Monograph

on

Theory of Machines

By

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Introduction to mechanisms, velocity and acceleration analysis of mechanisms

1.1 Introduction

Mechanics: It is that branch of scientific analysis which deals with motion, time and force.

Kinematics is the study of motion, without considering the forces which produce that motion. Kinematics of machines deals with the study of the relative motion of machine parts. It involves the study of position, displacement, velocity and acceleration of machine parts.

Dynamics of machines involves the study of forces acting on the machine parts and the motions resulting from these forces.

Plane motion: A body has plane motion, if all its points move in planes which are parallel to some reference plane. A body with plane motion will have only three degrees of freedom. I.e., linear along two axes parallel to the reference plane and rotational/angular about the axis perpendicular to the reference plane. (eg. linear along X and Z and rotational about Y.)The reference plane is called plane of motion. Plane motion can be of three types. 1) Translation 2) rotation and 3) combination of translation and rotation.

Kinematic link (or) element

A machine part or a component of a mechanism is called a kinematic link or simply a link. A link is assumed to be completely rigid, or under the action of forces it does not suffer any deformation, signifying that the distance between any two points on it remains constant. Although all real machine parts are flexible to some degree, it is common practice to assume that deflections are negligible and parts are rigid when analyzing a machine's kinematic performance.

Types of link

(a) Based on number of elements of link:

Binary link: Link which is connected to other links at two points. (Fig.1.3 a)

Ternary link: Link which is connected to other links at three points. (Fig.1.3 b)

Quaternary link: Link which is connected to other links at four points. (Fig1.3 c)

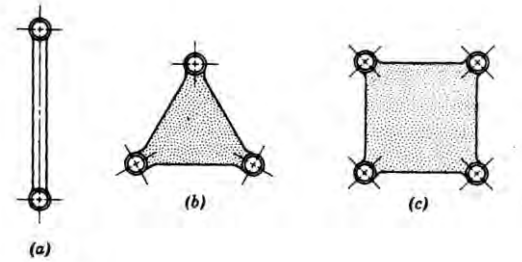


Fig.1.3

(a) Based on type of structural behavior:

Sometimes, a machine member may possess one-way rigidity and is capable of transmitting the force in one direction with negligible deformation. Examples are (a) chains, belts and ropes which are resistant to tensile forces, and (b) fluids which are resistant to compressive forces and are used as links in hydraulic presses, brakes and jacks. In order to transmit motion, the driver and the follower may be connected by the following three types of links:

1. *Rigid link.* A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. *Flexible link.* A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. *Fluid link.* A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

1.4 Structure

It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

Machine: A machine is a mechanism or collection of mechanisms, which transmit force from the source of power to the resistance to be overcome. Though all machines are mechanisms, all mechanisms are not machines. Many instruments are mechanisms but are not machines, because they do no useful work nor do they transform energy.

Difference between structure & machine

The following differences between a machine and a structure are important from the subject point of view:

1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another.

2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.

3. The links of a machine may transmit both power and motion, while the members of a structure transmits forces only.

Comparison of Mechanism, Machine and Structure

Mechanism	Machine	Structure
1. There is relative motion between the parts of a mechanism	Relative motion exists between parts of a machine.	There is no relative motion between the members of a structure. It is rigid as a whole.
2. A mechanism modifies and transmits motion.	A machine consists of one or more mechanisms and hence transforms motion	A structure does not transform motion.
3. A mechanism does not transmit forces and does not do work	A machine modifies energy or do some work	A structure does not do work. It only transmits forces.
4. Mechanisms are dealt with in kinematics.	Machines are dealt with in kinetics.	Structures are dealt with in statics.

1.6 Kinematic pair

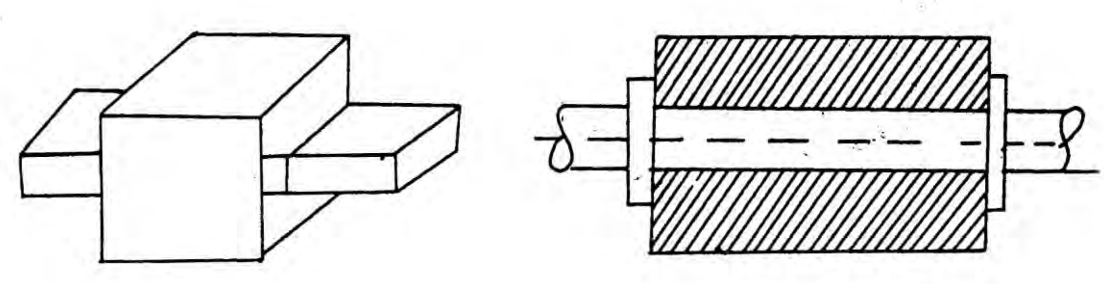
The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as **kinematic pair**.

Classification of kinematic pair

The kinematic pairs may be classified according to the following considerations :

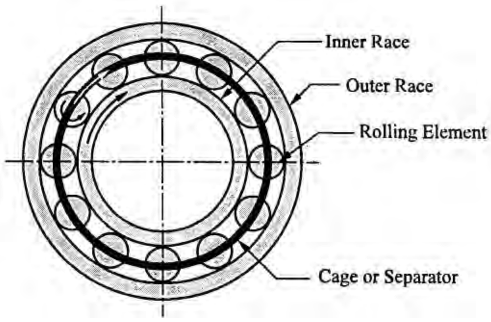
(i) Based on nature of contact between elements:

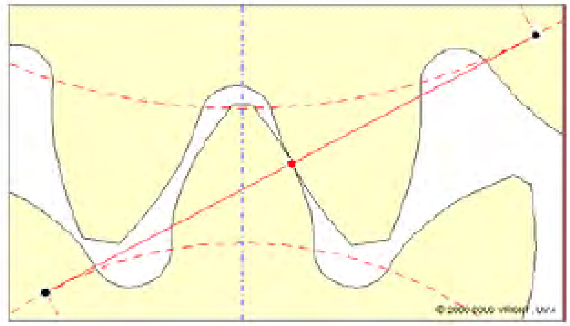
(a) Lower pair. If the joint by which two members are connected has surface contact, the pair is known as lower pair. Eg. pin joints, shaft rotating in bush, slider in slider crank mechanism.



Lower pairs

(b) Higher pair. If the contact between the pairing elements takes place at a point or along a line, such as in a ball bearing or between two gear teeth in contact, it is known as a higher pair.





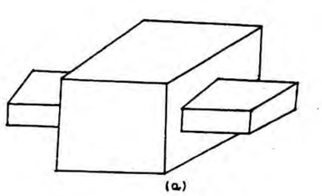
Higher pairs

(ii) Based on relative motion between pairing elements:

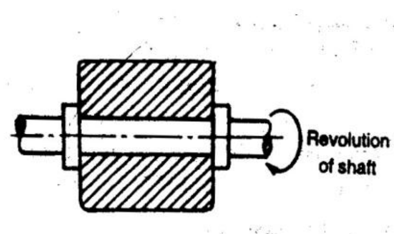
(a) Siding pair. Sliding pair is constituted by two elements so connected that one is constrained to have a sliding motion relative to the other. DOF = 1

(b) Turning pair (revolute pair). When connections of the two elements are such that only a constrained motion of rotation of one element with respect to the other is possible, the pair constitutes a turning pair. DOF = 1

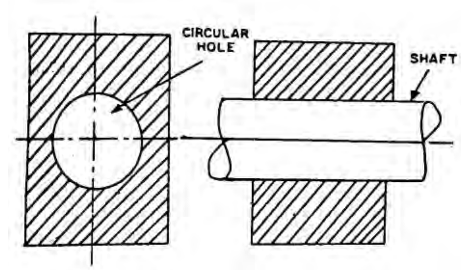
(c) Cylindrical pair. If the relative motion between the pairing elements is the combination of turning and sliding, then it is called as cylindrical pair. DOF = 2



Sliding pair

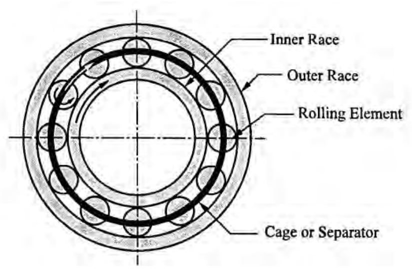


Turning pair

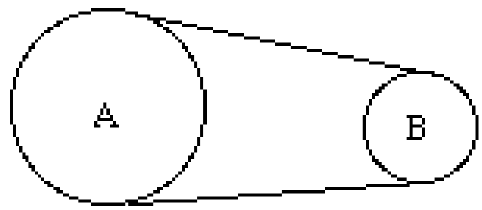


Cylindrical pair

(d) Rolling pair. When the pairing elements have rolling contact, the pair formed is called rolling pair. Eg. Bearings, Belt and pulley. DOF = 1



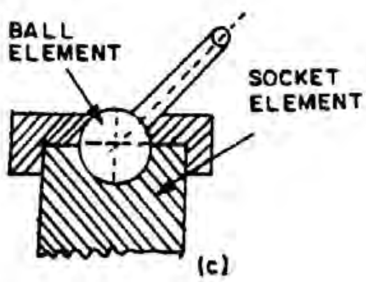
Ball bearing



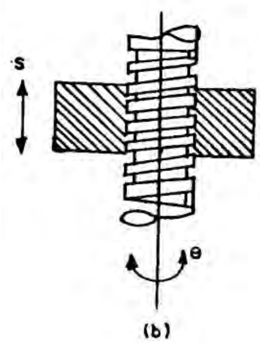
Belt and pulley

(e) Spherical pair. A spherical pair will have surface contact and three degrees of freedom. Eg. Ball and socket joint. DOF = 3

(f) Helical pair or screw pair. When the nature of contact between the elements of a pair is such that one element can turn about the other by screw threads, it is known as screw pair. Eg. Nut and bolt. DOF = 1



Ball and socket joint



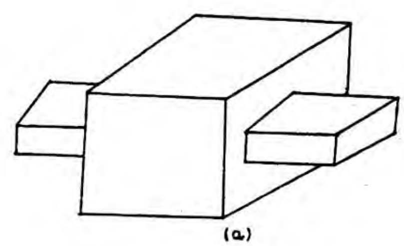
Screw pair

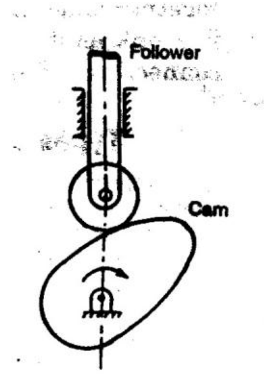
(a) Sliding pair (prismatic pair) eg. piston and cylinder, crosshead and slides, tail stock on lathe bed.
(b) Turning pair (Revolute pair): eg: cycle wheel on axle, lathe spindle in head stock.
(c) Cylindrical pair: eg. shaft turning in journal bearing.
(d) Screw pair (Helical pair): eg. bolt and nut, lead screw of lathe with nut, screw jack.
(e) Spherical pair: eg. penholder on stand, castor balls.

(iii) Based on the nature of mechanical constraint.

(a) Closed pair. Elements of pairs held together mechanically due to their geometry constitute a closed pair. They are also called form-closed or self-closed pair.

(b) Unclosed or force closed pair. Elements of pairs held together by the action of external forces constitute unclosed or force closed pair .Eg. Cam and follower.



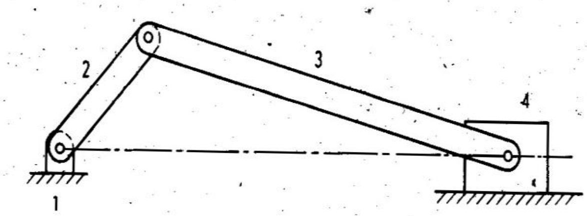


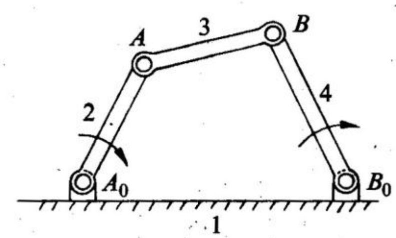
Closed pair Force closed pair (cam & follower)

Mechanism

When one of the links of a kinematic chain is fixed, the chain is known as ***mechanism***. A mechanism with four links is known as ***simple mechanism***, and the mechanism with more than four links is known as ***compound mechanism***. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a ***machine***.

A mechanism is a constrained kinematic chain. This means that the motion of any one link in the kinematic chain will give a definite and predictable motion relative to each of the others. Usually one of the links of the kinematic chain is fixed in a mechanism.



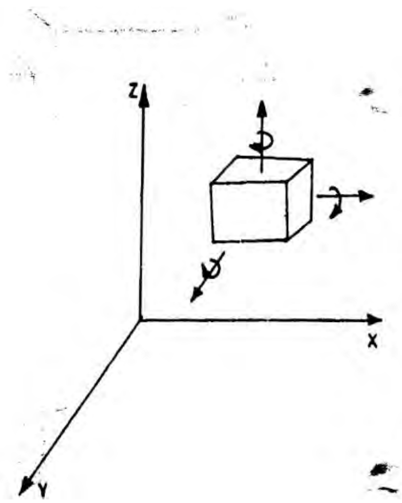


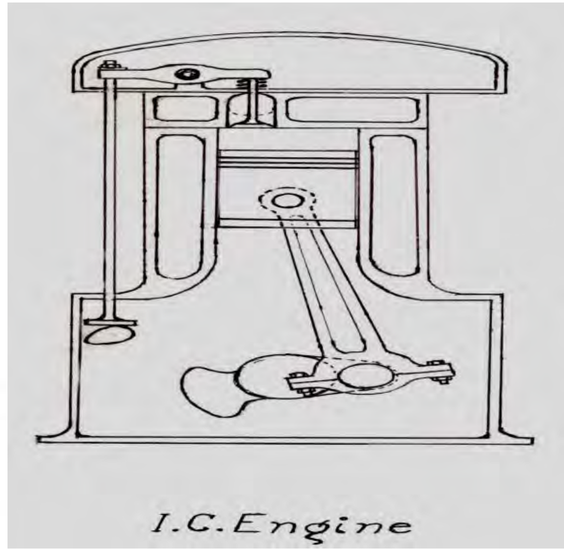
Slider crank and four bar mechanisms.

Number of degrees of freedom for plane mechanism

Degrees of freedom/mobility of a mechanism: The number of independent input parameters (or pair variables) that are needed to determine the position of all the links of the mechanism with respect to the fixed link is termed its degrees of freedom.

Degrees of freedom (DOF) is the number of independent coordinates required to describe the position of a body in space. A free body in space can have six degrees of freedom. I.e., linear positions along x, y and z axes and rotational/angular positions with respect to x, y and z axes. In a kinematic pair, depending on the constraints imposed on the motion, the links may loose some of the six degrees of freedom.





Planar mechanisms: When all the links of a mechanism have plane motion, it is called as a planar mechanism. All the links in a planar mechanism move in planes parallel to the reference plane.

Serial Mechanisms (Manipulators): Early manipulators were work holding devices in manufacturing operations so that the work piece could be manipulated or brought to different orientations with respect to the tool head. Welding robots of the auto industry and assembly robots of IC manufacture are examples.

Application of kutzbach criterion to Plane mechanisms

$$F = 3(n-1) - 2l - h$$

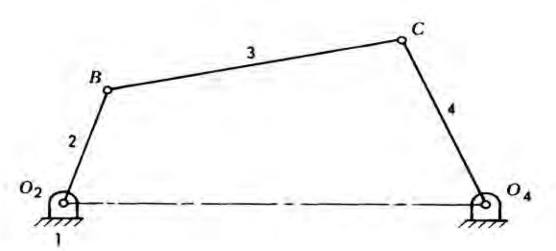
Where n=number of links; l= number of lower joints (or) pairs and h= number of higher pairs (or) joints

This is called the Kutzbach criterion for the mobility of a planar mechanism.

Inversion of Mechanism

A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism.

Inversions of Four Bar Chain



One of the most useful and most common mechanisms is the four-bar linkage. In this mechanism, the link which can make complete rotation is known as crank (link 2). The link which oscillates is known as rocker or lever (link 4). And the link connecting these two is known as coupler (link 3). Link 1 is the frame.

Inversions:

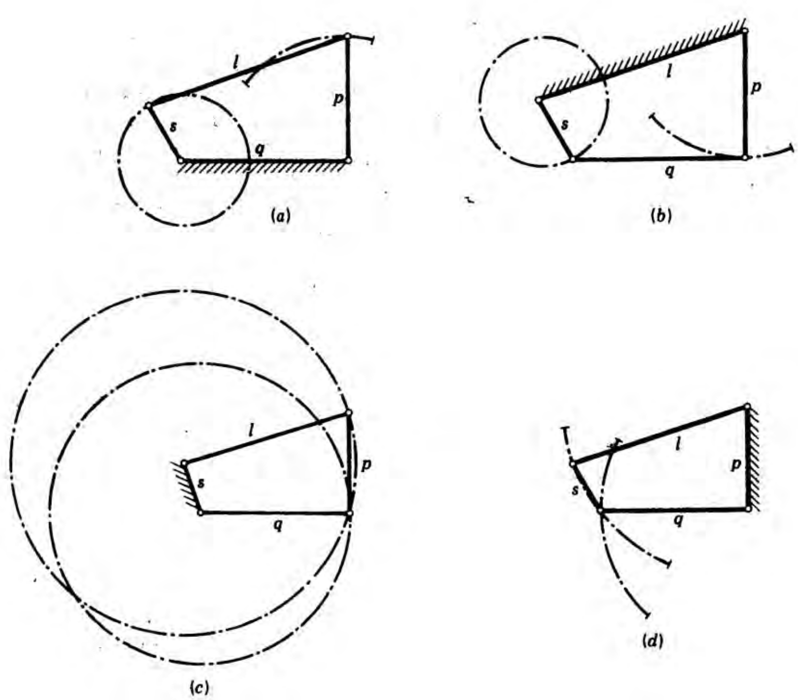


Fig.1.23 Inversions of four bar chain.

Crank-rocker mechanism: In this mechanism, either link 1 or link 3 is fixed. Link 2 (crank) rotates completely and link 4 (rocker) oscillates. It is similar to (a) or (b) of fig.1.23.

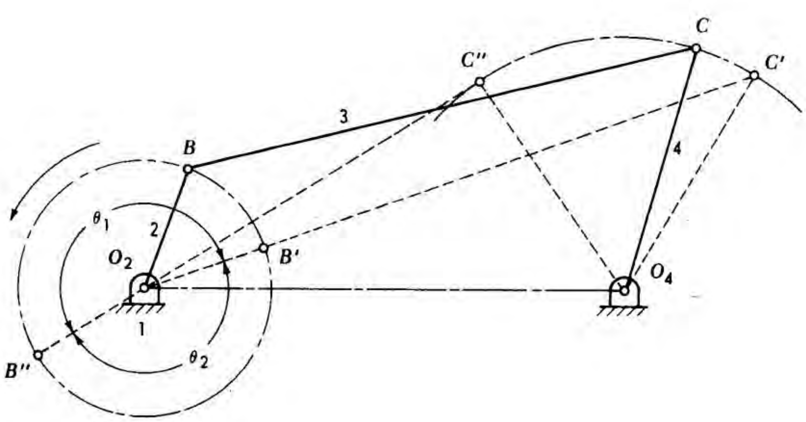


Fig.1.24

Double crank mechanism (Coupling rod of locomotive). This is one type of drag link mechanism, where, links 1& 3 are equal and parallel and links 2 & 4 are equal and parallel.

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links in the fig. in this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

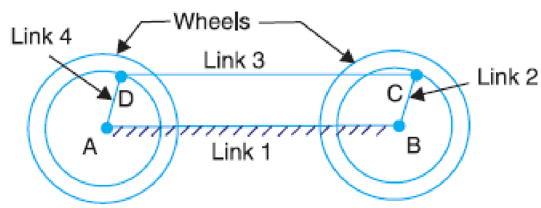


Fig.1.25

Double rocker mechanism. In this mechanism, link 4 is fixed. Link 2 makes complete rotation, whereas links 3 & 4 oscillate (Fig.1.23d)

Coupler Curves: The link connecting the driving crank with the follower crank in a four bar linkage is called the coupler. Similarly, in the case of a single slider crank mechanism the connecting rod is the coupler. During the motion of the mechanism any point attached to the coupler generates some path with respect to the fixed link. This path is called the coupler curve. The point, which generates the path is variously called the coupler point, trace point, tracing point, or tracer point.

An example of the coupler curves generated by different coupler points is given in figure below. Mechanisms can be designed to generate any curve.

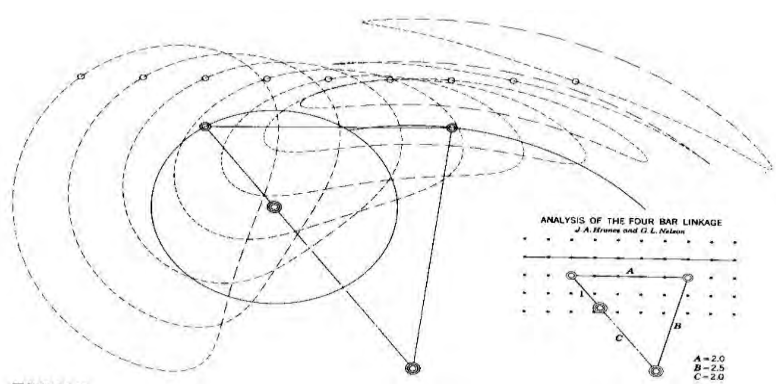


Fig. 1.26

Inversions of Single Slider Chain

Slider crank chain: This is a kinematic chain having four links. It has one sliding pair and three turning pairs. Link 2 has rotary motion and is called crank. Link 3 has got combined rotary and reciprocating motion and is called connecting rod. Link 4 has reciprocating motion and is called slider. Link 1 is frame (fixed). This mechanism is used to convert rotary motion to reciprocating and vice versa.

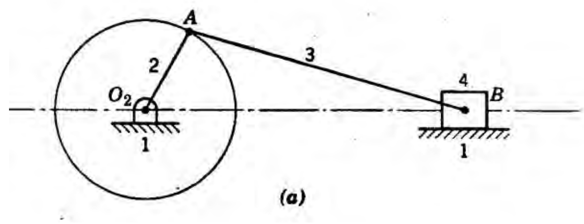
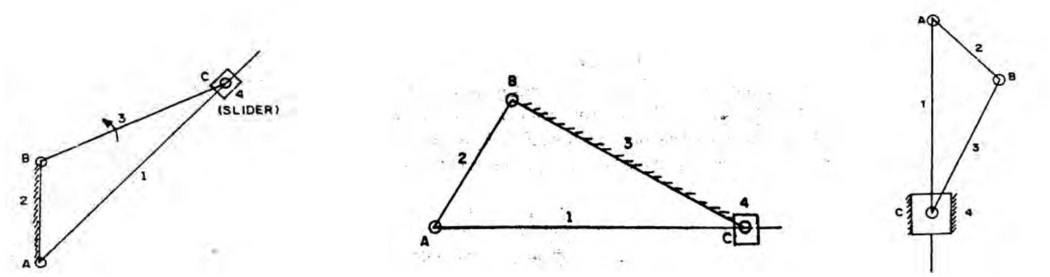


Fig1.27

Inversions of slider crank chain

Inversions of slider crank mechanism is obtained by fixing links 2, 3 and 4.



(a) crank fixed (b) connecting rod fixed (c) slider fixed

Fig.1.28

Quick return motion mechanisms.

