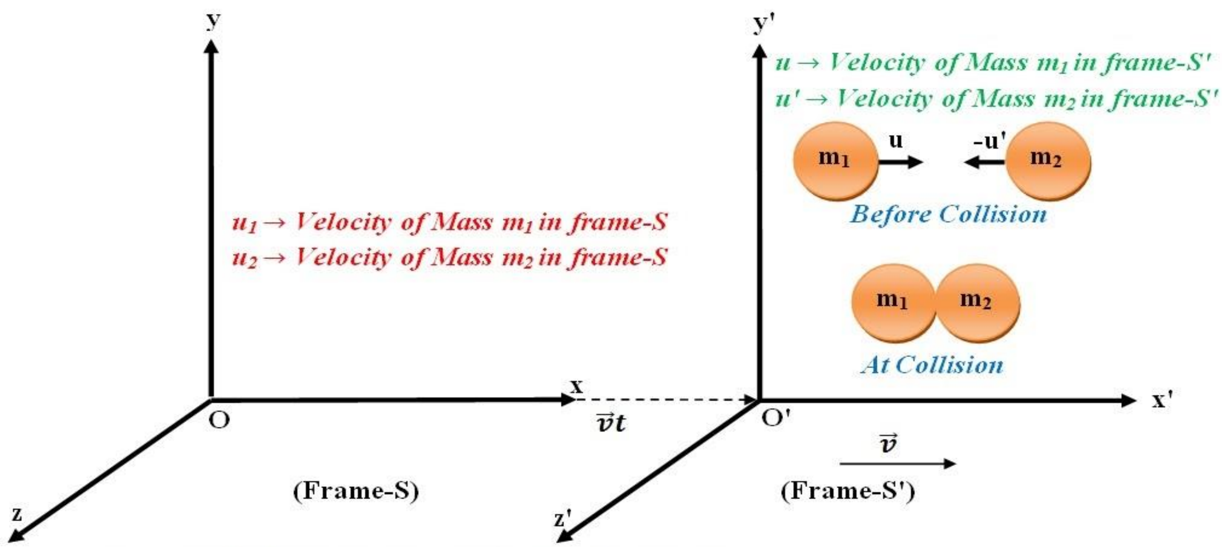
Therefore, the relativistic addition of velocities, is consistent with Einstein's second postulate of special theory of relativity. This also shows that Lorentz transformation equations are in accordance with the constancy of velocity of light.

Variation of mass with velocity: According to Newtonian mechanics – "The mass of a body does not change with velocity", but according to Einstein's theory "The mass of a body in motion is different from the mass of the body at rest".

Let the two identical bodies be moving with velocities u' and –u' parallel to x-axis in a frame S', which moving with a uniform velocity v relative to a frame S in the direction of +ve x-axis. The velocities of these two masses in frame S are

$u_1 = (u' + v)/(1 + u'v/c^2)$ ----------- (1)

$u_2 = (-u' + v)/(1 - u'v/c^2)$ ---------- (2)



In frame S let the masses of two bodies be m1 & m2 and since after collision the velocity of these bodies is zero (as shown in Fig. above) relative to S', it will be v relative to S.

Now applying the law of conservation of momentum in frame S, we obtain

$m1u1 + m2u2 = (m1 + m2)v$ ----------- (3)

Putting the value of u1 & u2 from equation (1) & (2) in equation (3), we get

$m1[(u' + v)/(1 + u'v/c^2)] + m2[(-u' + v)/(1- u'v/c^2)] = (m1 + m2)v$

$m1/m2 = (1 + u'v/c^2)/(1 – u'v/c^2)$ ----------- (4)

Now with the help of equation (1), we have

$1 – (u_1^2/c^2) = 1 – 1/c^2[(u' + v)/(1 + u'v/c^2)]^2$

$= [(1 + u'v/c2)^2 – (u' + v)/c^2]/(1 + u'v/c^2)^2]$

$= [(1 – v^2/c^2) – u'^2(1 – v^2/c^2)/c^2]/(1 + u'v/c^2)^2$

Or $1 – (u_1^2/c^2) = (1 – v^2/c^2)(1 – u'^2/c^2)/(1 + u'v/c^2)^2$

Or $(1 + u'v/c^2) = [(1 –v^2/c^2)(1 – u'^2/c^2)/(1 – u_1^2/c^2)]^{1/2}$ ------------ (5)

Similarly

$(1 – u'v/c2) = [(1 – v^2/c^2)(1 – u'^2/c^2)/(1 – u_2^2/c^2)]^{1/2}$ ------------ (6)

With the help of eqs. (5) & (6), eqn. (4) changes to

$m1/m2 = \sqrt{(1 - u_2^2)}/\sqrt{(1 - u_1^2/c^2)}$ ----------- (7)

If the mass m2 is considered to be initially at rest in frame S then m2 can be taken as the rest mass mO of the body i.e. $m2 = m_O$ and $u_2 = 0$, eqn. (7) becomes

Therefore $m1 = m_O/\sqrt{(1 - u_1^2/c^2)}$ --------(8)

Since the two bodies are considered to be identical the rest mass of m1 will also be m_O. Therefore, eqn.(8) is applicable for a single body of rest mass m_O, relativistic mass m1 moving with u_1 velocity. If we replace m1 by m and u_1 by v, the eqn. (8) modifies as

$m = m_O/\sqrt{(1 - v^2/c^2)}$ --------------- (9)

From this eqn. (9), it is clear that

1. As v of the moving particle increases, its mass also increases.
2. As v = c, then m = infinity, i.e. the mass of the particle becomes infinite.
3. When $v << c$ then v^2/c^2 = negligible and hence $m = m_O$.