Quick return mechanisms are used in machine tools such as shapers and power driven saws for the purpose of giving the reciprocating cutting tool a slow cutting stroke and a quick return stroke with a constant angular velocity of the driving crank.

Whitworth quick return motion mechanism–Inversion of slider crank mechanism.
This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link *CD* (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank *CA* (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at *A* slides along the slotted bar *PA* (link 1) which oscillates at a pivoted point *D*. The connecting rod *PR* carries the ram at *R* to which a cutting tool is fixed. The motion of the tool is constrained along the line *RD* produced, *i.e.* along a line passing through *D* and perpendicular to *CD*.

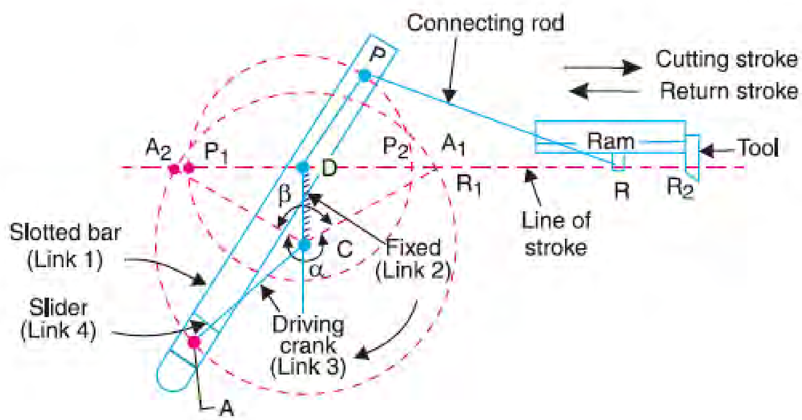


Fig.1.29

When the driving crank *CA* moves from the position *CA*1 to *CA*2 (or the link *DP* from the position *DP*1 to *DP*2) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2 *PD*.

Now when the driving crank moves from the position *CA*2 to *CA*1 (or the link *DP* from *DP*2 to *DP*1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (*i.e.* during forward or cutting stroke) will be equal to the time taken by the driving crank to move from *CA*1 to *CA*2. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from *CA*2 to *CA*1.

Since the crank link *CA* rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.

The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha}$$

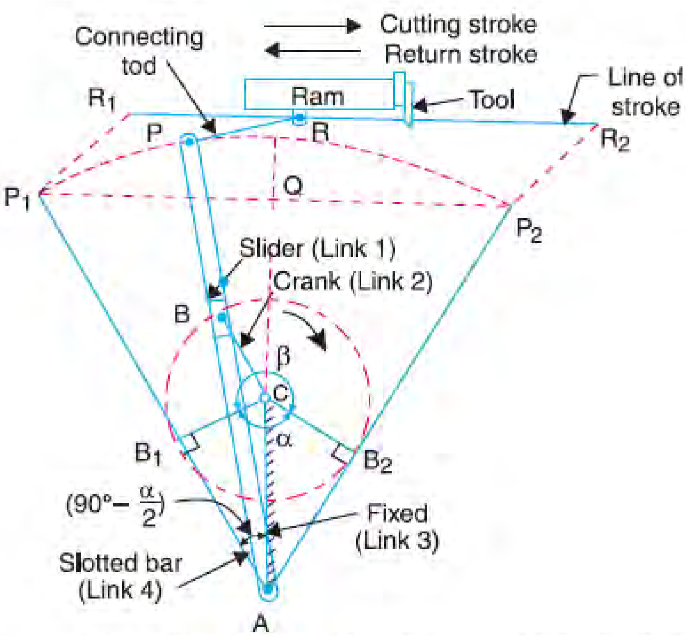
Crank and slotted lever quick return motion mechanism – Inversion of slider crank mechanism (connecting rod fixed).

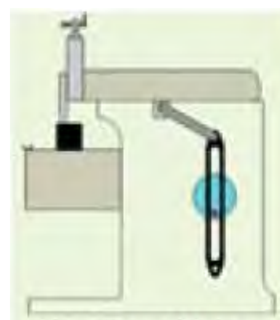
This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines. In this mechanism, the link *AC* (*i.e.* link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank *CB* revolves with uniform angular speed about the fixed centre *C*. A sliding block attached to the crank pin at *B* slides along the slotted bar *AP* and thus causes *AP* to oscillate about the pivoted point *A*. A short link *PR* transmits the motion from *AP* to the ram which carries the tool and reciprocates along the line of stroke $R1R2$. The line of stroke of the ram (*i.e.* $R1R2$) is perpendicular to AC produced.

We see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the return stroke is completed within shorter time. Thus it is called quick return motion mechanism.

In the extreme positions, $AP1$ and $AP2$ are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position $CB1$ to $CB2$ (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position $CB2$ to $CB1$ (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta}$$





The Shaping Machine

Since the tool travels a distance of $R_1 R_2$ during cutting and return stroke, therefore travel of the tool or length of stroke

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

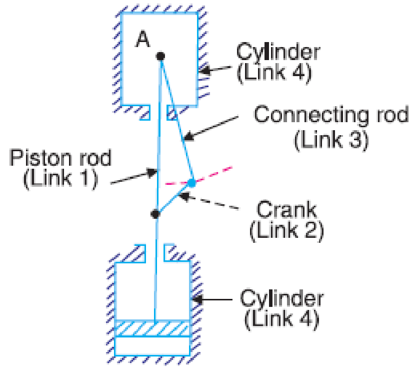
$$= 2AP_1 \sin\left(90^\circ - \frac{\alpha}{2}\right) = 2AP \cos \frac{\alpha}{2}$$

$$= 2AP \times \frac{CB_1}{AC}$$

$$= 2AP \times \frac{CB}{AC}$$

Pendulum pump or bull engine–Inversion of slider crank mechanism (slider fixed).

In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (*i.e.* sliding pair), as shown in Fig. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.



VELOCITY AND ACCELERATION ANALYSIS OF MECHANISMS

In this, we shall discuss the relative velocity method for determining the velocity of different points in the mechanism. The study of velocity analysis is very important for determining the acceleration of points in the mechanisms.

Kinematics deals with study of relative motion between the various parts of the machines. Kinematics does not involve study of forces. Thus motion leads study of displacement, velocity and acceleration of a part of the machine. As dynamic forces are a function of acceleration and acceleration is a function of velocities, study of velocity and acceleration will be useful in the design of mechanism of a machine. The mechanism will be represented by a line diagram which is known as configuration diagram. The analysis can be carried out both by graphical method as well as analytical method.

Displacement: All particles of a body move in parallel planes and travel by same distance is known, linear displacement and is denoted by 'x'. A body rotating about a fired point in such a way that all particular move in circular path angular displacement and is denoted by 'θ'.

Velocity: Rate of change of displacement is velocity. Velocity can be linear velocity of angular velocity.

Linear velocity is Rate of change of linear displacement= $V = \frac{dx}{dt}$

Angular velocity is Rate of change of angular displacement = $\omega = \frac{d\theta}{dt}$

Relation between linear velocity and angular velocity.

$x = r\theta$

$$\frac{dx}{dt} = r\frac{d\theta}{dt}$$

$\mathbf{V = r\omega}$

$$\boldsymbol{\omega} = \frac{d\theta}{dt}$$

<u>Acceleration</u>: Rate of change of velocity

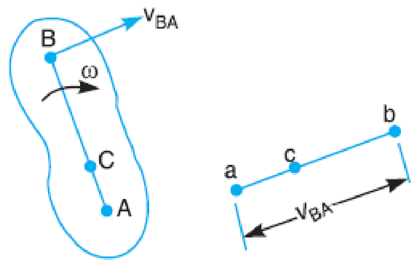
$$f = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$ Linear Acceleration (Rate of change of linear velocity)

Thirdly $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ Angular Acceleration (Rate of change of angular velocity)

Motion of a link

Consider two points A and B on a rigid link AB, as shown in Fig. (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.

Hence **velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.**



(*a*) (*b*)

The relative velocity of *B* with respect to *A* (*i.e.* *v*BA) is represented by the vector *ab* and is perpendicular to the line *AB* as shown in Fig. (*b*).

Let ω = Angular velocity of the link *A B* about *A*.

We know that the velocity of the point *B* with respect to *A*,

$$v_{BA} = \overline{ab} = \omega . AB$$

Similarly, the velocity of any point *C* on *A B* with respect to *A*,

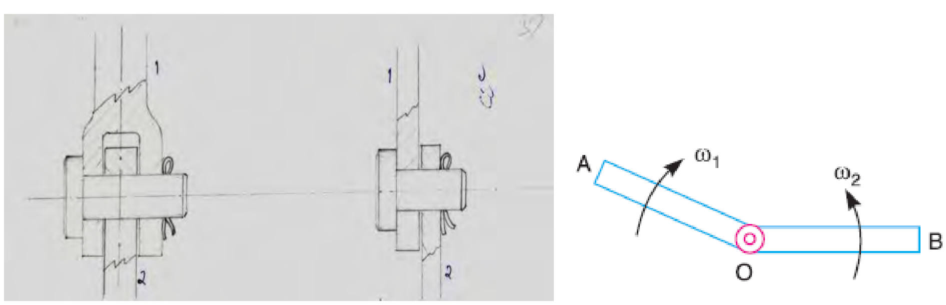
$$v_{CA} = \overline{ac} = \omega . AC$$

From the above two equations

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB}$$

Thus, we see from above equation that the point *c* on the vector *ab* divides it in the same ratio as *C* divides the link *AB*.

Rubbing Velocity at a Pin Joint



The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links *OA* and *OB* connected by a pin joint at *O* as shown in Fig.
Let
ω1 = Angular velocity of the link *OA* or the angular velocity of the point *A* with respect to O
ω2 = Angular velocity of the link *OB* or the angular velocity of the point *B* with respect to *O*, and
r = Radius of the pin.

According to the definition,

Rubbing velocity at the pin joint *O* is given by the formula
=(ω1-ω2)*r , if the links move in the same direction
= (ω1+ω2)*r, if the links move in the opposite direction
where ω_1 = angular velocity of link 1
ω_2 = angular velocity of link 2
r = radius of the pin

Note : When the pin connects one sliding member and the other turning member, the angular velocity of the sliding member is zero. In such cases,

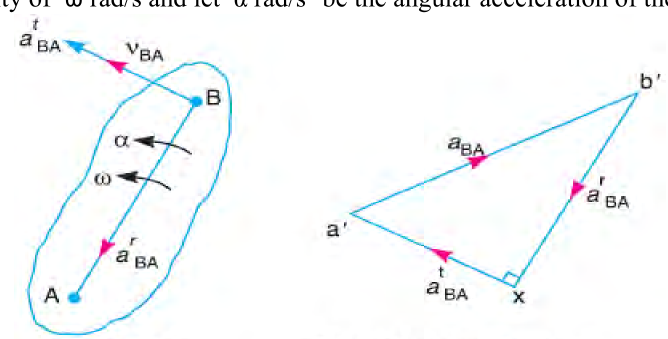
Rubbing velocity at the pin joint = ω*r*
where ω=Angular velocity of the turning member, and
r = Radius of the pin.

<u>Acceleration in mechanisms</u> (Introduction)

We have discussed in the previous chapter the velocities of various points in the mechanisms. Now we shall discuss the acceleration of points in the mechanisms. The acceleration analysis plays a very important role in the development of machines and mechanisms.

Acceleration diagram for a link

Consider two points *A* and *B* on a rigid link as shown in Fig. (*a*). Let the point *B* moves with respect to *A*, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link *AB*.



(*a*) Link. (*b*) Acceleration diagram.

We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components:
1. The *centripetal or radial component,* which is perpendicular to the velocity of the particle at the given instant.
2. The *tangential component,* which is parallel to the velocity of the particle at the given instant.
Thus for a link *AB*, the velocity of point *B* with respect to *A* (*i.e.* *v*BA) is perpendicular to the link *AB* as shown in Fig.(*a*). Since the point *B* moves with respect to *A* with an angular velocity of ω rad/s, therefore centripetal or radial component of the acceleration of *B* with respect to *A*,

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v^2_{BA} / AB \qquad \cdots \left(\because \ \omega = \frac{v_{BA}}{AB} \right)$$

This radial component of acceleration acts perpendicular to the velocity *v*BA, In other words, it acts *parallel* to the link *AB*.

We know that tangential component of the acceleration of *B* with respect to *A*,

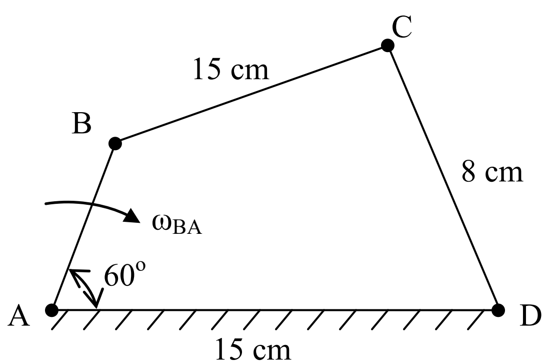
$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

This tangential component of acceleration acts parallel to the velocity *v*BA. In other words, it acts *perpendicular* to the link *AB*.

In order to draw the acceleration diagram for a link *AB*, as shown in Fig. (*b*), from any point *b'*, draw vector *b'x parallel to BA* to represent the radial component of acceleration of *B* with respect to *A i.e.* a^r_{BA} from point *x* draw vector *xa'* perpendicular to *B A* to represent the tangential component of acceleration of *B* with respect to *A i.e.* a^t_{BA}. *Join b' a'.* The vector *b' a'* (known as *acceleration image* of the link *AB*) represents the total acceleration of *B* with respect to *A* (*i.e.* *a*BA) and it is the vector sum of radial component (a^r_{BA}) and tangential component (a^t_{BA}) of acceleration.

Exercise Problems:

1. In a four bar chain ABCD link AD is fixed and in 15 cm long. The crank AB is 4 cm long rotates at 180 rpm (cw) while link CD rotates about D is 8 cm long BC = AD and $\underline{|\text{BAD}} = 60^o$. Find angular velocity of link CD.



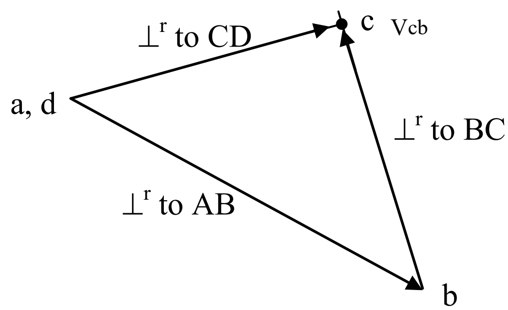
Configuration Diagram

<u>Velocity vector diagram</u>

$$V_b = \omega r = \omega_{ba} \text{ x AB} = \frac{2\pi \text{x} 120}{60} \text{x} 4 = 50.24 \text{ cm/sec}$$

<u>Choose a suitable scale</u>

$$1 \text{ cm} = 20 \text{ m/s} = \overrightarrow{ab}$$



$V_{cb} = \overrightarrow{bc}$

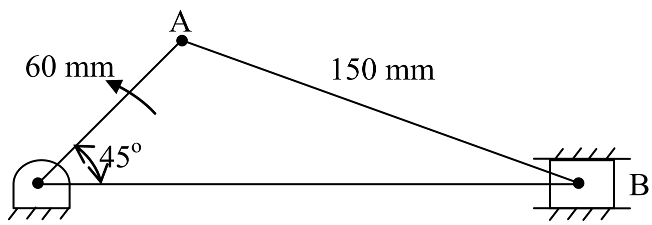
$V_c = \overrightarrow{dc} = 38$ cm/sec $= V_{cd}$

We know that $V = \omega R$

$V_{cd} = \omega_{CD} \times CD$

$\omega_{cD} = \dfrac{V_{cd}}{CD} = \dfrac{38}{8} = 4.75$ rad/sec (cw)

2. In a crank and slotted lever mechanism crank rotates of 300 rpm in a counter clockwise direction. Find
   (i) Angular velocity of connecting rod and
   (ii) Velocity of slider.



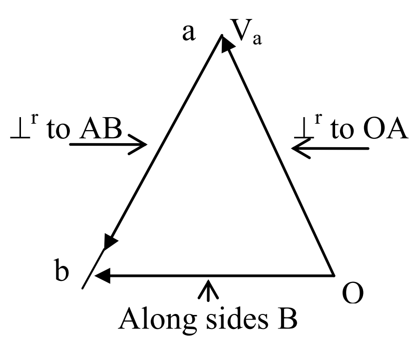
Configuration diagram

Step 1: Determine the magnitude and velocity of point A with respect to 0,

$V_A = \omega_{O1A} \times O_2A = \dfrac{2\pi \times 300}{60} \times 60$

$= 600\ \pi$ mm/sec

Step 2: Choose a suitable scale to draw velocity vector diagram.



Velocity vector diagram

$V_{ab} = \overrightarrow{ab} = 1300$mm/sec

$$\omega_{ba} = \frac{V_{ba}}{BA} = \frac{1300}{150} = 8.66 \text{ rad/sec}$$

$V_b = \vec{ob}$ velocity of slider

Note: Velocity of slider is along the line of sliding.

3. In a four bar mechanism, the dimensions of the links are as given below:

AB = 50 mm, BC = 66 mm

CD = 56 mm and AD = 100 mm

At a given instant when $\underline{|DAB} = 60^o$ the angular velocity of link AB is 10.5 rad/sec in CCW direction.

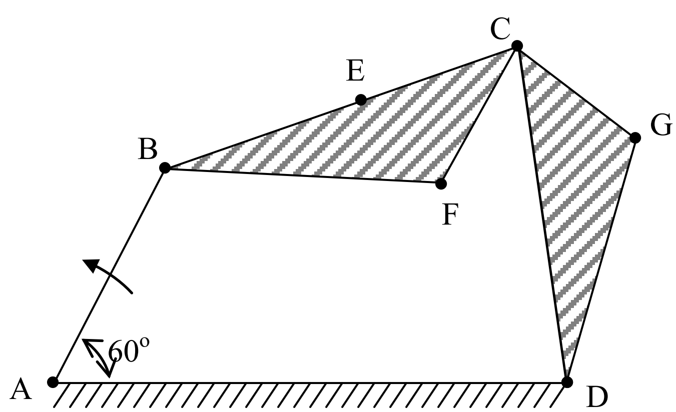
Determine,

i) Velocity of point C
ii) Velocity of point E on link BC when BE = 40 mm
iii) The angular velocity of link BC and CD
iv) The velocity of an offset point F on link BC, if BF = 45 mm, CF = 30 mm and BCF is read clockwise.
v) The velocity of an offset point G on link CD, if CG = 24 mm, DG = 44 mm and DCG is read clockwise.
vi) The velocity of rubbing of pins A, B, C and D. The ratio of the pins are 30 mm, 40 mm, 25 mm and 35 mm respectively.

Solution:

Step -1: Construct the configuration diagram selecting a suitable scale.

Scale: 1 cm = 20 mm



Step – 2: Given the angular velocity of link AB and its direction of rotation determine velocity of point with respect to A (A is fixed hence, it is zero velocity point).

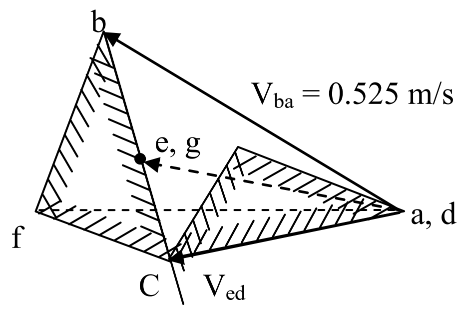
$V_{ba} = \omega_{BA} \text{ x } BA$

$= 10.5 \text{ x } 0.05 = 0.525 \text{ m/s}$

Step – 3: To draw velocity vector diagram choose a suitable scale, say 1 cm = 0.2 m/s.

- First locate zero velocity points.

- Draw a line $\perp^r$ to link AB in the direction of rotation of link AB (CCW) equal to 0.525 m/s.



- From b draw a line $\perp^r$ to BC and from d. Draw d line $\perp^r$ to CD to interest at C.
- V_{cb} is given vector bc V_{bc} = 0.44 m/s
- V_{cd} is given vector dc V_{cd} = 0.39 m/s

Step – 4: To determine velocity of point E (Absolute velocity) on link BC, first locate the position of point E on velocity vector diagram. This can be done by taking corresponding ratios of lengths of links to vector distance i.e.

$$\frac{be}{bc} = \frac{BE}{BC}$$

$$\therefore\ be = \frac{BE}{BC} \times V_{cb} = \frac{0.04}{0.066} \times 0.44 = 0.24 \text{ m/s}$$

Join e on velocity vector diagram to zero velocity points a, d / vector $\overrightarrow{de} = V_e$ = 0.415 m/s.

Step 5: To determine angular velocity of links BC and CD, we know V_{bc} and V_{cd}.

$$\therefore\ V_{bc} = \omega_{BC} \times BC$$

$$\therefore\ \omega_{BC} = \frac{V_{bc}}{BC} = \frac{0.44}{0.066} = 6.6\ r/s\ .\ (cw)$$

Similarly, $V_{cd} = \omega_{CD} \times CD$

$$\therefore\ \omega_{CD} = \frac{V_{cd}}{CD} = \frac{0.39}{0.056} = 6.96\ r/s\ \ (CCW)$$

Step – 6: To determine velocity of an offset point F

- Draw a line $\perp^r$ to CF from C on velocity vector diagram.
- Draw a line $\perp^r$ to BF from b on velocity vector diagram to intersect the previously drawn line at ‘f’.
- From the point f to zero velocity point a, d and measure vector fa to get

  V_f = 0.495 m/s.

Step – 7: To determine velocity of an offset point.

- Draw a line $\perp^r$ to GC from C on velocity vector diagram.
- Draw a line $\perp^r$ to DG from d on velocity vector diagram to intersect previously drawn line at g.
- Measure vector dg to get velocity of point G.

$$V_g = \overrightarrow{dg} = 0.305 \text{ m/s}$$

Step – 8: To determine rubbing velocity at pins

- Rubbing velocity at pin A will be

  $V_{pa} = \omega_{ab}$ x r of pin A

  $V_{pa} = 10.5 \times 0.03 = 0.315$ m/s
- Rubbing velocity at pin B will be

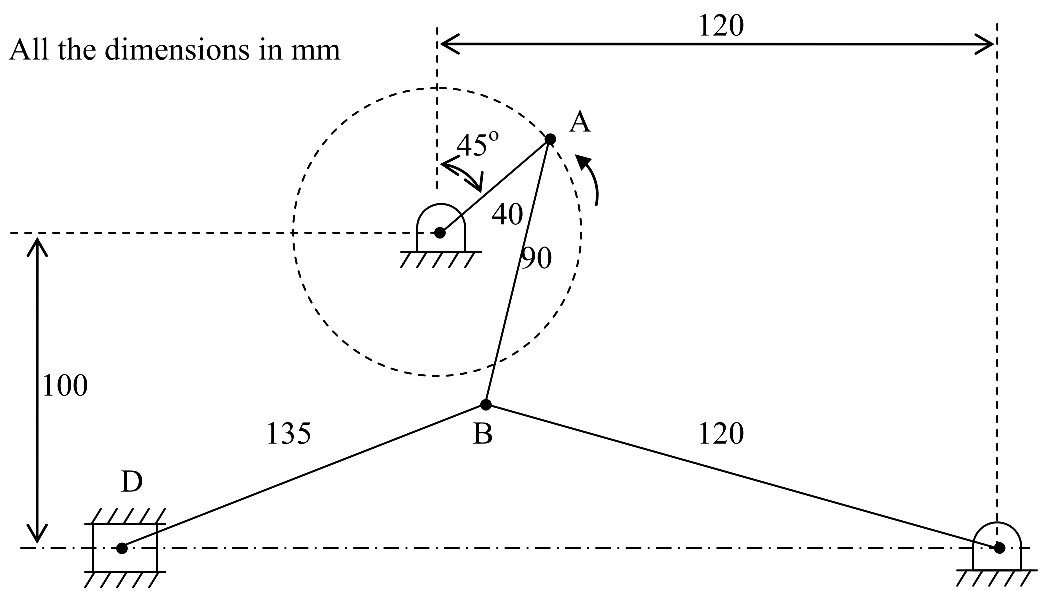
  $V_{pb} = (\omega_{ab} + \omega_{cb}) \times r_{pb}$ of point at B.

  [ω_{ab} CCW and ω_{cb}CW]

  $V_{pb} = (10.5 + 6.6) \times 0.04 = 0.684$ m/s.
- Rubbing velocity at point C will be = 6.96 x 0.035 = 0.244 m/s

4. Figure below shows a toggle mechanism in which the crank OA rotates at 120 rpm. Find the velocity and acceleration of the slider D.

Solution:



Configuration Diagram

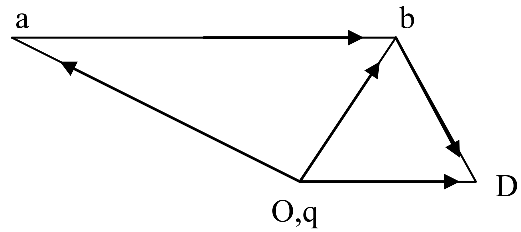
Step 1: Draw the configuration diagram choosing a suitable scale.
Step 2: Determine velocity of point A with respect to O.

$$V_{ao} = \omega_{OA} \times OA$$

$$V_{ao} = \frac{2\pi \times 120}{60} = 0.4 = 5.024 \text{ m/s}$$

Step 3: Draw the velocity vector diagram.

- Choose a suitable scale
- Mark zero velocity points O,q
- Draw vector $\overrightarrow{oa} \perp^r$ to link OA and magnitude = 5.024 m/s.



Velocity vector diagram

- From a draw a line $\perp^r$ to AB and from q draw a line $\perp^r$ to QB to intersect at b.

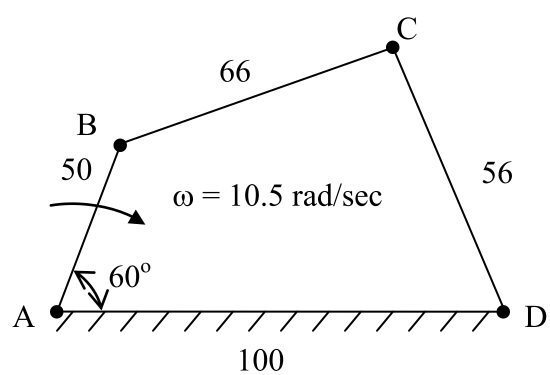
$$\overrightarrow{ab} = V_{ba} \text{ and } \overrightarrow{qb} = V_{bq}.$$

- Draw a line $\perp^r$ to BD from b from q draw a line along the slide to intersect at d.

$$\overrightarrow{dq} = V_d \text{ (slider velocity)}$$

5. For a 4-bar mechanism shown in figure draw velocity and acceleration diagram.

All dimensions are in mm



Solution:

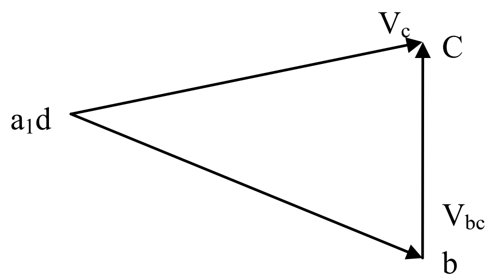
Step 1: Draw configuration diagram to a scale.

Step 2: Draw velocity vector diagram to a scale.

$V_b = \omega_2 \times AB$

$V_b = 10.5 \times 0.05$

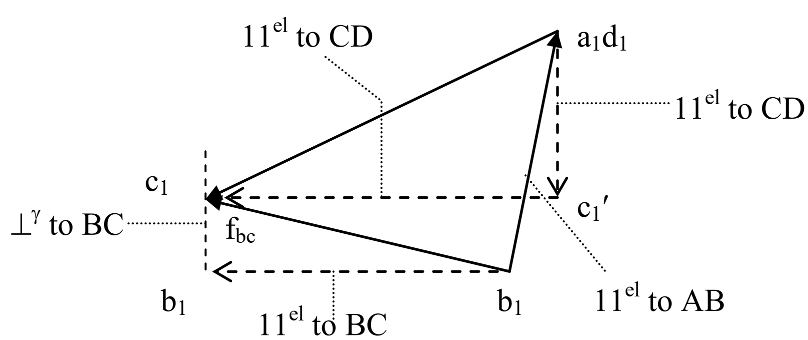
$V_b = 0.525$ m/s



Step 3: Prepare a table as shown below:

Sl. No.	Link	Magnitude	Direction	Sense
1.	AB	$f^c = \omega^2_{AB}r$ $f^c = (10.5)^2/0.525$ $f^c = 5.51\ m/s^2$	Parallel to AB	→ A
2.	BC	$f^c = \omega^2_{BC}r$ $f^c = 1.75$ $f^t = \alpha r$	Parallel to BC $\perp^r$ to BC	→ B –
3.	CD	$f^c = \omega^2_{CD}r$ $f^c = 2.75$ $f^t =$?	Parallel to DC $\perp^r$ to DC	→ D –

Step 4: Draw the acceleration diagram.



- Choose a suitable scale to draw acceleration diagram.
- Mark the zero acceleration point a_1d_1.
- Link AB has only centripetal acceleration. Therefore, draw a line parallel to AB and toward A from a_1d_1 equal to 5.51 m/s^2 i.e. point b_1.
- From b_1 draw a vector parallel to BC points towards B equal to 1.75 m/s^2 ($b^1{}_1$).

- From b^1_1 draw a line $\perp^r$ to BC. The magnitude is not known.
- From a_1d_1 draw a vector parallel to AD and pointing towards D equal to 2.72 m/s^2 i.e. point c_1.
- From c^1_1 draw a line $\perp^r$ to CD to intersect the line drawn $\perp^r$ to BC at c_1, $\overrightarrow{d_1c_1}$ = f_{CD} and $\overrightarrow{b_1c_1} = f_{bc}$.

  To determine angular acceleration.

$$\alpha_{BC} = \frac{f^t_{bc}}{BC} = \frac{\overrightarrow{c_1b^1_1}}{BC} = 34.09\,\text{rad/sec (CCW)}$$

$$\alpha_{CD} = \frac{f^t_{cd}}{CD} = \frac{\overrightarrow{c_1c^1_1}}{CD} = 79.11\,\text{rad/sec(CCW)}$$

BELT DRIVES

Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The conditions under which the belt is used.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

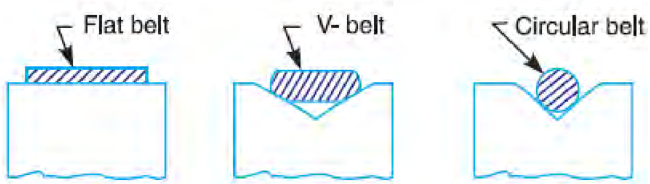
1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and
8. Service conditions.

Types of Belt Drives

The belt drives are usually classified into the following three groups :

1. ***Light drives***. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.
2. ***Medium drives***. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.
3. ***Heavy drives***. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

Types of Belts



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

Though there are many types of belts used these days, yet the following are important from the subject point of view:

1. ***Flat belt.*** The flat belt, as shown in Fig. (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.
2. ***V-belt.*** The V-belt, as shown in Fig. (*b*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

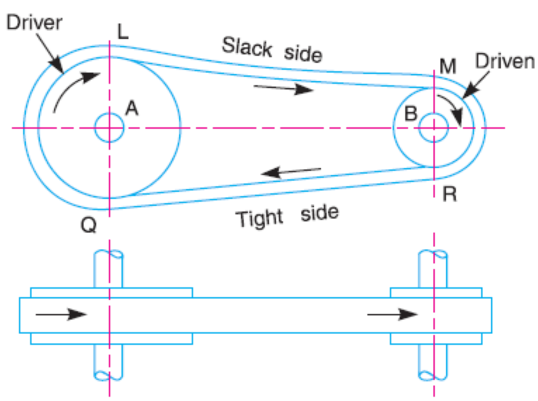
3. ***Circular belt or rope***. The circular belt or rope, as shown in Fig. (*c*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.

If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

Types of Flat Belt Drives

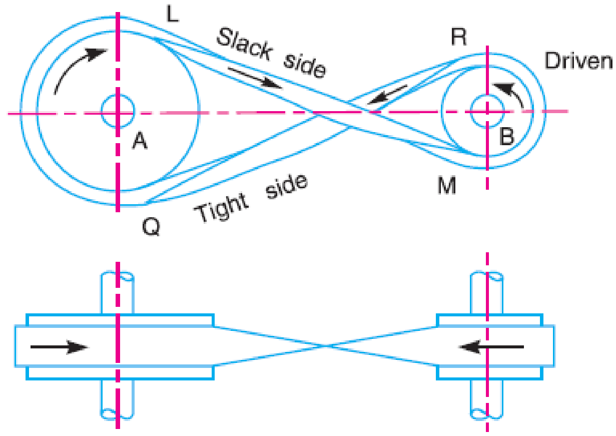
The power from one pulley to another may be transmitted by any of the following types of belt drives:

1. ***Open belt drive***. The open belt drive, as shown in Fig. 11.3, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver *A* pulls the belt from one side (*i.e.* lower side *RQ*) and delivers it to the other side (*i.e.* upper side *LM*). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as ***tight side*** whereas the upper side belt (because of less tension) is known as ***slack side***, as shown in Fig.

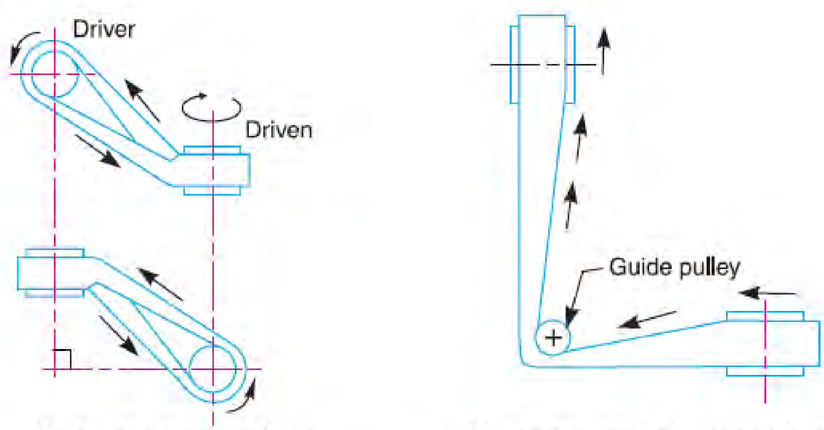


2. ***Crossed or twist belt drive***. The crossed or twist belt drive, as shown in Fig. 11.4, is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belt from one side (*i.e. RQ*) and delivers it to the other side (*i.e. LM*). Thus the tension in the belt *RQ* will be more than that in the belt *LM*. The belt *RQ* (because of more tension) is known as ***tight side*,** whereas the belt *LM* (because of less tension) is known as ***slack side***, as shown in Fig.

A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 *b*, where *b* is the width of belt and the speed of the belt should be less than 15 m/s.



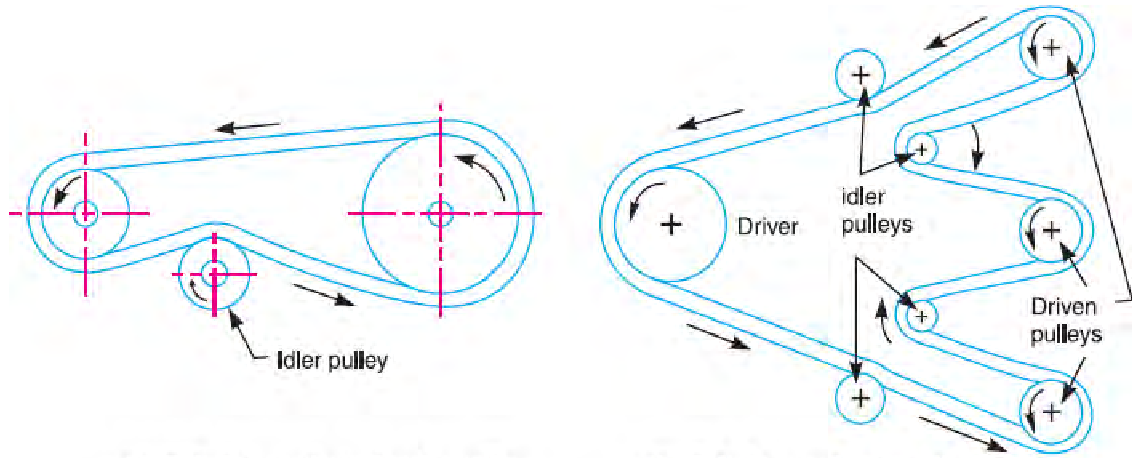
3. ***Quarter turn belt drive.*** The quarter turn belt drive also known as right angle belt drive, as shown in Fig. (*a*), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 *b*, where *b* is the width of belt. In case the pulleys cannot be arranged, as shown in Fig. (*a*), or when the reversible is desired, then a ***quarter turn belt drive with guide pulley***, as shown in Fig. (*b*), may be used.



(*a*) Quarter turn belt drive. (*b*) Quarter turn belt drive with guide pulley.

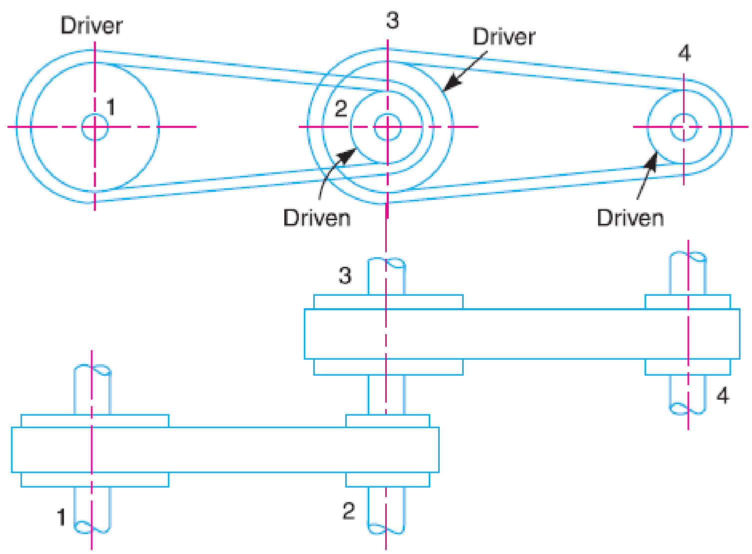
4. ***Belt drive with idler pulleys***. A belt drive with an idler pulley, as shown in Fig. (*a*), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.

When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. (*b*), may be employed.

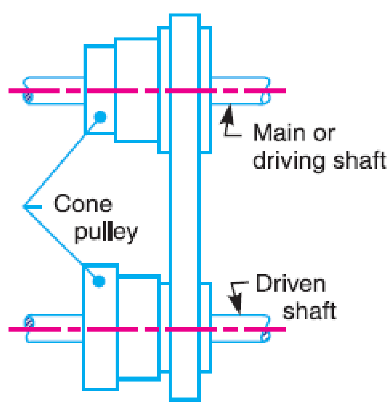


(*a*) Belt drive with single idler pulley. (*b*) Belt drive with many idler pulleys.

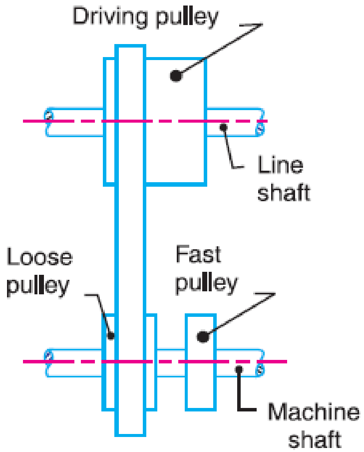
5. ***Compound belt drive***. A compound belt drive, as shown in Fig., is used when power is transmitted from one shaft to another through a number of pulleys.



6. ***Stepped or cone pulley drive***. A stepped or cone pulley drive, as shown in Fig, is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.



7. *Fast and loose pulley drive*. A fast and loose pulley drive, as shown in Fig., is used when the driven or machine shaft is to be started or stopped when ever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called ***fast pulley*** and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.



Velocity Ratio of Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let $d1$ = Diameter of the driver,

$d2$ = Diameter of the follower,

$N1$ = Speed of the driver in r.p.m., and

$N2$ = Speed of the follower in r.p.m.

Length of the belt that passes over the driver, in one minute = $\pi\, d1N1$

Similarly, length of the belt that passes over the follower, in one minute = $\pi\, d2\, N2$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$\pi\, d1\, N1 = \pi\, d2\, N2$

Velocity ratio, $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

When the thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

The velocity ratio of a belt drive may also be obtained as discussed below :

We know that peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi d_1 . N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven or follower pulley,

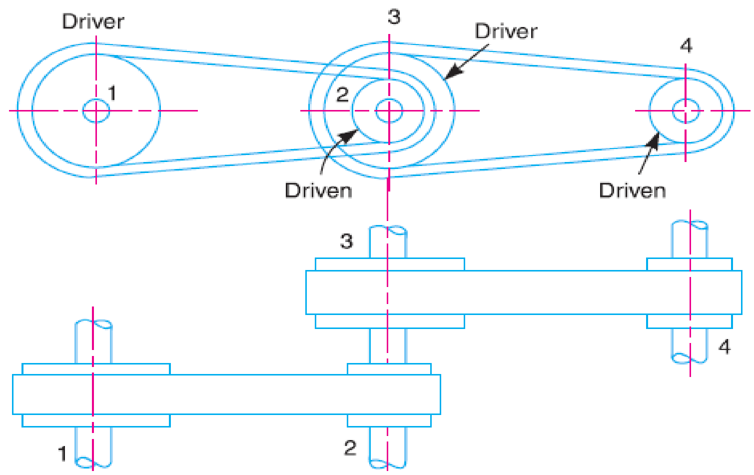
$$v_2 = \frac{\pi d_2 . N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi d_1 . N_1}{60} = \frac{\pi d_2 . N_2}{60} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys, as shown in fig. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.



Let

$d1$ = Diameter of the pulley 1,

$N1$ = Speed of the pulley 1 in r.p.m.,

$d2$, $d3$, $d4$, and $N2$, $N3$, $N4$= Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4}$$

Multiplying the above equations gives

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{...}(\because N_2 = N_3, \text{ being keyed to the same shaft})$$

Slip of Belt

In the previous articles, we have discussed the motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called ***slip of the belt*** and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance.

Let $s_1\%$ = Slip between the driver and the belt, and

$s_2\%$ = Slip between the belt and the follower.

∴ Velocity of the belt passing over the driver per second

$$v = \frac{\pi d_1 . N_1}{60} - \frac{\pi d_1 . N_1}{60} \times \frac{s_1}{100} = \frac{\pi d_1 . N_1}{60}\left(1 - \frac{s_1}{100}\right)$$

and velocity of the belt passing over the follower per second,

$$\frac{\pi d_2 . N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (i),

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \qquad \ldots\left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

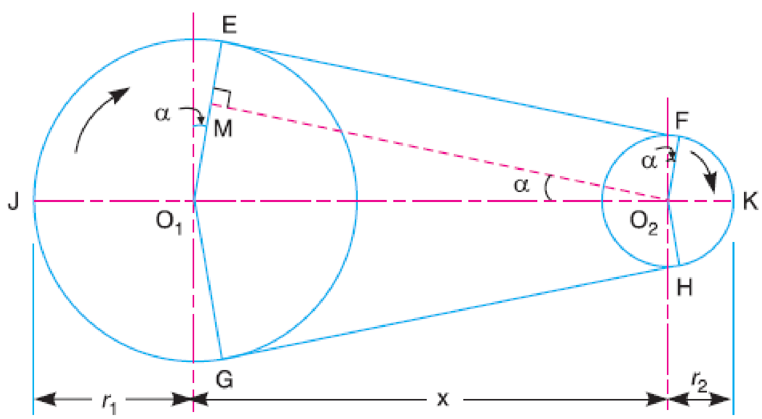
$$= \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

... (where $s = s_1 + s_2$, *i.e.* total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Length of an open Belt Drive



We have already discussed that in an open belt drive, both the pulleys rotate in the ***same*** direction as shown in Fig.

Let

r_1 and r_2 = Radii of the larger and smaller pulleys,
x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and
L = Total length of the belt.

Let the belt leaves the larger pulley at *E* and *G* and the smaller pulley at *F* and *H* as shown in Fig. Through O_2, draw $O_2 M$ parallel to *FE*.

From the geometry of the figure, we find that $O_2 M$ will be perpendicular to $O_1 E$.
Let the angle $MO_2 O_1$ = α radians.

We know that the length of the belt,
L = Arc *GJE* + *EF* + Arc *FKH* + *HG*

= 2 (Arc *JE* + *EF* + Arc *FK*)

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - EM}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x}$$

$$\therefore \quad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right)$$

Similarly $$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right)$$

and $$EF = MO_2 = \sqrt{(O_1 O_2)^2 - (O_1 M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + ...\right] = x - \frac{(r_1 - r_2)^2}{2x}$$

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 - r_2}{x}$

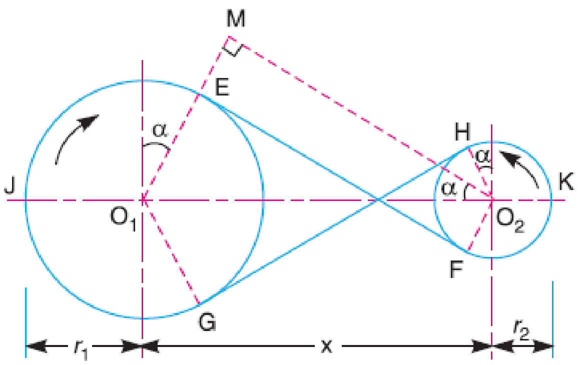
$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{...(In terms of pulley diameters)}$$

Length of a Cross Belt Drive



We have already discussed that in a cross belt drive, both the pulleys rotate in ***opposite*** directions as shown in Fig.

Let $r1$ and $r2$ = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O1\ O2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at *E* and *G* and the smaller pulley at *F* and *H*, as shown in Fig. Through $O2$, draw $O2M$ parallel to FE.

From the geometry of the figure, we find that $O2M$ will be perpendicular to $O1E$.

Let the angle $MO2\ O1$ = α radians

We know that the length of the belt,

L = Arc $GJE + EF$ + Arc $FKH + HG$

= 2 (Arc $JE + EF$ + Arc FK)

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x}$$

$$\therefore \qquad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right)$$

$$\text{Similarly} \qquad \text{Arc } FK = r_2\left(\frac{\pi}{2} + \alpha\right)$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 + r_2)^2}{2x}$$

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 + r_2}{x}$

$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \qquad \text{...(In terms of pulley diameters)}$$

It may be noted that the above expression is a function of ($r1$ + $r2$). It is thus obvious that if sum of the radii of the two pulleys be constant, then length of the belt required will also remain constant, provided the distance between centres of the pulleys remain unchanged.