Mass Energy Equivalence: In Newtonian Mechanics, the force is defined as the time rate of change of linear momentum. This definition of force is valid in relativistic mechanics as well. Thus, the force applied on a particle moving with relativistic velocity v (nealy equal to c) is given by

$F = dp/dt = d(mv)/dt$ ----------(1)

where m is the relativistic mass of the particle and is variable quantity, eqn. (1) may be written as

$F = mdv/dt + vdm/dt$ -----------(2)

The increase in kinetic energy of this particle, when it displaces through a distance ds in time dt under the influence of the force F, will be equal to the work done by the force on it i.e.

$dE_k = F.ds = [mdv/dt + vdm/dt]ds$ using eqn(2)

$dE_k = mdv.ds/dt + vdm.ds/dt = mvdv + v^2dm$ ---------- (3) because $ds/dt = v$

If mO is the rest mass of the particle, then

$m = mO/\sqrt{(1 - v2/c2)}$ --------- (4)

Differentiate eqn. (4)

$dm = m_O(-1/2)(1 - v^2/c^2)^{-3/2}(-2v/c^2)dv$

or $dm = m_Ovdv/c^2(1 - v^2/c^2)^{3/2} = m_Ovdv/(1 - v^2/c^2)^{1/2}c^2(1 - v^2/c^2)$

or $dm = mvdv/c^2(1 - v^2/c^2) = mvdv/(c^2 - v^2)$

or $dm(c^2 - v^2) = mvdv$

or $c^2dm - v^2dm = mvdv$

or $c^2dm = mvdv + v^2dm$ ---------- (5)

correlate the eqns. (3) & (5), we get

$dE_k = c^2dm$ -------------- (6)

$E_k = \int dE_k = \int_{m_O}^{m} c^2 dm = c^2 \int_{m_o}^{m} dm = c^2(m - m_O) = mc^2 - m_Oc^2$

$E_k + m_Oc^2 = mc^2$ ------------ (7)

Eqn.(7) obtained by integrating eqn (6) between the limits (m, mO) the total increase in kinetic energy is given by eqn.(7). Eqn.(7) may be written as

$E = mc^2$ ---------- (8)

Where $E = E_k + m_Oc^2$, E_k = kinetic energy, m_Oc^2 = rest mass energy, mc^2 = total energy. Eqn. (8) represents the mass-energy relation.

Examples of mass-energy equivalence:

1. ***Pair production phenomenon:*** When a gamma ray photon of suitable energy is absorbed by a nucleus, the photon disappears giving rise to the production of electron –positron pair
   $\gamma\ (h\nu) \rightarrow {}_{-1}e^0 + {}_{+1}e^0$
   1.02 0.51 MeV + 0.51 MeV
2. ***Annihilation phenomenon:*** When an electron and a positron combine together they annihilate and produce two γ-ray photons.
   ${}_{-1}e^0 + {}_{+1}e^0 \rightarrow h\nu + h\nu \rightarrow 2\gamma$
   0.51 MeV + 0.51 MeV → 1.02 MeV

3. ***Nuclear fission:*** The formation of two lighter nuclei due to disintegration of a

heavy nucleus is known as nuclear fission. For example, U235 nucleus disintegrate into barium and krypton nuclei

$_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + \text{Energy}$

4. ***Nuclear fusion:*** The formation of heavy nucleus due to combination of two lighter nuclei is known as nuclear fusion. For example, formation of helium nucleus due to combination of two hydrogen nuclei

$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + \text{Energy}$

Relation between relativistic momentum and relativistic energy: We know that the relation between the rest mass mO and the relativistic mass m of a particle is given by

$m = m_O/\sqrt{(1 - v^2/c^2)} \rightarrow mc^2 = m_Oc^2/\sqrt{(1 - v^2/c^2)}$ --------- (1)

or $(mc^2)^2 = m_O{}^2c^4/(1 - v^2/c^2)$ or $E^2 = m_O{}^2c^4/(1 - m^2v^2/m^2c^2)$ because $mc^2 = E$

or $E^2 = m_O{}^2c^4/(1 - p^2/m^2c^2) = m_O{}^2c^4/(1 - p^2c^2/m^2c^4) = m_O{}^2c^4/(1 - p^2c^2/E^2)$

$E^2(1 - p^2c^2/E^2) = m_O{}^2c^4$ or $E^2 - p^2c^2 = m_O{}^2c^4$ or $E^2 = m_O{}^2c^4 + p^2c^2$ -------(2)

Equation (2) is the required relation between the relativistic momentum and the relativistic energy.

Concept of rest mass of photon: A particle which has zero rest mass (m_O) is called a massless particle. In classical physics the existence of massless particle is impossible. However, in relativistic mechanics, a particle with zero rest mass can exist. According to the relativistic relation between energy and momentum

$E^2 - p^2c^2 = m_O{}^2c^4$

For massless particle, $m_O = 0$. Therefore

$E = pc$ or $p = E/c$

Thus, we can say that massless particle has energy pc and momentum E/c and moves with the velocity of light

$E = pc$ or $mc^2 = pc$ or $mc = mv$

Or $c = v$ in case of photon

Photon is a massless particle.