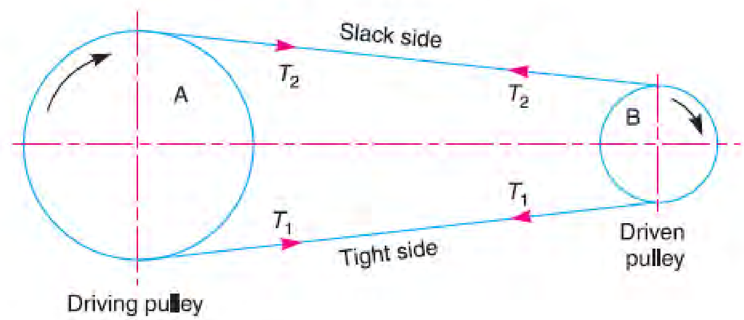
Power transmitted by a Belt

Fig. shows the driving pulley (or driver) *A* and the driven pulley (or follower) *B*. We have already discussed that the driving pulley pulls the belt from one side and delivers the same to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side (*i.e.* slack side) as shown in Fig.

Let T_1 and T_2 = Tensions in the tight and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driver and follower respectively, and

v = Velocity of the belt in m/s.



The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

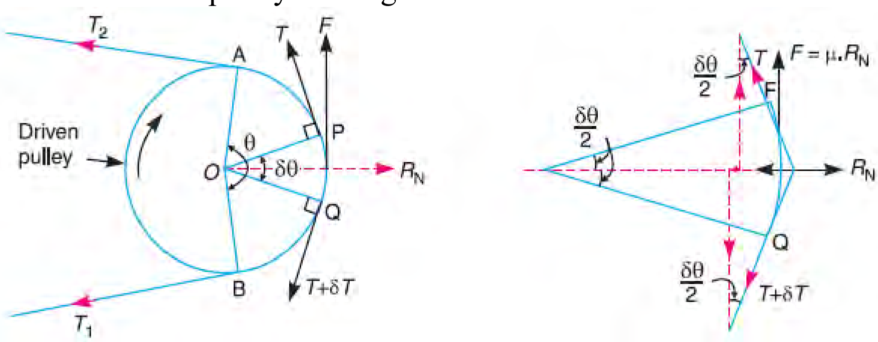
$\therefore$ Work done per second = $(T_1 - T_2)\, v$ N-m/s

and power transmitted, $P = (T_1 - T_2)\, v$ W ...($\because$ 1 N-m/s = 1 W)

A little consideration will show that the torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$. Similarly, the torque exerted on the driven pulley *i.e.* follower is $(T_1 - T_2)\, r_2$.

Ratio of Driving Tensions for Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig.



Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc AB, along which the belt touches the pulley at the centre).

Now consider a small portion of the belt PQ, subtending an angle $\delta\theta$ at the centre of the pulley as shown in Fig. 11.15. The belt PQ is in equilibrium under the following forces :

1. Tension T in the belt at P,
2. Tension $(T + \delta T)$ in the belt at Q,
3. Normal reaction R_N, and
4. Frictional force, $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \qquad ...(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin \delta\theta / 2 = \delta\theta / 2$ in equation (*i*),

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta \qquad ...(ii)$$

$$...\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad ...(iii)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta / 2 = 1$ in equation (*iii*),

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{or} \quad R_N = \frac{\delta T}{\mu} \qquad ...(iv)$$

Equating the values of R_N from equations (*ii*) and (*iv*),

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

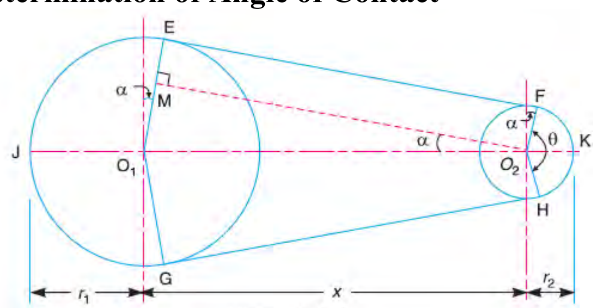
i.e. $$\int_{T_2}^{T_1}\frac{\delta T}{T} = \mu\int_0^{\theta}\delta\theta \quad \text{or} \quad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta}$$

Equation (*v*) can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

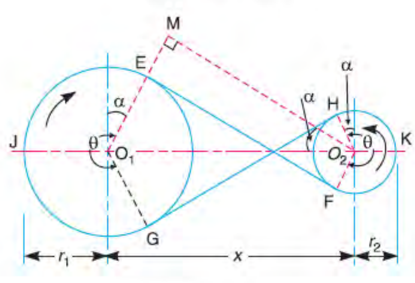
$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

Determination of Angle of Contact



(*a*) Open belt drive.



(*b*) Crossed belt drive.

When the two pulleys of different diameters are connected by means of an open belt as shown in Fig. (*a*), then the angle of contact or lap (θ) at the smaller pulley must be taken into consideration.

Let r_1 = Radius of larger pulley,

r_2 = Radius of smaller pulley, and

x = Distance between centres of two pulleys (*i.e.* $O_1 O_2$).

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E - ME}{O_1 O_2} = \frac{r_1 - r_2}{x} \qquad \ldots(\because ME = O_2 F = r_2)$$

∴ Angle of contact or lap,

$$\theta = (180° - 2\alpha)\frac{\pi}{180}\ \text{rad}$$

A little consideration will show that when the two pulleys are connected by means of a crossed belt as shown in Fig. (*b*), then the angle of contact or lap (θ) on both the pulleys is same

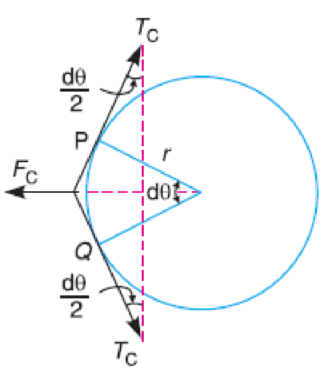
$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + ME}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

∴ Angle of contact or lap, $\theta = (180° + 2\alpha)\frac{\pi}{180}$ rad

Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension***. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion *PQ* of the belt subtending an angle $d\theta$ the centre of the pulley as shown in Fig.



Let m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$$= r.\,d\theta$$

and mass of the belt PQ $= m.\,r.\,d\theta$

$\therefore$ Centrifugal force acting on the belt PQ,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

The centrifugal tension T_C acting tangentially at P and Q keeps the belt in equilibrium.

Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally and equating the same, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

Since the angle $d\theta$ is very small, therefore, putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$, in the above expression,

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \quad \text{or} \quad T_C = m\,.\,v^2$$

When the centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_C$$

Power transmitted, $P = (T_{t1} - T_{t2})\,v$ ...(in watts)

$= [(T_1 + T_C) - (T_2 + T_C)]\,v = (T_1 - T_2)\,v$ ...(same as before)

Thus we see that centrifugal tension has no effect on the power transmitted.

The ratio of driving tensions may also be written as

$$2.3\log\left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

where T_{t1} = Maximum or total tension in the belt.

Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let σ = Maximum safe stress in N/mm^2,

b = Width of the belt in mm, and

t = Thickness of the belt in mm.

We know that maximum tension in the belt,

T = Maximum stress × cross-sectional area of belt = $\sigma.\,b.\,t$

When centrifugal tension is neglected, then

T (or T_{t1}) = T_1, *i.e.* Tension in the tight side of the belt

and when centrifugal tension is considered, then

$$T \text{ (or } T_{t1}) = T_1 + T_C$$

Condition for the Transmission of Maximum Power

We know that power transmitted by a belt,

$$P = (T_1 - T_2)\,v \qquad ...(i)$$

where T_1 = Tension in the tight side of the belt in newtons,

T_2 = Tension in the slack side of the belt in newtons, and

v = Velocity of the belt in m/s.

From Art. 11.14, we have also seen that the ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu.\theta}} \qquad ...(ii)$$

Substituting the value of T_2 in equation *(i)*,

$$P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu.\theta}}\right)v = T_1.v.C \qquad ...(iii)$$

where

$$C = 1 - \frac{1}{e^{\mu.\theta}}$$

We know that $T_1 = T - T_C$

where T = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of T_1 in equation *(iii)*,

$$P = (T - T_C)\,v.C$$

$$= (T - m.v^2)\,v.C = (T.v - m\,v^3)\,C \quad \text{... (Substituting } T_C = m.\,v^2\text{)}$$

For maximum power, differentiate the above expression with respect to v and equate to zero, *i.e.*

$$\frac{dP}{dv} = 0 \qquad \text{or} \qquad \frac{d}{dv}(T.v - mv^3)\,C = 0$$

$$\therefore \quad T - 3\,m\,.\,v^2 = 0$$

or

$$T - 3\,T_C = 0 \quad \text{or} \quad T = 3\,T_C \qquad ...(iv)$$

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

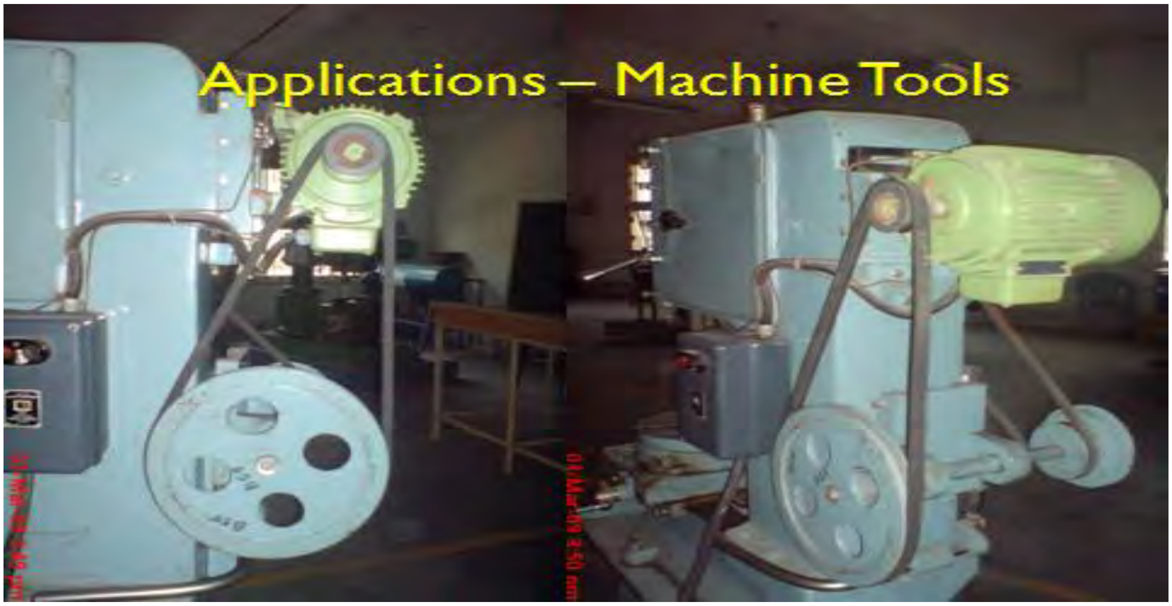
We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$$T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

From equation *(iv)*, the velocity of the belt for the maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

Applications:



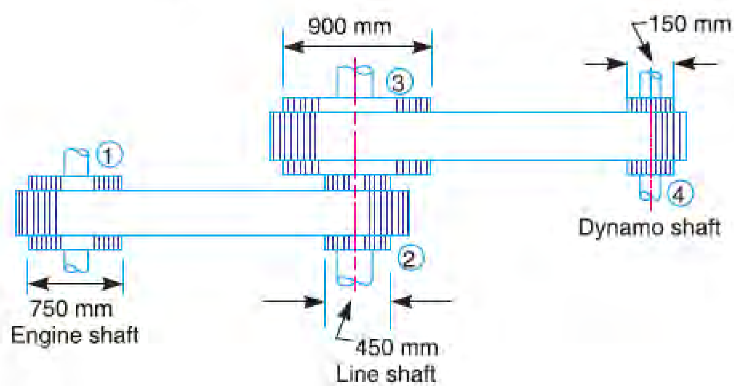
Exercise Problems:

1) An engine, running at 150 r.p.m., drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive.

Solution:

Given : $N_1 = 150$ r.p.m. ; $d_1 = 750$ mm ; $d_2 = 450$ mm ; $d_3 = 900$ mm ; $d_4 = 150$ mm

Let N_4 = Speed of the dynamo shaft .



1. When there is no slip

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

$$N_4 = 150 \times 10 = 1500 \text{ r.p.m.}$$

2. When there is a slip of 2% at each drive

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$$N_4 = 150 \times 9.6 = 1440 \text{ r.p.m.}$$

2) Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 r.p.m. The coefficient of friction between the belt and the pulley is 0.25, angle of lap 160° and maximum tension in the belt is 2500 N.

Solution:

Given: $d = 600$ mm = 0.6 m ; $N = 200$ r.p.m. ; $\mu = 0.25$; $\theta = 160° = 160 \times \pi/180 = 2.793$ rad ; $T_1 = 2500$ N

We know that velocity of the belt,

$$v = \frac{\pi d . N}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.284 \text{ m/s}$$

Let T_2 = Tension in the slack side of the belt.

We know that $$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta = 0.25 \times 2.793 = 0.6982$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.6982}{2.3} = 0.3036$$

$$\frac{T_1}{T_2} = 2.01 \qquad \text{...(Taking antilog of 0.3036)}$$

$$T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} = 1244 \text{ N}$$

We know that power transmitted by the belt,

$$P = (T_1 - T_2)\,v = (2500 - 1244)\,6.284 = 7890 \text{ W}$$
$$= 7.89 \text{ kW}$$

3) A casting weighing 9 kN hangs freely from a rope which makes 2.5 turns round a drum of 300 mm diameter revolving at 20 r.p.m. The other end of the rope is pulled by a man. The coefficient of friction is 0.25. Determine **1.** The force required by the man, and **2.** The power to raise the casting.

Solution:

Given : $W = T_1 = 9$ kN = 9000 N ; $d = 300$ mm = 0.3 m ; $N = 20$ r.p.m. ; $\mu = 0.25$

1. *Force required by the man*

Let T_2 = Force required by the man.

Since the rope makes 2.5 turns round the drum, therefore angle of contact,

$\theta = 2.5 \times 2\,\pi = 5\,\pi$ rad

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 5\pi = 3.9275$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{3.9275}{2.3} = 1.71 \quad \text{or} \quad \frac{T_1}{T_2} = 51$$

...(Taking antilog of 1.71)

$$T_2 = \frac{T_1}{51} = \frac{9000}{51} = 176.47 \text{ N}$$

2. *Power to raise the casting*

We know that velocity of the rope,

$$v = \frac{\pi\,d.N}{60} = \frac{\pi \times 0.3 \times 20}{60} = 0.3142 \text{ m/s}$$

Power to raise the casting,

$$P = (T_1 - T_2)\,v = (9000 - 176.47)\,0.3142 = 2772 \text{ W}$$
$$= 2.772 \text{ kW}$$

4) Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25 ?

Solution:

Given : $d_1 = 450$ mm = 0.45 m or $r_1 = 0.225$ m ; $d_2 = 200$ mm = 0.2 m or $r_2 = 0.1$ m ; $x = 1.95$ m ; $N_1 = 200$ r.p.m. ; $T_1 = 1$ kN = 1000 N ; $\mu = 0.25$

We know that speed of the belt,

$$v = \frac{\pi d_1.N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

We know that length of the crossed belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975\,\text{m}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667 \quad \text{or} \quad \alpha = 9.6°$$

$$\theta = 180° + 2\alpha = 180° + 2 \times 9.6° = 199.2°$$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad}$$

Power transmitted

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8692$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8692}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387 \qquad \text{...(Taking antilog of 0.378)}$$

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\,v = (1000 - 419)\,4.714 = 2740 \text{ W} = 2.74 \text{ kW}$$

5) A shaft rotating at 200 r.p.m. drives another shaft at 300 r.p.m. and transmits 6 kW through a belt. The belt is 100 mm wide and 10 mm thick. The distance between the shafts is 4m. The smaller pulley is 0.5 m in diameter. Calculate the stress in the belt, if it is **1.** an open belt drive, and **2.** a cross belt drive. Take μ =0.3.

Solution:

Given : N_1 = 200 r.p.m. ; N_2 = 300 r.p.m. ; P = 6 kW = 6×10^3 W ; b = 100 mm ; t = 10 mm ; x = 4 m ; d_2 = 0.5 m ; μ = 0.3

Let σ= Stress in the belt.

1. *Stress in the belt for an open belt drive*

First of all, let us find out the diameter of larger pulley (d_1). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_1 = \frac{N_2.d_2}{N_1} = \frac{300 \times 0.5}{200} = 0.75\,\text{m}$$

and velocity of the belt,

$$v = \frac{\pi\, d_2.N_2}{60} = \frac{\pi \times 0.5 \times 300}{60} = 7.855 \text{ m/s}$$

Now let us find the angle of contact on the smaller pulley. We know that, for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.75 - 0.5}{2 \times 4} = 0.03125 \quad \text{or} \quad \alpha = 1.8°$$

$\therefore$ Angle of contact, $\theta = 180° - 2\,\alpha = 180 - 2 \times 1.8 = 176.4°$

$= 176.4 \times \pi / 180 = 3.08$ rad

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu . \theta = 0.3 \times 3.08 = 0.924$$

$$\therefore \quad \log \left(\frac{T_1}{T_2} \right) = \frac{0.924}{2.3} = 0.4017 \quad \text{or} \quad \frac{T_1}{T_2} = 2.52$$

We also know that power transmitted (P),

$$6 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 7.855$$

$$\therefore \quad T_1 - T_2 = 6 \times 10^3 / 7.855 = 764 \text{ N}$$

By solving the above two equations

T1= 1267 N and T2= 503 N

We know that maximum tension in the belt (T_1),

$$1267 = \sigma . b. t = \sigma \times 100 \times 10 = 1000\, \sigma$$

$$\sigma = 1267 / 1000 = 1.267 \text{ N/mm}^2 = 1.267 \text{ MPa}$$

...[$\because$ 1 MPa = 1 MN/m^2 = 1 N/mm^2]

Stress in the belt for a cross belt drive

We know that for a cross belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{d_1 + d_2}{2x} = \frac{0.75 + 0.5}{2 \times 4} = 0.1562 \quad \text{or} \quad \alpha = 9°$$

$\therefore$ Angle of contact, $\theta = 180° + 2\alpha = 180 + 2 \times 9 = 198°$

$= 198 \times \pi / 180 = 3.456$ rad

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu . \theta = 0.3 \times 3.456 = 1.0368$$

$$\log \left(\frac{T_1}{T_2} \right) = \frac{1.0368}{2.3} = 0.4508 \quad \text{or} \quad \frac{T_1}{T_2} = 2.82$$

By solving the above equations

T_1 = 1184 N and T_2 = 420 N

We know that maximum tension in the belt (T_1),

$$1184 = \sigma . b. t = \sigma \times 100 \times 10 = 1000\, \sigma$$

$$\sigma = 1184 / 1000 = 1.184 \text{ N/mm}^2 = 1.184 \text{ MPa}$$

6) Determine the width of a 9.75 mm thick leather belt required to transmit 15 kW from a motor running at 900 r.p.m. The diameter of the driving pulley of the motor is 300 mm. The driven pulley runs at 300 r.p.m. and the distance between the centre of two pulleys is 3 metres. The density of the leather is 1000 kg/m3. The maximum allowable stress in the leather is 2.5 MPa. The coefficient of friction between the leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt.

Solution:

Given: t = 9.75 mm = 9.75×10^{-3} m ; P = 15 kW = 15×10^3 W ; N_1 = 900 r.p.m. ; d_1 = 300 mm = 0.3 m ; N_2 = 300 r.p.m. ; x = 3m ; ρ= 1000 kg/m^3 ; σ= 2.5 MPa = 2.5×10^6 N/m^2 ; μ = 0.3

First of all, let us find out the diameter of the driven pulley (d_2). We know that

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{or} \quad d_2 = \frac{N_1 \times d_1}{N_2} = \frac{900 \times 0.3}{300} = 0.9 \text{ m}$$

velocity of the belt,
$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.3 \times 900}{60} = 14.14 \text{ m/s}$$

For an open belt drive,

$$\sin\alpha = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{0.9 - 0.3}{2 \times 3} = 0.1 \qquad \ldots(\because d_2 > d_1)$$

$$\alpha = 5.74°$$

∴ Angle of lap, $\theta = 180° - 2\alpha = 180 - 2 \times 5.74 = 168.52°$

$= 168.52 \times \pi / 180 = 2.94$ rad

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.94 = 0.882$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.882}{2.3} = 0.3835 \quad \text{or} \quad \frac{T_1}{T_2} = 2.42$$

We also know that power transmitted (P),

$$15 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.14$$

∴ $T_1 - T_2 = 15 \times 10^3 / 14.14 = 1060$ N

On solving above two equations we get T1= 1806 N

Let b = Width of the belt in metres.

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho$$
$$= b \times 9.75 \times 10^{-3} \times 1 \times 1000 = 9.75\, b \text{ kg}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 9.75\, b\, (14.14)^2 = 1950\ b \text{ N}$$

Maximum tension in the belt,

$$T = \sigma.\, b.\, t = 2.5 \times 10^6 \times b \times 9.75 \times 10^{-3} = 24\,400\, b \text{ N}$$

We know that $T = T_1 + T_C$ or $T - T_C = T_1$

$$24\,400\, b - 1950\, b = 1806 \quad \text{or} \quad 22\,450\, b = 1806$$

∴ $b = 1806 / 22\,450 = 0.080$ m = 80 mm

CAM

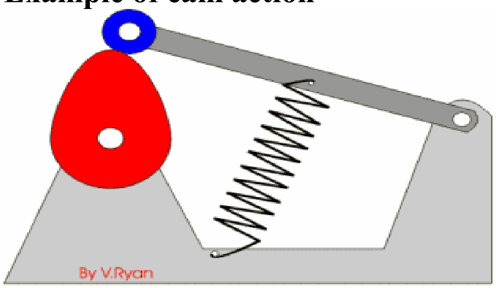
Introduction

Cam - A mechanical device used to transmit motion to a follower by direct contact.
Where Cam – driver member
Follower - driven member.
The cam and the follower have line contact and constitute a higher pair.
In a cam - follower pair, the cam normally rotates at uniform speed by a shaft, while the follower may is predetermined, will translate or oscillate according to the shape of the cam. A familiar example is the camshaft of an automobile engine, where the cams drive the push rods (the followers) to open and close the valves in synchronization with the motion of the pistons.

Applications:

The cams are widely used for operating the inlet and exhaust valves of Internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes.

Example of cam action



Classification of Followers

(i) Based on surface in contact. **(Fig.3.1)**

(a) Knife edge follower

(b) Roller follower

(c) Flat faced follower

(d) Spherical follower

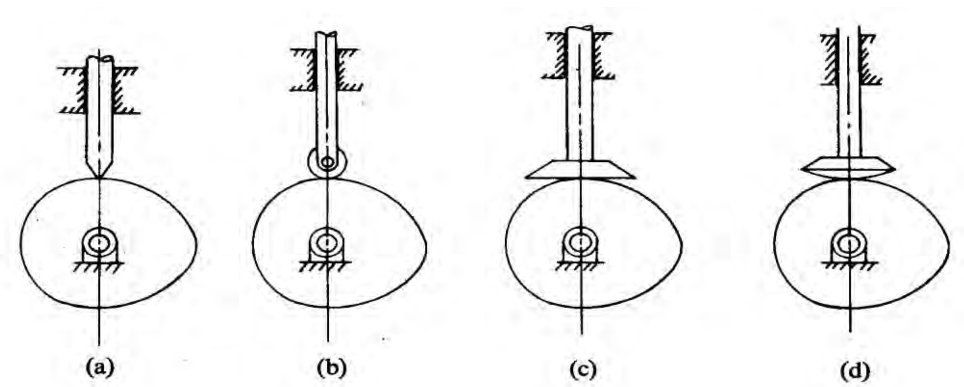


Fig. 3.1 Types of followers

(ii) Based on type of motion: **(Fig. 3.2)**

(a) Oscillating follower

(b) Translating follower

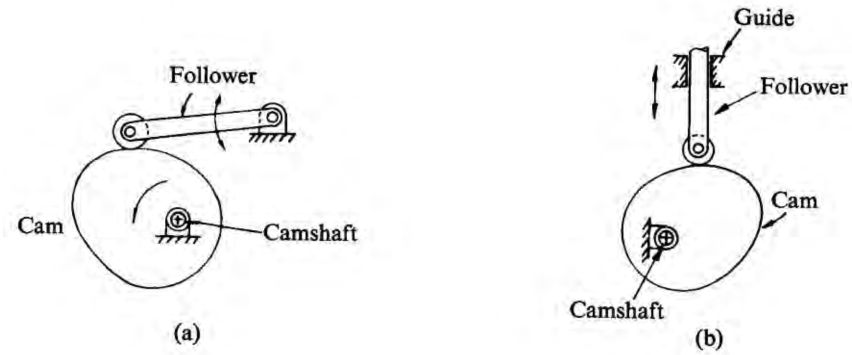


Fig.3.2

(iii) Based on line of motion:

(a) Radial follower: The lines of movement of in-line cam followers pass through the centers of the camshafts (Fig. 3.1a, b, c, and d).

(b) Off-set follower: For this type, the lines of movement are offset from the centers of the camshafts (Fig. 3.3a, b, c, and d).

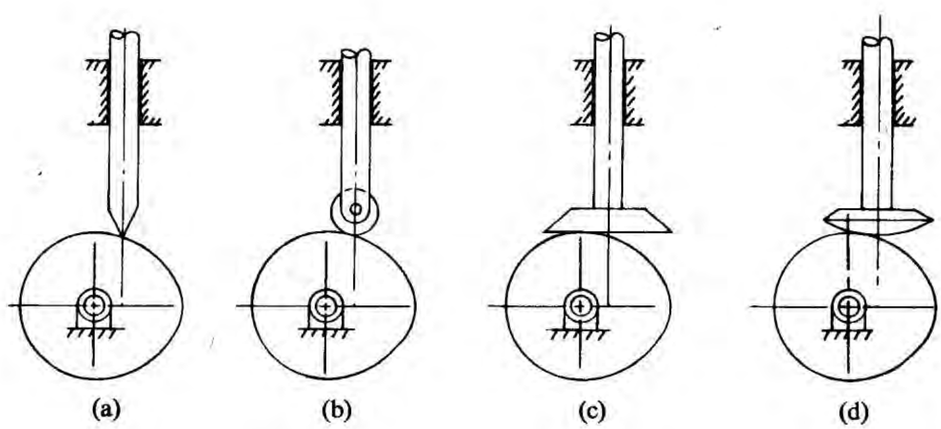


Fig.3.3 Off set followers

Classification of Cams

Cams can be classified based on their physical shape.

a) Disk or plate cam (Fig. 3.4 a and b): The disk (or plate) cam has an irregular contour to impart a specific motion to the follower. The follower moves in a plane perpendicular to the axis of rotation of the camshaft and is held in contact with the cam by springs or gravity.

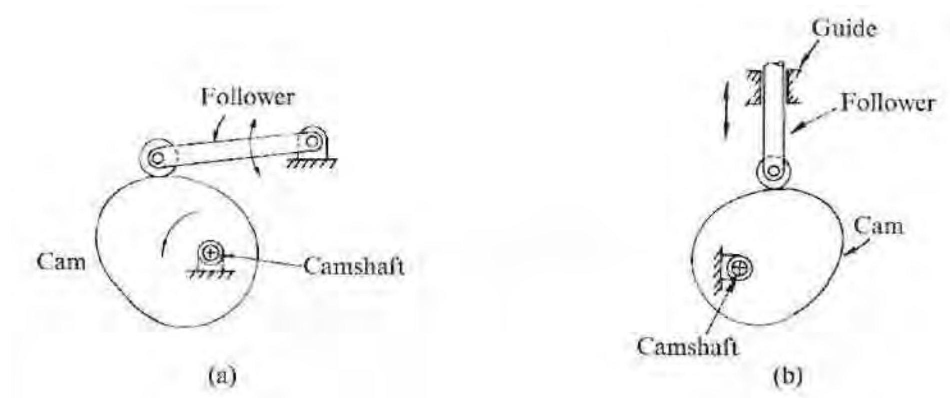


Fig. 3.4 Plate or disk cam.

b) Cylindrical cam (Fig. 3.5): The cylindrical cam has a groove cut along its cylindrical surface. The roller follows the groove, and the follower moves in a plane parallel to the axis of rotation of the cylinder.

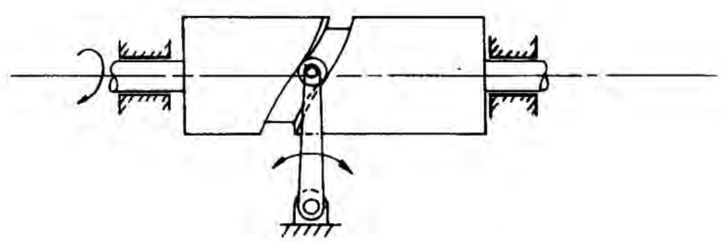


Fig. 3.5 Cylindrical cam.

c) Translating cam (Fig. 3.6a and b). The translating cam is a contoured or grooved plate sliding on a guiding surface(s). The follower may oscillate (Fig. 3.6(a)) or reciprocate (Fig. 3.6(b)). The contour or the shape of the groove is determined by the specified motion of the follower.

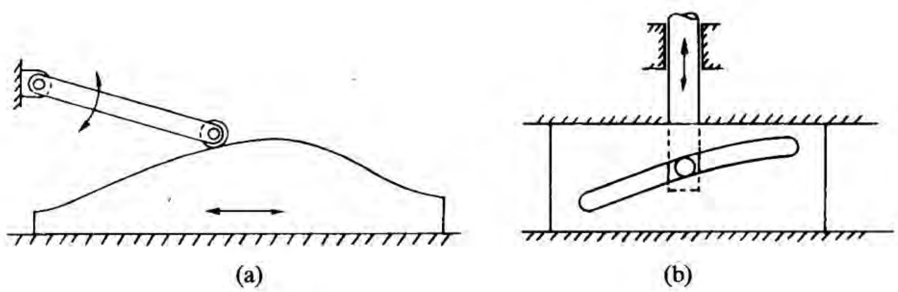


Fig. 3.6 Translating cam

Terms Used in Radial Cams

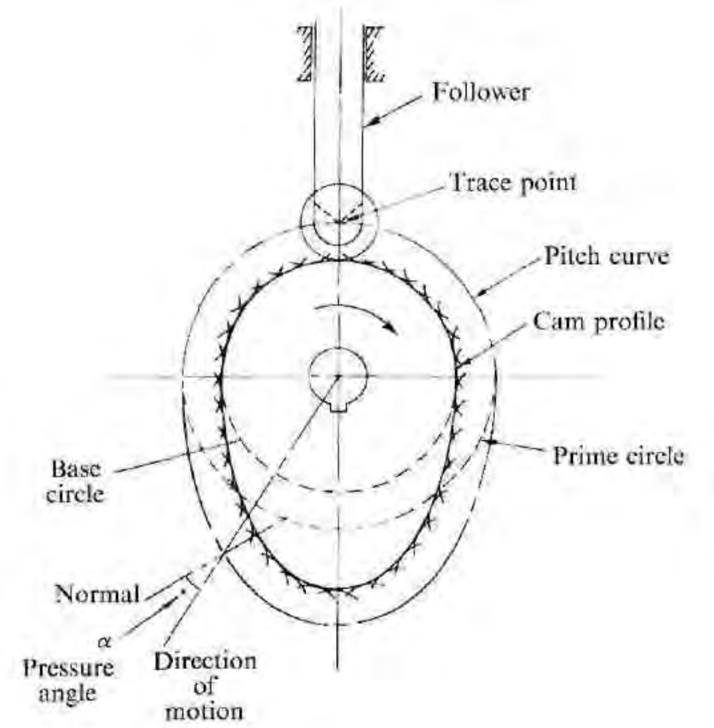


Fig.3.7

Pressure angle: It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the angle is too large, a reciprocating follower will jam in its bearings.

Base circle: It is the smallest circle that can be drawn to the cam profile.

Trace point: It is the reference point on the follower and is used to generate the pitch curve. In the case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In the roller follower, the centre of the roller represents the trace point.

Pitch point: It is a point on the pitch curve having the maximum pressure angle.

Pitch circle: It is a circle drawn from the centre of the cam through the pitch points.

Pitch curve: It is the curve generated by the trace point as the follower moves relative to the cam.For a knife edge follower, the pitch curve and the cam profile are same where as for a roller follower; they are separated by the radius of the follower.

Prime circle: It is the smallest circle that can be drawn from the centre of the cam and tangent to the point.For a knife edge and a flat face follower, the prime circle and the base circle and the base circle are identical.For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

Lift (or) stroke: It is the maximum travel of the follower from its lowest position to the topmost position.

Motion of the Follower

Cam follower systems are designed to achieve a desired oscillatory motion. Appropriate displacement patterns are to be selected for this purpose, before designing the cam surface. The cam is assumed to rotate at a constant speed and the follower raises, dwells, returns to its original position and dwells again through specified angles of rotation of the cam, during each revolution of the cam.

Some of the standard follower motions are as follows:

They are, follower motion with,

(a) Uniform velocity

(b) Modified uniform velocity

(c) Uniform acceleration and deceleration

(d) Simple harmonic motion

Displacement diagrams: In a cam follower system, the motion of the follower is very important. Its displacement can be plotted against the angular displacement θ of the cam and it is called as the displacement diagram. The displacement of the follower is plotted along the y-axis and angular displacement θ of the cam is plotted along x-axis. From the displacement diagram, velocity and acceleration of the follower can also be plotted for different angular displacements θ of the cam. The displacement, velocity and acceleration diagrams are plotted for one cycle of operation i.e., one rotation of the cam. Displacement diagrams are basic requirements for the construction of cam profiles. Construction of displacement diagrams and calculation of velocities and accelerations of followers with different types of motions are discussed in the following sections.

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity

Fig.3.8 shows the displacement, velocity and acceleration patterns of a follower having uniform velocity type of motion. Since the follower moves with constant velocity, during rise and fall, the displacement varies linearly with θ. Also, since the velocity changes from zero to a finite value, within no time, theoretically, the acceleration becomes infinite at the beginning and end of rise and fall.

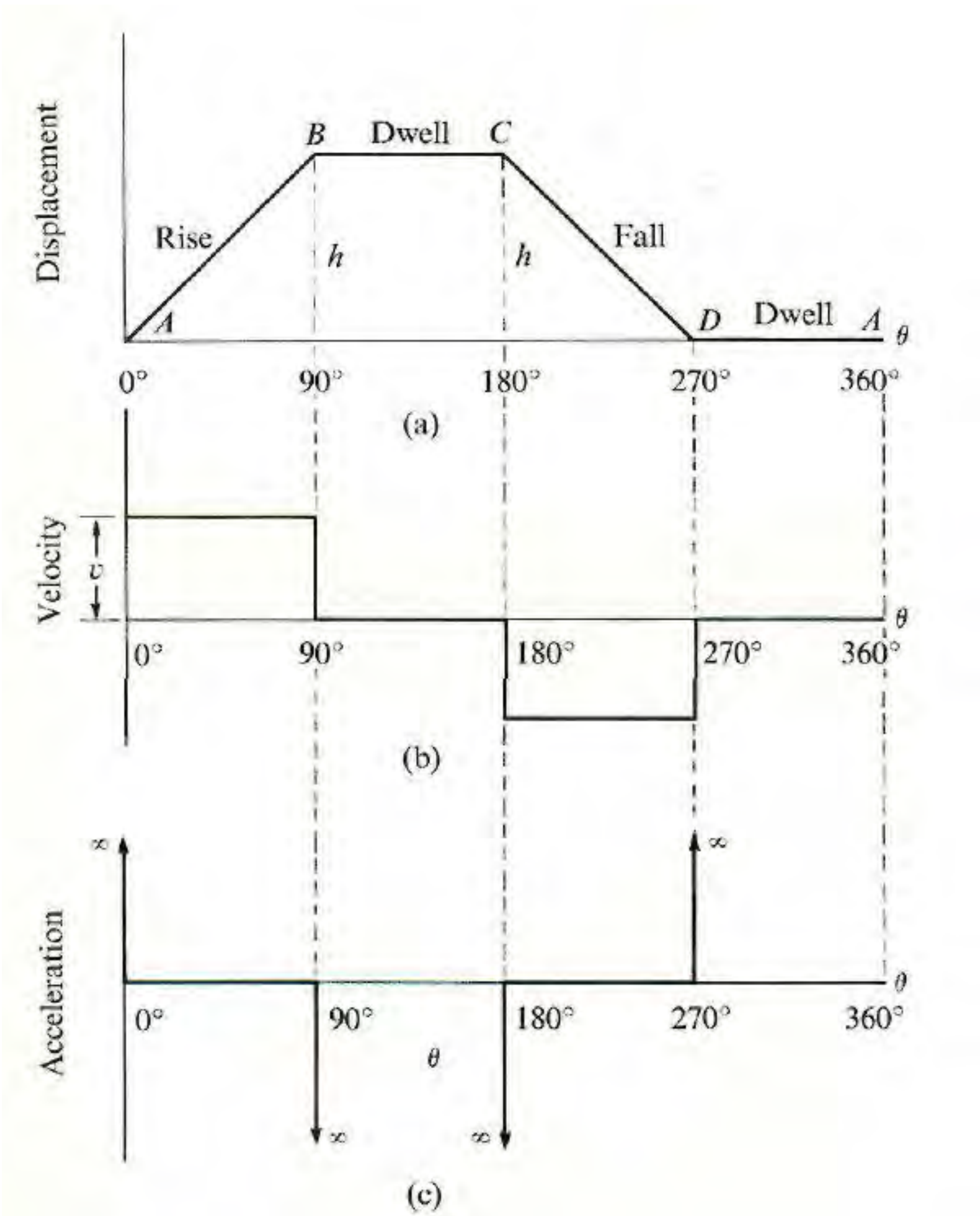


Fig.3.8

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

In fig.3.9, the motion executed by point P^l, which is the projection of point P on the vertical diameter is called simple harmonic motion. Here, P moves with uniform angular velocity ω_p, along a circle of radius r (r = s/2).

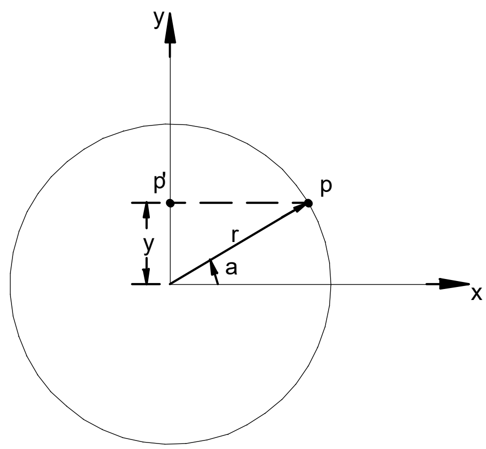


Fig.3.9

Displacement = $y = r\sin\alpha = r\sin\omega_p t$; $y_{\max} = r$ [eq.1]

Velocity = $\dot{y} = \omega_p r\cos\omega_p t$; $\dot{y}_{\max} = r\omega_p$ [eq.2]

Acceleration = $\ddot{y} = -\omega_p{}^2 r\sin\omega_p t = -\omega_p{}^2 y$; $\ddot{y}_{\max} = -r\omega_p{}^2$ [eq.3]

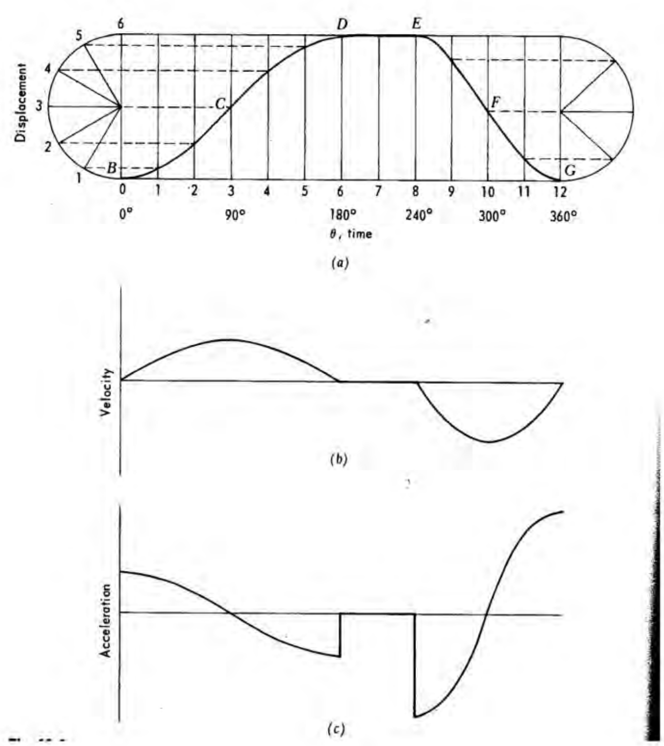


Fig.3.10

s= Stroke or displacement of the follower.

θ_o = Angular displacement during outstroke.

θ_r = Angular displacement during return stroke

ω = Angular velocity of cam.

t_o = Time taken for outstroke = $\dfrac{\theta_o}{\omega}$

t_r = Time taken for return stroke = $\dfrac{\theta_r}{\omega}$

Max. velocity of follower during outstroke = $vo_{max} = r\omega_p = \dfrac{s}{2}\dfrac{\pi}{t_o} = \dfrac{\pi\omega s}{2\theta_o}$

Similarly Max. velocity of follower during return stroke = , $vr_{max} = \dfrac{s}{2}\dfrac{\pi}{t_r} = \dfrac{\pi\omega s}{2\theta_r}$

Max. acceleration during outstroke = $ao_{max} = r\omega^2_p$ (from d3) = $\dfrac{s}{2}\left(\dfrac{\pi}{t_o}\right)^2 = \dfrac{\pi^2\omega^2 s}{2\theta_o^{\ 2}}$

Similarly, Max. acceleration during return stroke = $ar_{max} = \dfrac{s}{2}\left(\dfrac{\pi}{t_r}\right)^2 = \dfrac{\pi^2\omega^2 s}{2\theta^2{}_r}$

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

Here, the displacement of the follower varies parabolically with respect to angular displacement of cam. Accordingly, the velocity of the follower varies uniformly with respect to angular displacement of cam. The acceleration/retardation of the follower becomes constant accordingly. The displacement, velocity and acceleration patterns are shown in **fig. 3.11**.

s = Stroke of the follower

θ_o and θ_r = Angular displacement of the cam during outstroke and return stroke.

ω = Angular velocity of cam.

Time required for follower outstroke = $t_o = \dfrac{\theta_o}{\omega}$

Time required for follower return stroke = $t_r = \dfrac{\theta_r}{\omega}$

Average velocity of follower = $\dfrac{s}{t}$

Average velocity of follower during outstroke = $\dfrac{s/2}{t_o/2} = \dfrac{s}{t_o} = \dfrac{vo_{min} + vo_{max}}{2}$

$vo_{min} = 0$

$\therefore vo_{max} = \dfrac{2s}{t_o} = \dfrac{2\omega s}{\theta_o}$ = Max. velocity during outstroke.

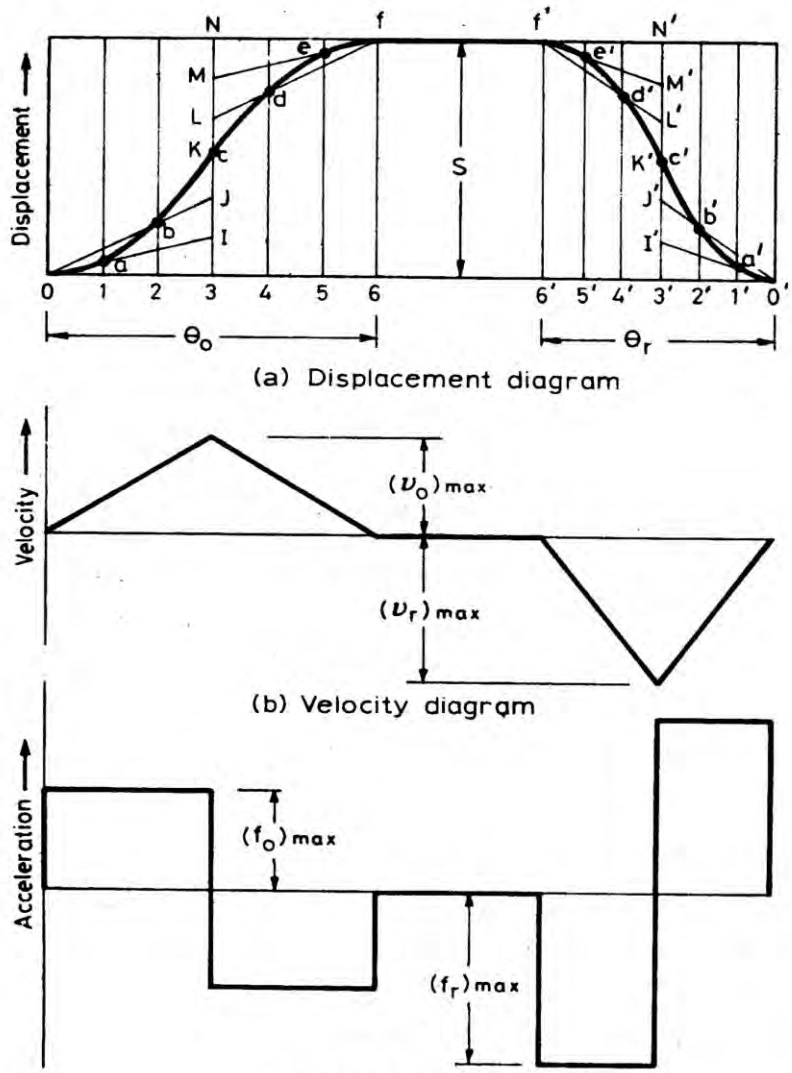
Average velocity of follower during return stroke = $\dfrac{s/2}{t_r/2} = \dfrac{s}{t_r} = \dfrac{vr_{\min} + vr_{\max}}{2}$

$vr_{\min} = 0$

$\therefore vr_{\max} = \dfrac{2s}{t_r} = \dfrac{2\omega s}{\theta_r}$ = Max. velocity during return stroke.

Acceleration of the follower during outstroke = $a_o = \dfrac{vo_{\max}}{t_o/2} = \dfrac{4\omega^2 s}{\theta_o^2}$

Similarly acceleration of the follower during return stroke = $a_r = \dfrac{4\omega^2 s}{\theta_r^2}$



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

Fig.3.11

Construction of Cam Profile for a Radial Cam

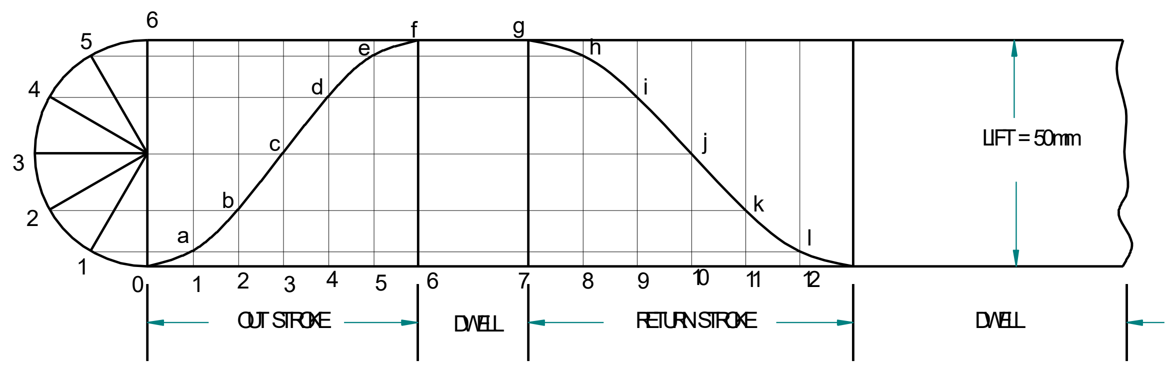
In order to draw the cam profile for a radial cam, first of all the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.
In constructing the cam profile, the principle of kinematic inversion is used, i.e. the cam is imagined to be stationary and the follower is allowed to rotate in the **opposite direction** to the **cam rotation.**
The construction of cam profiles for different types of follower with different types of motions are discussed in the following examples.

Practise problems:

(1) Draw the cam profile for following conditions:

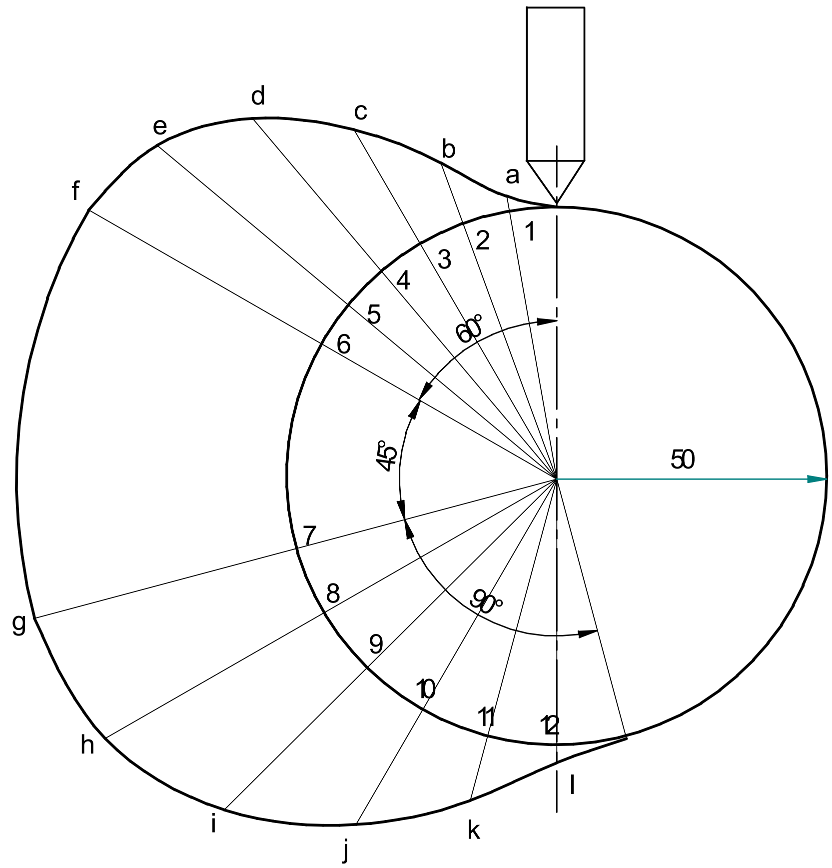
Follower type = Knife edged, in-line; lift = 50mm; base circle radius = 50mm; out stroke with SHM, for 60^0 cam rotation; dwell for 45^0 cam rotation; return stroke with SHM, for 90^0 cam rotation; dwell for the remaining period. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 1000 rpm in clockwise direction.

Displacement diagram:



Cam profile:

- Construct base circle.
- Mark points 1,2,3…..in direction opposite to the direction of cam rotation.
- Transfer points a,b,c…..l from displacement diagram to the cam profile and join them by a smooth free hand curve.
- This forms the required cam profile.



Calculations:

Angular velocity of cam = $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1000}{60}$ =**104.76 rad/sec**

Max. velocity of follower during outstroke = $vo_{max} = \frac{\pi \omega s}{2\theta_o}$

$= \frac{\pi \times 104.76 \times 50}{2 \times \pi/3}$ =7857mm/sec =**7.857m/sec**

Similarly Max. velocity of follower during return stroke = , $vr_{max} = \frac{\pi \omega s}{2\theta_r}$

$= \frac{\pi \times 104.76 \times 50}{2 \times \pi/2}$ = 5238mm/sec = **5.238m/sec**

Max. acceleration during outstroke = $ao_{max} = r\omega^2_p$ (from d3) = $\frac{\pi^2 \omega^2 s}{2\theta_o^{\ 2}}$

$$= \frac{\pi^2 \times (104.76)^2 \times 50}{2 \times \left(\pi/3\right)^2} = 2469297.96 \text{mm/sec}^2 = \mathbf{2469.3 m/sec^2}$$

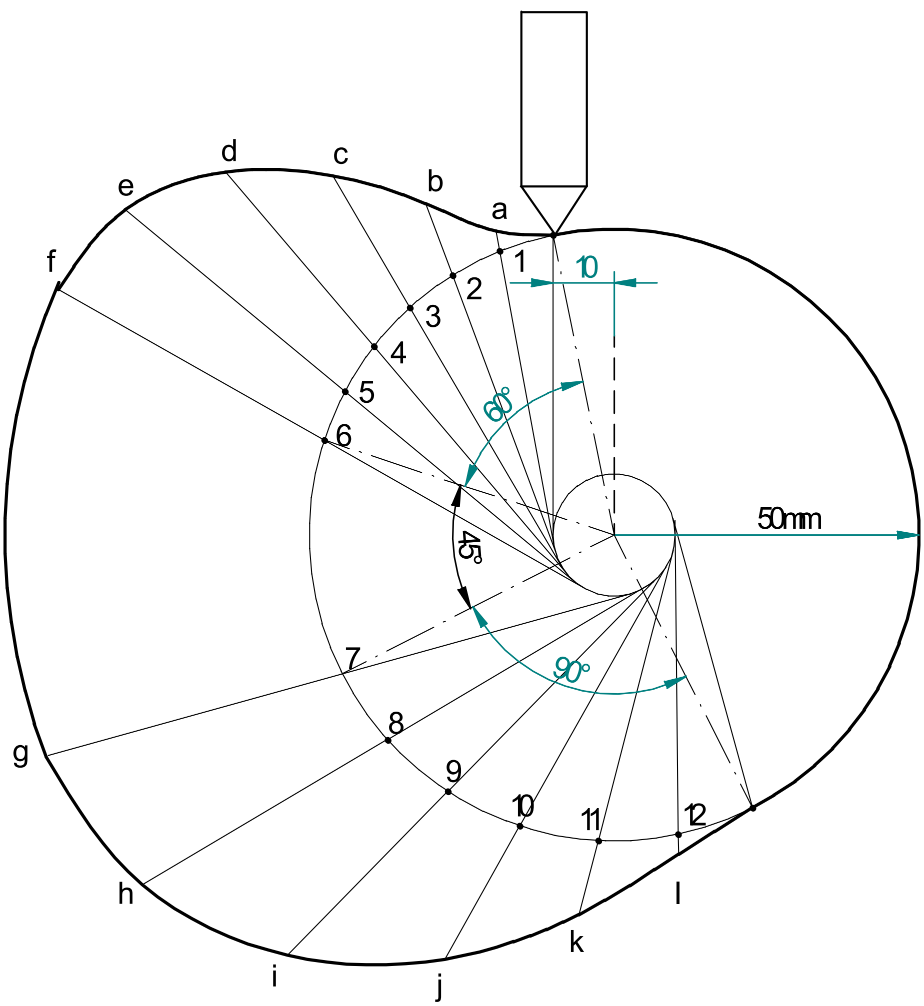
Similarly, Max. acceleration during return stroke = $ar_{max} = \frac{\pi^2 \omega^2 s}{2\theta^2_r} =$

$$= \frac{\pi^2 \times (104.76)^2 \times 50}{2 \times \left(\pi/2\right)^2} = 1097465.76 \text{mm/sec}^2 = \mathbf{1097.5 m/sec^2}$$

(2.) Draw the cam profile for the same operating conditions of problem (1), with the follower off set by 10 mm to the left of cam center.

Displacement diagram: Same as previous problem.

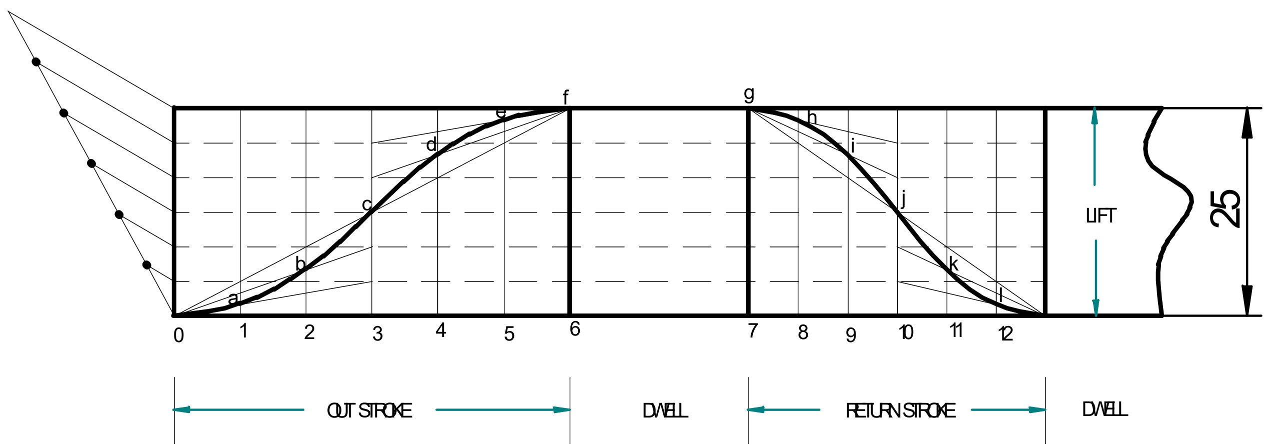
Cam profile: Construction is same as previous case, except that the lines drawn from 1,2,3…. are tangential to the offset circle of 10mm dia. as shown in the fig.



(3) Draw the cam profile for following conditions:

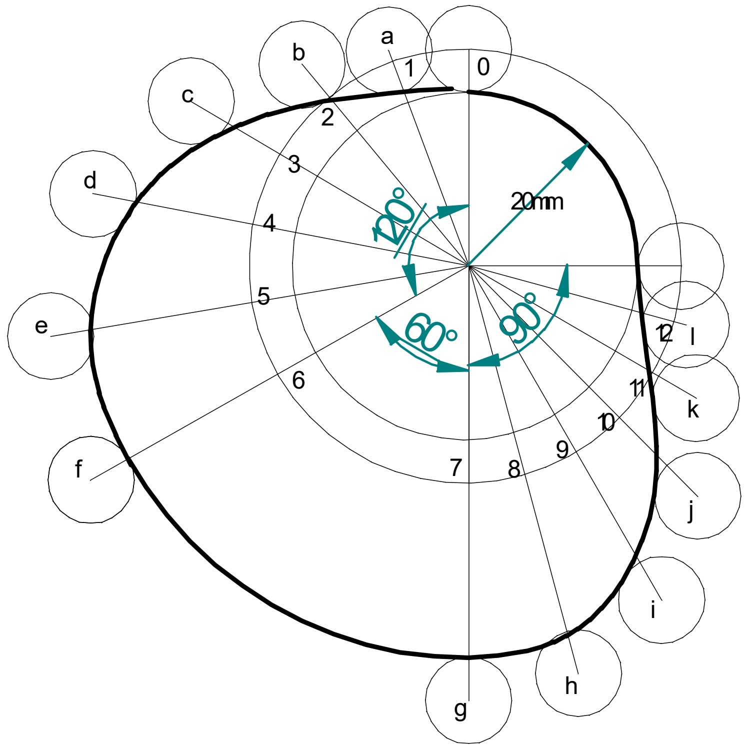
Follower type = roller follower, in-line; lift = 25mm; base circle radius = 20mm; roller radius = 5mm; out stroke with UARM, for 120^0 cam rotation; dwell for 60^0 cam rotation; return stroke with UARM, for 90^0 cam rotation; dwell for the remaining period. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 1200 rpm in clockwise direction.

Displacement diagram:



Cam profile:

- Construct base circle and prime circle (25mm radius).
- Mark points 1,2,3…..in direction opposite to the direction of cam rotation, on prime circle.
- Transfer points a,b,c…..l from displacement diagram.
- At each of these points a,b,c… draw circles of 5mm radius, representing rollers.
- Starting from the first point of contact between roller and base circle, draw a smooth free hand curve, tangential to all successive roller positions.
- This forms the required cam profile.



Calculations:

Angular velocity of the cam = $\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times1200}{60} =$ **125.71rad/sec**

Max. velocity during outstroke $= vo_{\max} = \frac{2s}{t_o} = \frac{2\omega s}{\theta_o} =$

$= \frac{2\times125.71\times25}{2\times\pi/3}$ =2999.9mm/sec =**2.999m/sec**

Max. velocity during return stroke = $vr_{\max} = \frac{2s}{t_r} = \frac{2\omega s}{\theta_r} = \frac{2\times125.71\times25}{\pi/2} =$

= 3999.86mm/sec = **3.999m/sec**

Acceleration of the follower during outstroke = $a_o = \frac{vo_{\max}}{t_o/2} = \frac{4\omega^2 s}{\theta_o^2} =$

$= \frac{4\times(125.71)^2\times25}{(2\times\pi/3)^2}$ =359975mm/sec^2 = **359.975m/sec^2**

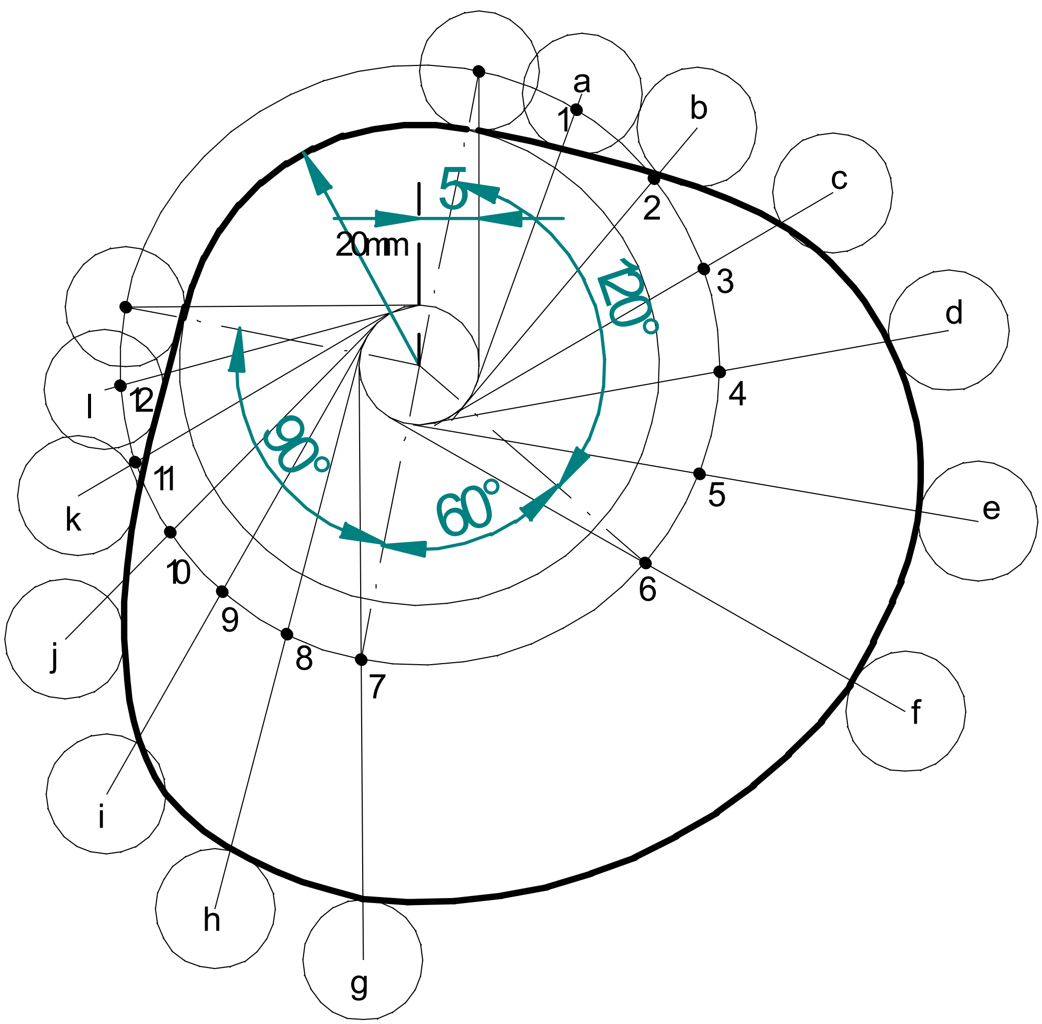
Similarly acceleration of the follower during return stroke = $a_r = \dfrac{4\omega^2 s}{\theta_r^{\ 2}} =$

$$= \frac{4\times(125.71)^2\times 25}{\left(\pi/2\right)^2} = 639956\text{mm/sec}^2 = \mathbf{639.956m/sec^2}$$

(4.) Draw the cam profile for conditions same as in (3), with follower off set to right of cam center by 5mm and cam rotating counter clockwise.

Displacement diagram: Same as previous case.

Cam profile: Construction is same as previous case, except that the lines drawn from 1,2,3…. are tangential to the offset circle of 10mm dia. as shown in the fig.



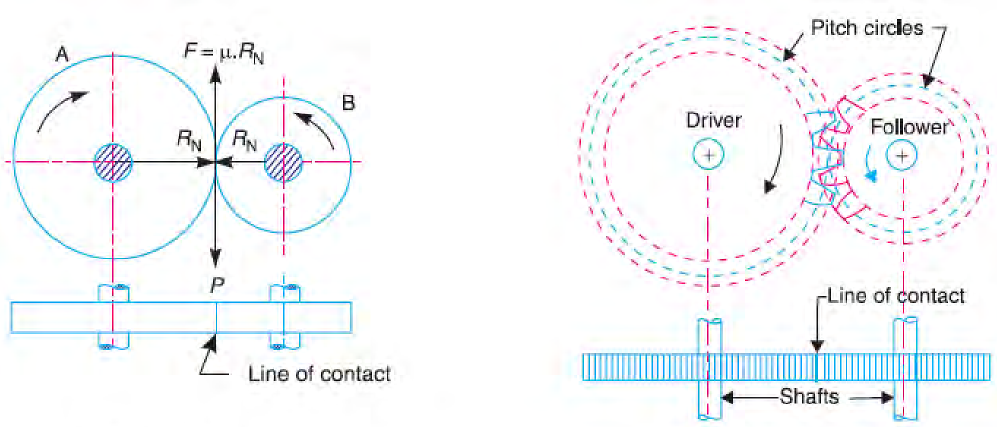
GEAR DRIVE

Introduction

The slip and creep in the belt or rope drives is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slip is to reduce the velocity ratio of the drive. In precision machine, in which a definite velocity ratio is importance (as in watch mechanism, special purpose machines..etc), the only positive drive is by means of gears or toothed wheels.

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers. Gears are highly efficient (nearly 95%) due to primarily rolling contact between the teeth, thus the motion transmitted is considered as positive. Gears essentially allow positive engagement between teeth so high forces can be transmitted while still undergoing essentially rolling contact. Gears do not depend on friction and do best when friction is minimized.

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated. A little consideration will show, that when the wheel A is rotated by a rotating shaft, it will rotate the wheel B in the opposite direction as shown in Fig. 4.1 (a). The wheel B will be rotated (by the wheel A) so long as the tangential force exerted by the wheel A does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.



(*a*) Friction wheels. (*b*) Toothed wheels.

Fig. 4.1.

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. 5.1 (b), are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as **toothed wheel or gear.** The usual connection to show the toothed wheels is by their pitch circles.

Advantages and Disadvantages of Gear Drive

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages:

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages:

1. The manufacture of gears requires special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

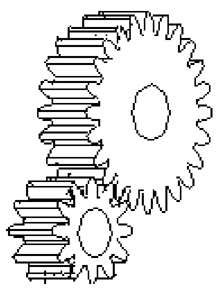
Classification of Toothed Wheels

Gears may be classified according to the relative position of the axes of revolution. The axes may be

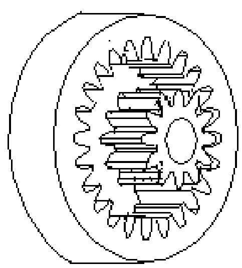
1. Gears for connecting parallel shafts,
2. Gears for connecting intersecting shafts,
3. Gears for neither parallel nor intersecting shafts.

Gears for connecting parallel shafts

1. **Spur gears:** Spur gears are the most common type of gears. They have straight teeth, and are mounted on parallel shafts. Sometimes, many spur gears are used at once to create very large gear reductions. Each time a gear tooth engages a tooth on the other gear, the teeth collide, and this impact makes a noise. It also increases the stress on the gear teeth. To reduce the noise and stress in the gears, most of the gears in your car are



External contact



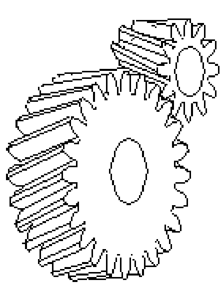
Internal contact

Spur gears

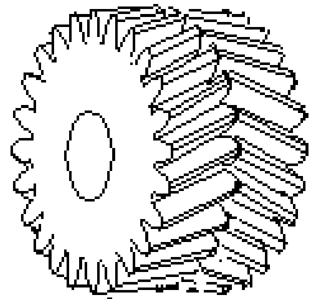
Spur gears are the most commonly used gear type. They are characterized by teeth, which are perpendicular to the face of the gear. Spur gears are most commonly available, and are generally the least expensive.

- **Limitations:** Spur gears generally cannot be used when a direction change between the two shafts is required.
- **Advantages:** Spur gears are easy to find, inexpensive, and efficient.

2. **Parallel helical gears:** The teeth on helical gears are cut at an angle to the face of the gear. When two teeth on a helical gear system engage, the contact starts at one end of the tooth and gradually spreads as the gears rotate, until the two teeth are in full engagement.



Helical gears



Herringbone gears
(or double-helical gears)

This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears. For this reason, helical gears are used in almost all car transmission.

Because of the angle of the teeth on helical gears, they create a thrust load on the gear when they mesh. Devices that use helical gears have bearings that can support this thrust load.

One interesting thing about helical gears is that if the angles of the gear teeth are correct, they can be mounted on perpendicular shafts, adjusting the rotation angle by 90 degrees.

Helical gears to have the following differences from spur gears of the same size:

- Tooth strength is greater because the teeth are longer,
- Greater surface contact on the teeth allows a helical gear to carry more load than a spur gear
- The longer surface of contact reduces the efficiency of a helical gear relative to a spur gear

Rack **and** ***pinion***: (The rack is like a gear whose axis is at infinity mathematically but practically a gear of larger length.)

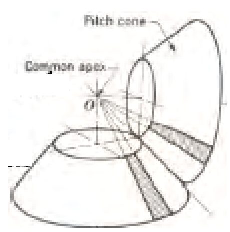
Racks are straight gears that are used to convert rotational motion to translational motion by means of a gear mesh. (They are in theory a gear with an infinite pitch diameter). In theory, the torque and angular velocity of the pinion gear are related to the Force and the velocity of the rack by the radius of the pinion gear, as is shown.

Perhaps the most well-known application of a rack is the rack and pinion steering system used on many cars in the past.

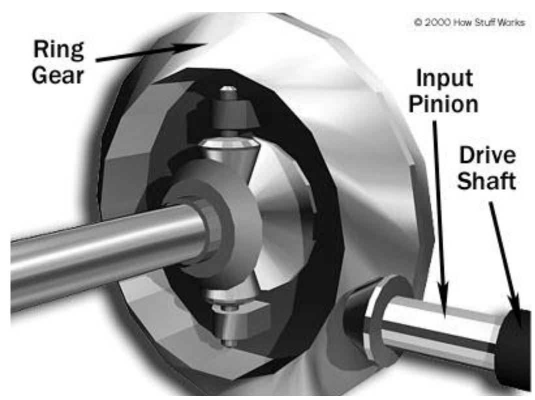
Gears for connecting intersecting shafts: Bevel gears are useful when the direction of a shaft's rotation needs to be changed. They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well.

The teeth on bevel gears can be straight, spiral or hypoid. Straight bevel gear teeth actually have the same problem as straight spur gear teeth, as each tooth engages; it impacts the corresponding tooth all at once.

Just like with spur gears, the solution to this problem is to curve the gear teeth. These spiral teeth engage just like helical teeth: the contact starts at one end of the gear and progressively spreads across the whole tooth.



Straight bevel gears *Spiral bevel* gears

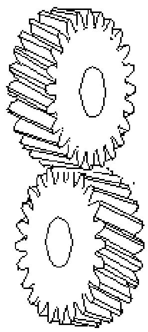


Hypoid gears

On straight and spiral bevel gears, the shafts must be perpendicular to each other, but they must also be in the same plane. The hypoid gear, can engage with the axes in different planes.

This feature is used in many car differentials. The ring gear of the differential and the input pinion gear are both hypoid. This allows the input pinion to be mounted lower than the axis of the ring gear. Figure shows the input pinion engaging the ring gear of the differential. Since the driveshaft of the car is connected to the input pinion, this also lowers the driveshaft. This means that the driveshaft doesn't pass into the passenger compartment of the car as much, making more room for people and cargo.

Neither parallel nor intersecting shafts: Helical gears may be used to mesh two shafts that are not parallel, although they are still primarily use in parallel shaft applications. A special application in which helical gears are used is a crossed gear mesh, in which the two shafts are perpendicular to each other.



Crossed-helical gears

***Worm and worm gear**:* Worm gears are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater.

Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm. This is because the angle on the worm is so shallow that when the gear tries to spin it, the friction between the gear and the worm holds the worm in place.

This feature is useful for machines such as conveyor systems, in which the locking feature can act as a brake for the conveyor when the motor is not turning. One other very interesting usage of worm gears is in the Torsen differential, which is used on some high-performance cars and trucks.

Terms Used in Gears

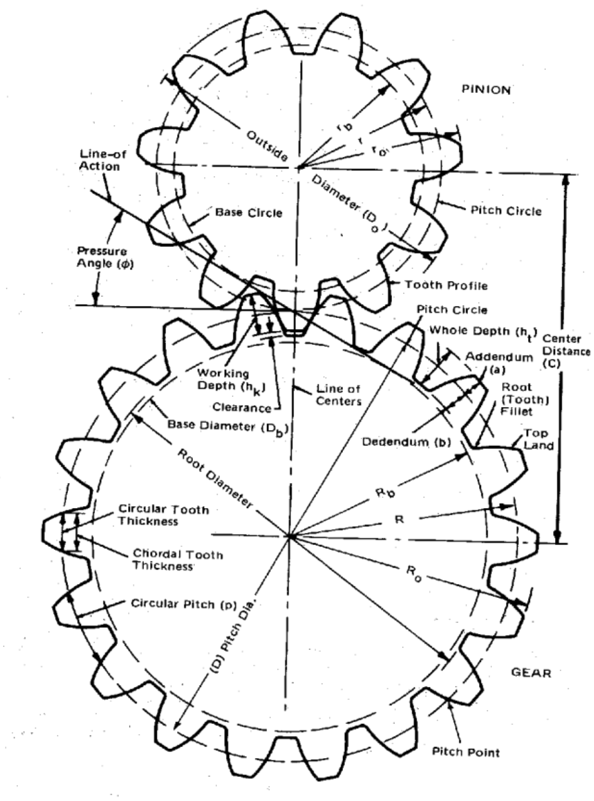
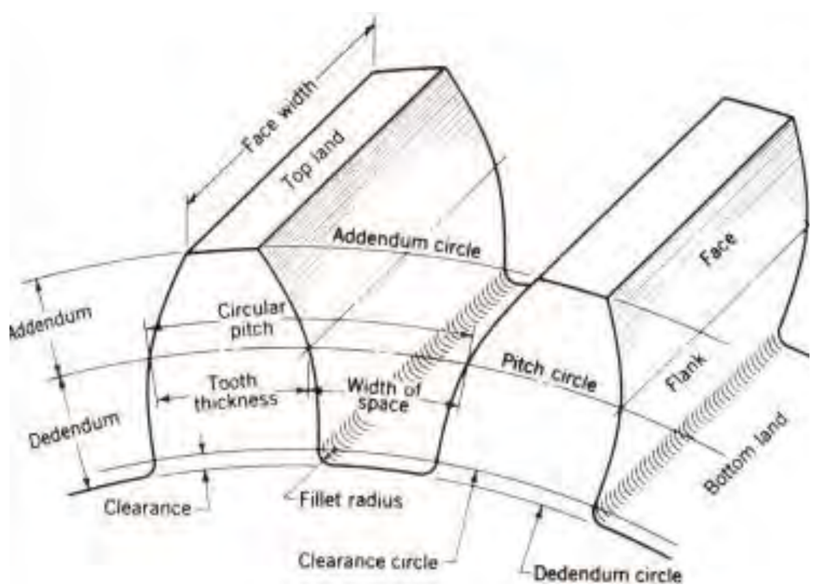


Fig. 4.2. Spur Gear and Pinion pair

Terms used in Gear



Addendum: The radial distance between the Pitch Circle and the top of the teeth.

Dedendum: The radial distance between the bottom of the tooth to pitch circle.

Base Circle: The circle from which is generated the involute curve upon which the tooth profile is based.

Center Distance: The distance between centers of two gears.

Circular Pitch: Millimeter of Pitch Circle circumference per tooth.

Circular Thickness: The thickness of the tooth measured along an arc following the Pitch Circle

Clearance: The distance between the top of a tooth and the bottom of the space into which it fits on the meshing gear.

Contact Ratio: The ratio of the length of the Arc of Action to the Circular Pitch.

Diametral Pitch: Teeth per mm of diameter.

Face: The working surface of a gear tooth, located between the pitch diameter and the top of the tooth.

Face Width: The width of the tooth measured parallel to the gear axis.

Flank: The working surface of a gear tooth, located between the pitch diameter and the bottom of the teeth

Gear: The larger of two meshed gears. If both gears are the same size, they are both called "gears".

Land: The top surface of the tooth.

Line of Action: That line along which the point of contact between gear teeth travels, between the first point of contact and the last.

Module: Millimeter of Pitch Diameter to Teeth.

Pinion: The smaller of two meshed gears.

Pitch Circle: The circle, the radius of which is equal to the distance from the center of the gear to the pitch point.

Diametral pitch: Teeth per millimeter of pitch diameter.

Pitch Point: The point of tangency of the pitch circles of two meshing gears, where the Line of Centers crosses the pitch circles.

Pressure Angle: Angle between the Line of Action and a line perpendicular to the Line of Centers.

Root Circle: The circle that passes through the bottom of the tooth spaces.

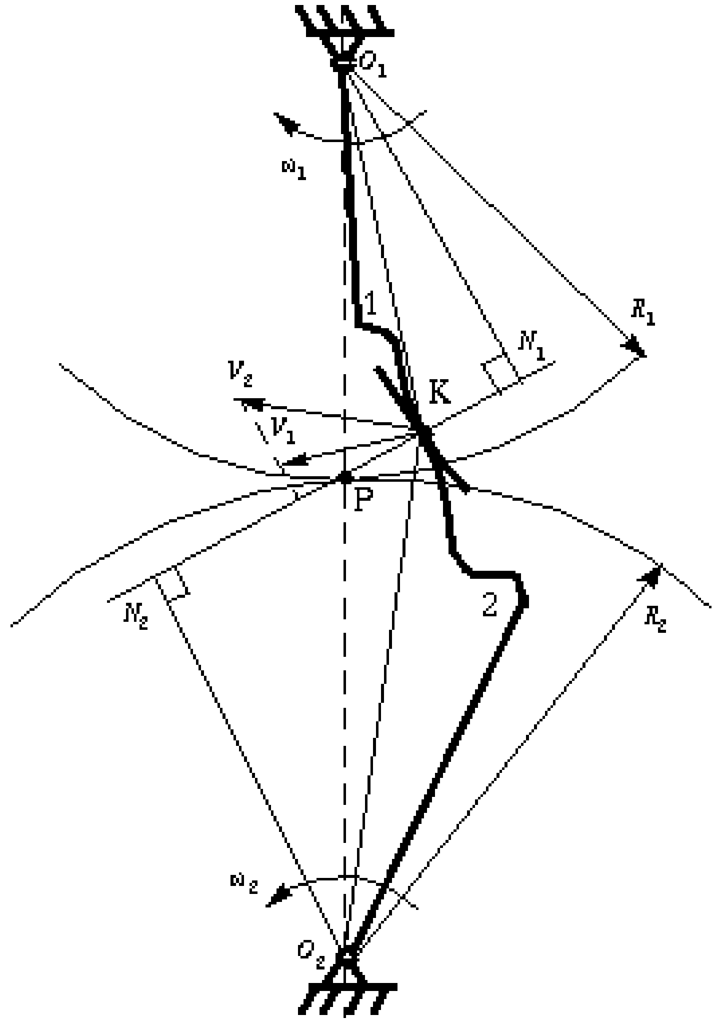
Working Depth: The depth to which a tooth extends into the space between teeth on the mating gear.

Gear-Tooth Action

Fundamental Law of Gear-Tooth Action

Figure shows two mating gear teeth, in which

- Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point *K*.
- N_1N_2 is the common normal of the two profiles.
- N_1 is the foot of the perpendicular from O_1 to N_1N_2
- N_2 is the foot of the perpendicular from O_2 to N_1N_2.



Although the two profiles have different velocities $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ at point *K*, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

$$O_1N_1\,\omega_1 = O_2N_2\,\omega_2$$

or

$$\frac{\omega_1}{\omega_2} = \frac{O_2N_2}{O_1N_1}$$

We notice that the intersection of the tangency N_1N_2 and the line of center O_1O_2 is point *P*, and from the similar triangles,

$$\Delta O_1N_1\,P = \Delta O_2N_2\,P$$

Thus, the relationship between the angular velocities of the driving gear to the driven gear, or **velocity ratio**, of a pair of mating teeth is

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

Point *P* is very important to the velocity ratio, and it is called the **pitch point**. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the **fundamental law of gear-tooth action**.

Constant Velocity Ratio

For a constant velocity ratio, the position of *P* should remain unchanged. In this case, the motion transmission between two gears is equivalent to the motion transmission between two imagined slip-less cylinders with radius R_1 and R_2 or diameter D_1 and D_2. We can get two circles whose centers are at O_1 and O_2, and through pitch point *P*. These two circles are termed **pitch circles**. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

The **fundamental law of gear-tooth action** may now also be stated as follow (for gears with fixed center distance)

A common normal (the line of action) to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point.
Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves, and the relative rotation speed of the gears will be constant(constant velocity ratio).

Conjugate Profiles

To obtain the expected *velocity ratio* of two tooth profiles, the normal line of their profiles must pass through the corresponding pitch point, which is decided by the *velocity ratio*. The two profiles which satisfy this requirement are called **conjugate profiles**. Sometimes, we simply termed the tooth profiles which satisfy the *fundamental law of gear-tooth action* the *conjugate profiles*.

Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the *cycloidal* and *involute* profiles. The involute has important advantages; it is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required when using the involute profile. The most commonly used *conjugate* tooth curve is the *involute curve.*

conjugate action : It is essential for correctly meshing gears, the size of the teeth (the module) must be the same for both the gears.

Another requirement - the shape of teeth necessary for the speed ratio to remain constant during an increment of rotation; this behavior of the contacting surfaces (ie. the teeth flanks) is known as ***conjugate action.***

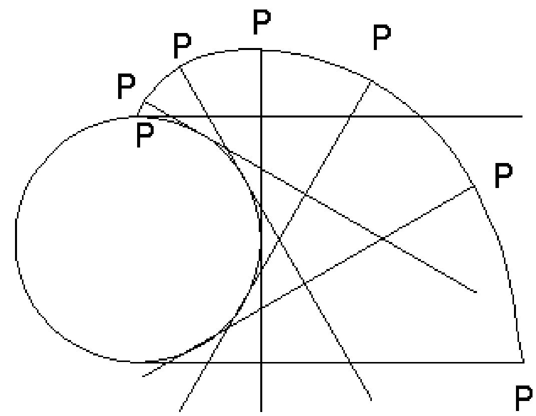
Forms of Teeth

Involute Profile

The following examples are involute spur gears. We use the word *involute* because the contour of gear teeth curves inward. Gears have many terminologies, parameters and principles. One of the important concepts is the *velocity ratio,* which is the ratio of the rotary velocity of the driver gear to that of the driven gears.

Generation of the Involute Curve

The curve most commonly used for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string, which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the **base circle**.



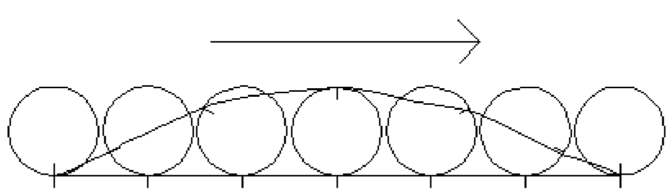
Involute curve

The involute profile of gears has important advantages;

1. It is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required. The most commonly used *conjugate* tooth curve is the *involute curve.*
2. In involute gears, the pressure angle, remains constant between the point of tooth engagement and disengagement. It is necessary for smooth running and less wear of gears.
3. The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively. Thus the involute teeth are easy to manufacture than cycloidal teeth.

In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

Cycloidal profile:



A ***cycloid*** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epi-cycloid***. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypo-cycloid***.

Advantages of Cycloidal gear teeth:

1. Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.

2. In cycloidal gears, the contact takes place between a convex flank and a concave surface, where as in involute gears the convex surfaces are in contact. This condition results in less wear in cycloidal gears as compared to involute gears. However the difference in wear is negligible

3. In cycloidal gears, the interference does not occur at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

S.No.	Involute tooth gears	Cycloid tooth gears
1.	The profile of involute gears is the single curvature.	The profile of cycloidal gears is double curvature i.e. epicycloid and hypocycloid.
2.	The pressure angle from start of engagement of teeth to the end of engagement remains constant, which results into smooth running.	The pressure angle varies from start of engagement to end of engagement, which results into less smooth running.
3.	Manufacturing of involute gears is easy due to single curvature of tooth profile.	Manufacturing of cycloidal gears is difficult due to double curvature of tooth profile.
4.	The involute gears have interference problem.	The cycloidal gears do not have interference problem.
5.	More wear of tooth surface.	Less wear as convex face engages with concave flank.
6.	The strength of involute teeth is less due to radial flanks	The strength of cyloidal teeth is comparatively more due to wider flanks.

Systems of Gear Teeth

The following four systems of gear teeth are commonly used in practice:

1. 14 ½^{O} Composite system
2. 14 ½^{O} Full depth involute system
3. 20^{O} Full depth involute system
4. 20^{O} Stub involute system

The 14½^{O} *composite system* is used for general purpose gears.

It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion.

The teeth are produced by formed milling cutters or hobs.

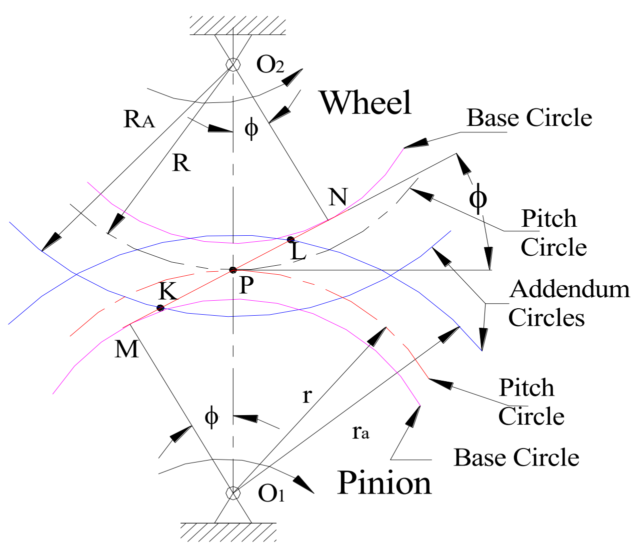
The tooth profile of the 14½^{O} *full depth involute system* was developed using gear hobs for spur and helical gears.

The tooth profile of the 20^{o} *full depth involute system* may be cut by hobs.

The increase of the pressure angle from 14½^{o} to 20^{o} results in a stronger tooth, because the tooth acting as a beam is wider at the base.

The *20^{o} stub involute system* has a strong tooth to take heavy loads.

Length of Path of Contact



Consider a pinion driving wheel as shown in figure. When the pinion rotates in clockwise, the contact between a pair of involute teeth begins at *K* (on the near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at *L* (outer end of the tooth face on the pinion or on the flank near the base circle of wheel).

MN is the common normal at the point of contacts and the common tangent to the base circles. The point *K* is the intersection of the addendum circle of wheel and the common tangent. The point *L* is the intersection of the addendum circle of pinion and common tangent.

The length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion. Thus the length of part of contact is *KL* which is the sum of the parts of path of contacts *KP* and *PL*. Contact length *KP* is called as **path of approach** and contact length *PL* is called as **path of recess**.

$r_a = O_1L$ = Radius of addendum circle of pinion,and

$R_A = O_2K$ = Radius of addendum circle of wheel

$r = O_1P$ = Radius of pitch circle of pinion,

and $R = O_2P$ = Radius of pitch circle of wheel.

Radius of the base circle of pinion = $O_1M = O_1P \cos\phi = r \cos\phi$

and radius of the base circle of wheel = $O2N = O2P \cos\phi = R \cos\phi$

From right angle triangle O_2KN

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2}$$
$$= \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

Path of approach: *KP*

$$KP = KN - PN$$
$$= \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angle triangle O_1ML

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2}$$
$$= \sqrt{(r_a)^2 - r^2\cos^2\phi}$$
$$MP = O_1P\sin\phi = r\sin\phi$$

Path of recess: *PL*

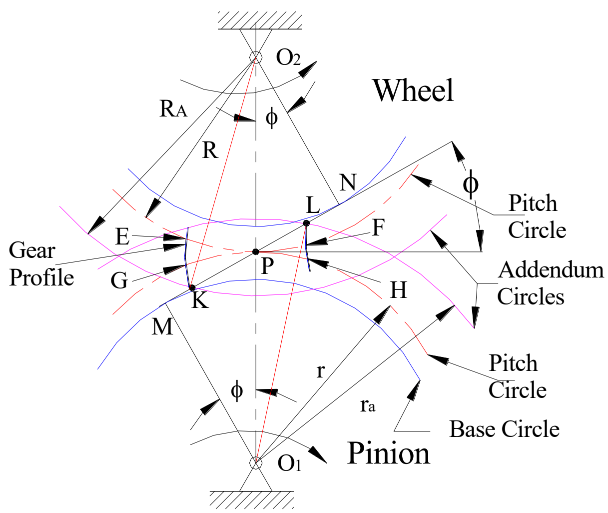
$$PL = ML - MP$$
$$= \sqrt{(r_a)^2 - r^2\cos^2\phi} - r\sin\phi$$

Length of path of contact = *KL*

$$KL = KP + PL$$
$$= \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_a)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

Length of Arc of Contact

Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Figure, the arc of contact is *EPF* or *GPH*.



Considering the arc of contact *GPH.*

The arc *GP* is known as *arc of approach* and the arc *PH* is called *arc of recess*. The angles subtended by these arcs at O_1 are called *angle of approach* and *angle of recess* respectively.

Length of arc of approach = arc $GP = \dfrac{Lenghtof\ pathof\ approach}{\cos\phi} = \dfrac{KP}{\cos\phi}$

Length of arc of recess = arc $PH = \dfrac{Lenghtof\ pathof\ recess}{\cos\phi} = \dfrac{PL}{\cos\phi}$

Length of arc contact = arc *GPH* = *arc GP* + *arc PH* $= \dfrac{KP}{\cos\phi} + \dfrac{PL}{\cos\phi} = \dfrac{KL}{\cos\phi} = \dfrac{Lengthof\ pathof\ contact}{\cos\phi}$

Contact Ratio (or) Number of Pairs of Teeth in Contact

The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

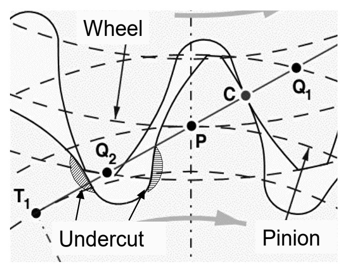
Mathematically, $Contatratio = \frac{Length\,of\,the\,arc\,of\,\,contact}{P_C}$

Where: $P_C = Circular\ pitch = \pi \times m$ and m = Module.

Interference in Involute Gears

The tooth tip of the pinion will then undercut the tooth on the wheel at the root and damages part of the involute profile. This effect is known as *interference*, and occurs when the teeth are being cut and weakens the tooth at its root.

In general, the phenomenon, when the tip of tooth undercuts the root on its mating gear is known as interference.



Similarly, if the radius of the addendum circles of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called interference points. The interference may only be prevented, if the point of contact between the two teeth is always on the involute profiles and if the addendum circles of the two mating gears cut the common tangent to the base circles at the points of tangency.

1. Height of the teeth may be reduced.
2. Under cut of the radial flank of the pinion.
3. Centre distance may be increased. It leads to increase in pressure angle.

Minimum number of teeth on the pinion avoid Interference 't'

$$t = \frac{2a_p}{\left[\left(1 + G(G+2)\sin^2\phi\right)^{\frac{1}{2}} - 1\right]}$$

Minimum number of teeth on the wheel avoid Interference 'T'

$$T = \frac{2a_w}{\left[\left(1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi\right)^{\frac{1}{2}} - 1\right]}$$

Backlash:

The gap between the non-drive face of the pinion tooth and the adjacent wheel tooth is known as ***backlash***. Backlash is the error in motion that occurs when gears change direction. The term "backlash" can also be used to refer to the size of the gap, not just the phenomenon it causes; thus, one could speak of a pair of gears as having, for example, "0.1 mm of backlash."

Practise problems:

1) Two gears in mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 teeth while the pinion has 23 teeth. If the addenda on pinion and gear wheel are equal to one module (1*m*), find
a. The number of pairs of teeth in contact and
b. The angle of action of the pinion and the gear wheel.

Solution:

Data: t=23; T=57; addendum = 1*m= 8mm* and f=20°

$$Pitch\ circle\ radius\ of\ the\ pinion = r = \frac{mt}{2} = \frac{8\times 23}{2} = 92mm$$

$$Pitch\ circle\ radius\ of\ the\ gear = R = \frac{mT}{2} = \frac{8\times 57}{2} = 228mm$$

$$Addendum\ circle\ radius\ of\ the\ pinion = r_a = r + addendum$$

$$r_a = 92 + 8 = 100mm$$

$$Addendum\ circle\ radius\ of\ the\ gear = R_A = R + addendum$$

$$R_A = 228 + 8 = 236mm$$

$$Length\ of\ path\ of\ contact = KL = KP + PL$$

$$= \sqrt{(R_A)^2 - R^2\cos^2\varphi} + \sqrt{(r_a)^2 - r^2\cos^2\varphi} - (R + r)\sin\varphi$$

$$= \sqrt{(236)^2 - (228)^2\cos^2 20} + \sqrt{(100)^2 - (92)^2\cos^2 20}$$

$$-(228 + 92)\sin 20$$

$$= 39.76mm$$

$$Length\ of\ arc\ of\ contact = \frac{Length\ of\ path\ of\ contact}{\cos\varphi}$$

$$= \frac{39.76}{\cos 20} = 42.31mm$$

$$Number\ of\ pairs\ of\ teeth\ in\ contact = \frac{Length\ of\ arc\ of\ contact}{circular\ pitch}$$

$$= \frac{Length\ of\ arc\ of\ contact}{p_c} = \frac{42.31}{\pi\, m} = 1.684 \approx 2$$

$$Angle\ of\ action\ of\ gear\ wheel = \frac{Length\ of\ arc\ of\ contact}{2\pi \times R} \times 360^o$$

$$= \frac{42.31}{2\pi \times 228} \times 360 = 10.637^o$$

$$Angle\ of\ action\ of\ pinion = \frac{Length\ of\ arc\ of\ contact}{2\pi \times r} \times 360^o$$

$$= \frac{42.31}{2\pi \times 92} \times 360 = 26.36^o$$

2.) Two gear wheels mesh externally and are to give a velocity ratio of 3 to 1. The teeth are of involute form ; module = 6 mm, addendum = one module, pressure angle = 20°. The pinion rotates at 90 r.p.m. Determine: **1.** The number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel, **2.** The length of path and arc of contact, **3.**The number of pairs of teeth in contact, and **4.** The maximum velocity of sliding.

Solution:

Given : $G = T / t = 3$; $m = 6$ mm ; $A_P = A_W = 1$ module $= 6$ mm ; $\phi = 20°$; $N_1 = 90$ r.p.m. or $\omega_1 = 2\pi \times 90 / 60 = 9.43$ rad/s

We know that number of teeth on the pinion to avoid interference,

$$t = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1} = \frac{2 \times 6}{\sqrt{1 + 3(3+2)\sin^2 20°} - 1}$$

$$= 18.2 \text{ say } 19$$

and corresponding number of teeth on the wheel,

$$T = G.t = 3 \times 19 = 57$$

Length of path and arc of contact:

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 6 \times 19/2 = 57 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum on pinion } (A_P) = 57 + 6 = 63 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T / 2 = 6 \times 57 / 2 = 171 \text{ mm}$$

∴ Radius of addendum circle of wheel,

$$R_A = R + \text{Addendum on wheel } (A_W) = 171 + 6 = 177 \text{ mm}$$

We know that the path of approach (*i.e.* path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(177)^2 - (171)^2\cos^2 20°} - 171\sin 20° = 74.2 - 58.5 = 15.7 \text{ mm}$$

and the path of recess (*i.e.* path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$
$$= \sqrt{(63)^2 - (57)^2 \cos^2 20^\circ} - 57 \sin 20^\circ = 33.17 - 19.5 = 13.67 \text{ mm}$$

∴ Length of path of contact,

$$KL = KP + PL = 15.7 + 13.67 = 29.37 \text{ mm}$$

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos \phi} = \frac{29.37}{\cos 20^\circ} = 31.25 \text{ mm}$$

Number of pairs of Teeth in contact:

We know that circular pitch,

$$p_c = \pi \times m = \pi \times 6 = 18.852 \text{ mm}$$

∴ Number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{p_c} = \frac{31.25}{18.852} = 1.66$$

Maximum velocity of sliding:

Let ω_2 = Angular speed of wheel in rad/s.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t}$ or $\omega_2 = \omega_1 \times \frac{t}{T} = 9.43 \times \frac{19}{57} = 3.14$ rad/s

∴ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\ KP$$
$$= (9.43 + 3.14)\ 15.7 = 197.35 \text{ mm/s}$$

GOVERNORS

5.1 Governors

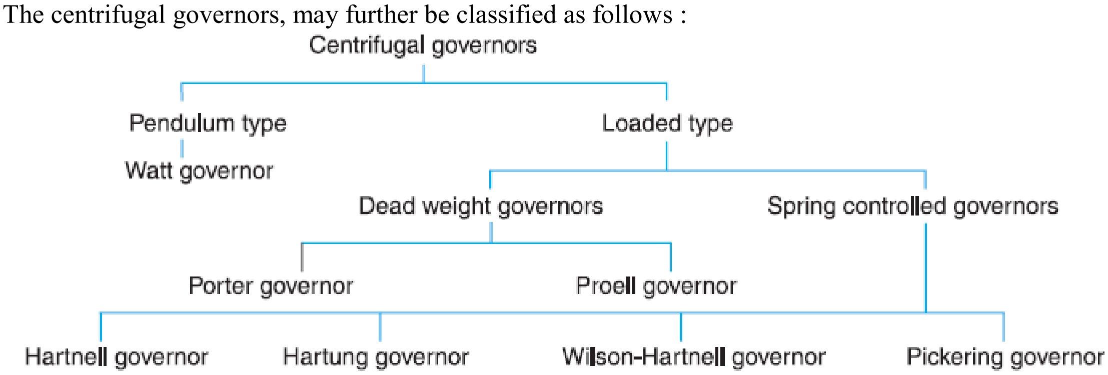
The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g. when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. A little consideration will show, that when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid; **conversely,** when the load decreases, the engine speed increases and the governor decreases the supply of working fluid. We can observe that the function of a flywheel in an engine is entirely different from that of a governor. It controls the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation. It does not control the speed variations caused by a varying load. The varying demand for power is met by the governor regulating the supply of working fluid.

5.2 Types of Governors

The governors may, broadly, be classified as

1. Centrifugal governors, and
2. Inertia governors.

The centrifugal governors, may further be classified as follows :

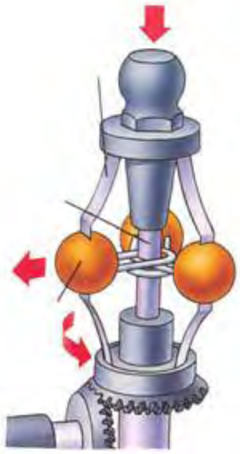


5.3 Centrifugal Governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the **controlling force**. Centrifugal governor consists of two balls of equal mass, which are attached to the arms as shown in Fig. 5.1. These balls are known as **governor balls or fly balls.** The balls revolve with a spindle, which is driven by the engine through bevel gears.

The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops S, S are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards.

The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced. The controlling force is provided either by the action of gravity as in Watt governor or by a spring as in case of Hartnell governor.




A governor controls engine speed. As it rotates, the weights swing outwards, pulling down a spindle that reduces the fuel supply at high speed. When the balls rotate at uniform speed, controlling force is equal to the centrifugal force and they balance each other.

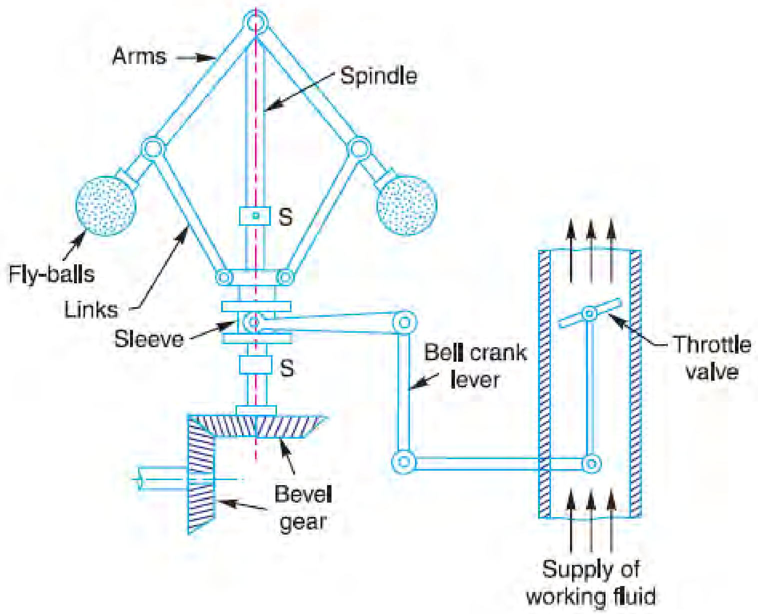


Fig. 5.1.

Terms Used in Governors

The following terms used in governors are important from the subject point of view ;

1. Height of a governor. It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by h.

2. Equilibrium speed. It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. Mean equilibrium speed. It is the speed at the mean position of the balls or the sleeve.

4. Maximum and minimum equilibrium speeds. The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the minimum
equilibrium speeds.

5. Sleeve lift. It is the vertical distance which the sleeve travels due to change in equilibrium speed.

5.4 Watt Governor

The simplest form of a centrifugal governor is a Watt governor, as shown in Fig. 5.2. It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways :

1. The pivot P, may be on the spindle axis as shown in Fig. 5.2 (a).
2. The pivot P, may be offset from the spindle axis and the arms when produced intersect at O, as shown in Fig. 5.2 (b).
3. The pivot P, may be offset, but the arms cross the axis at O, as shown in Fig. 5.2 (c).

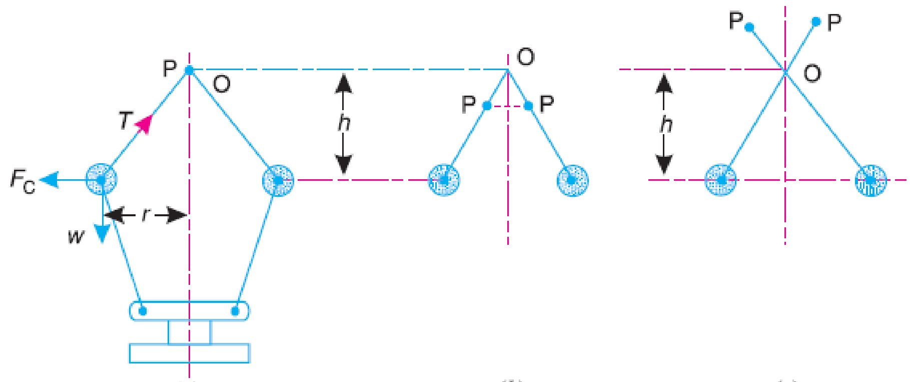


Fig. 5.2.

Let m = Mass of the ball in kg,

w = Weight of the ball in newtons = $m.g$,

T = Tension in the arm in newtons,

ω = Angular velocity of the arm and ball about the spindle axis in rad/s,

r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$, and

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of

1. the centrifugal force (FC) acting on the ball,
2. the tension (T) in the arm, and
3. the weight (w) of the ball.

Taking moments about point O, we have

$$F_C \times h = w \times r = m.g.r$$

$$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g/\omega^2 \quad \text{. . . (i)}$$

When g is expressed in m/s2 and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

$$\omega = 2\pi N/60$$

$$\therefore \quad h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres} \quad \text{. . . } (\because g = 9.81 \text{ m/s}^2) \quad \text{. . . (ii)}$$

Note : We see from the above expression that the height of a governor h, is inversely proportional to N2.Therefore at high speeds, the value of h is small. At such speeds, the change in the value of h corresponding to a small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply. This governor may only work satisfactorily at relatively low speeds i.e.from 60 to 80 r.p.m.

Practise Problem:

(1.) Calculate the vertical height of a Watt governor when it rotates at 60 r.p.m. Also find the change in vertical height when its speed increases to 61 r.p.m.

Solution:

Given :

N_1 = 60 r.p.m.

N_2 = 61 r.p.m.

Initial height

We know that initial height,

$$h_1 = \frac{895}{(N_1)^2} = \frac{895}{(60)^2} = 0.248 \text{ m}$$

Change in vertical height

We know that final height,

$$h_2 = \frac{895}{(N_2)^2} = \frac{895}{(61)^2} = 0.24 \text{ m}$$

Change in vertical height = $h_1 - h_2$ = 0.248 – 0.24 = 0.008 m = 8 mm

5.5 Porter Governor

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. 5.3 (a). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level. Consider the forces acting on one-half of the governor as shown in Fig. 6.3 (b).

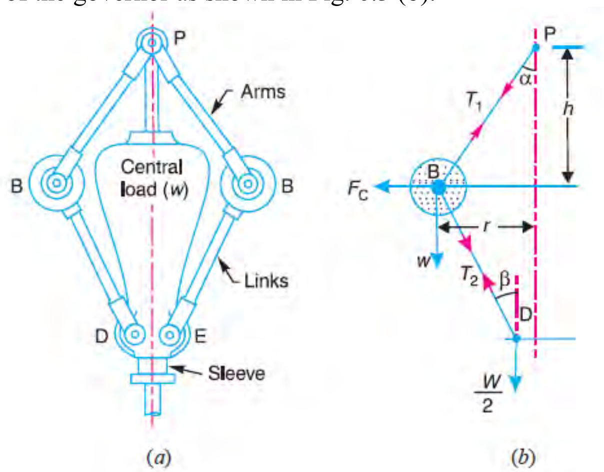


Fig. 5.3.

Let m = Mass of each ball in kg,

w = Weight of each ball in newtons = m.g,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = M.g,

r = Radius of rotation in metres,

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω = Angular speed of the balls in rad/s = $2\pi N/60$ rad/s,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$,

T_1 = Force in the arm in newtons,

T_2 = Force in the link in newtons,

α = Angle of inclination of the arm (or upper link) to the vertical, and

β = Angle of inclination of the link (or lower link) to the vertical.

Though there are several ways of determining the relation between the height of the governor (h) and the angular speed of the balls (ω), yet the following two methods are important from the subject point of view :

1. Method of resolution of forces ; and

2. Instantaneous centre method.

1. Method of resolution of forces

Considering the equilibrium of the forces acting at D, we have

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2}$$

$$T_2 = \frac{M.g}{2\cos\beta}$$

Again, considering the equilibrium of the forces acting on B. The point B is in equilibrium under the action of the following forces, as shown in Fig. 6.3 (b).

(i) The weight of ball (w = m.g),

(ii) The centrifugal force (FC),

(iii) The tension in the arm (T1), and

(iv) The tension in the link (T2).

Resolving the forces vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g$$

$$\left(\because\ T_2 \cos\beta = \frac{M.g}{2}\right)$$

Resolving the forces horizontally,

$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C \qquad \ldots\left(\because\ T_2 = \frac{M.g}{2\cos\beta}\right)$$

$$T_1 \sin\alpha + \frac{M.g}{2} \times \tan\beta = F_C$$

$$T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta$$

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

$$\left(\frac{M.g}{2} + m.g\right)\tan\alpha = F_C - \frac{M.g}{2} \times \tan\beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

Substituting $\frac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \frac{r}{h}$, we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q \qquad \ldots(\because F_C = m.\omega^2.r)$$

$$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$$h = \left[m.g + \frac{M.g}{2}(1+q) \right] \frac{1}{m.\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

$$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q) \right] \frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$$\left(\frac{2\pi N}{60} \right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h} \left(\frac{60}{2\pi} \right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

Notes : 1. When the length of arms are equal to the length of links and the points P and D lie on the same vertical line, then

$$\tan\alpha = \tan\beta \qquad \text{or} \qquad q = \tan\alpha / \tan\beta = 1$$

Therefore the above equation becomes

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}$$

2. When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.
If F = Frictional force acting on the sleeve in newtons, then the above equations get reduced as

$$N^2 = \frac{m.g + \left(\frac{M.g \pm F}{2} \right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{m.g + (M.g \pm F)}{m.g} \times \frac{895}{h} \qquad \ldots \text{(When } q = 1\text{)}$$

The + sign is used when the sleeve moves upwards or the governor speed increases and negative sign is used when the sleeve moves downwards or the governor speed decreases.

2. Instantaneous centre method

In this method, equilibrium of the forces acting on the link BD are considered. The instantaneous centre I lies at the point of intersection of PB produced and a line through D perpendicular to the spindle axis, as shown in Fig. 6.4. Taking moments about the point I,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID$$

$$\therefore \qquad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2} \times \frac{ID}{BM}$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2} \left(\frac{IM + MD}{BM} \right)$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2} \left(\frac{IM}{BM} + \frac{MD}{BM} \right)$$

$$= m.g \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)$$

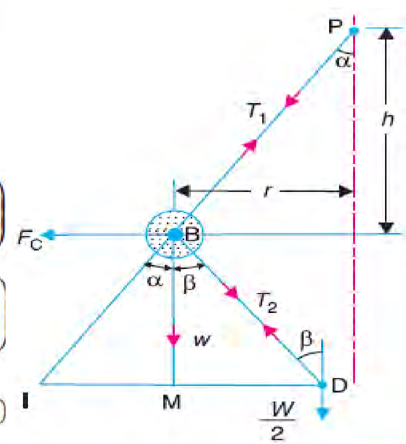


Fig. 6.4.

$$\ldots\left(\because \frac{IM}{BM} = \tan\alpha, \text{ and } \frac{MD}{BM} = \tan\beta\right)$$

Dividing throughout by tan α,

$$\frac{F_C}{\tan\alpha} = m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) = m.g + \frac{M.g}{2}(1+q) \qquad \ldots\left(\because q = \frac{\tan\beta}{\tan\alpha}\right)$$

We know that $F_C = m.\omega^2.r$, and $\tan\alpha = \frac{r}{h}$

$$\therefore\; m\omega^2.r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$$

or

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

. . . (Same as before)

When $\tan\alpha = \tan\beta$ or $q = 1$, then

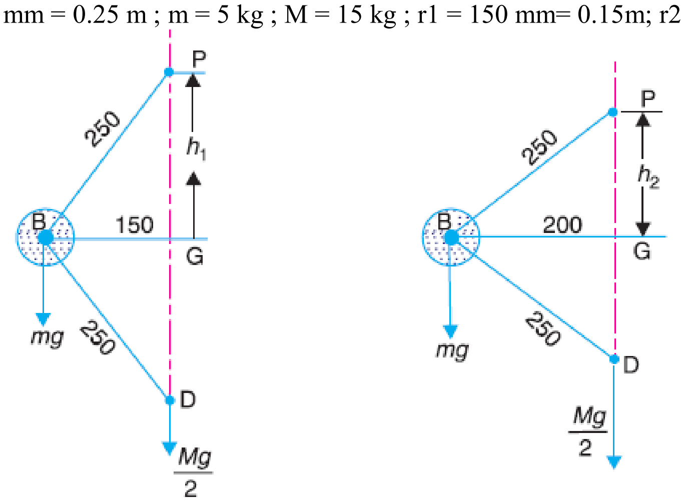
$$h = \frac{m+M}{m} \times \frac{g}{\omega^2}$$

Practise Problems:

(2.) Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.

Solution:

Given: BP = BD = 250 mm = 0.25 m ; m = 5 kg ; M = 15 kg ; r1 = 150 mm= 0.15m; r2 = 200 mm = 0.2 m



(*a*) Minimum position. (*b*) Maximum position.

Fig. 6.5.

The minimum and maximum positions of the governor are shown in Fig. 6.5 (a) and (b) respectively.

Minimum speed when r1 = BG = 0.15 m

Let N1 = Minimum speed.

From Fig. 6.5 (a), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.15)^2} = 0.2 \text{ m}$$

We know that

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+15}{5} \times \frac{895}{0.2} = 17\,900$$

N_1 = 133.8 r.p.m.

Maximum speed when r_2 = BG = 0.2 m
Let N_2 = Maximum speed.
From Fig. 6.5 (b), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.2)^2} = 0.15 \text{ m}$$

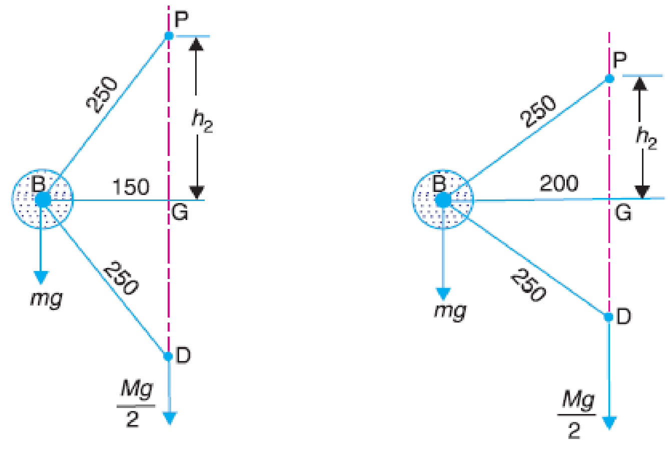
We know that

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+15}{5} \times \frac{895}{0.15} = 23\ 867$$

N_2 = 154.5 r.p.m.
We know that range of speed = N2 – N1 = 154.4 – 133.8 = 20.7 r.p.m.

(3.) The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.

Solution:
Given : BP = BD = 250 mm ; m = 5 kg ; M = 30 kg ; r1 = 150 mm ; r2 = 200 mm
First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 6.6 (a) and (b) respectively.
Let N1 = Minimum speed when r1 = BG = 150 mm, and
N2 = Maximum speed when r2 = BG = 200 mm.



(*a*) Minimum position. (*b*) Maximum position.

Fig. 6.6.

Speed range of the governor
From Fig. 6.6 (a), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

We know that

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+30}{5} \times \frac{895}{0.2} = 31\ 325$$

N1 = 177 r.p.m.
From Fig. 6.6 (b), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

We know that

$$(N_2)^2 = \frac{m + M}{m} \times \frac{895}{h_2} = \frac{5 + 30}{5} \times \frac{895}{0.15} = 41\,767$$

N2 = 204.4 r.p.m.

We know that speed range of the governor = N2 – N1 = 204.4 – 177 = 27.4 r.p.m.

Speed range when friction at the sleeve is equivalent of 20 N of load (i.e. when F = 20 N)

We know that when the sleeve moves downwards, the friction force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2} = 29\,500$$

N1 = 172 r.p.m.

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m\,.\,g} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{895}{0.15} = 44\,200$$

N2 = 210 r.p.m.

We know that speed range of the governor = N2 – N1 = 210 – 172 = 38 r.p.m.

(4.) A Porter governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 mm. Determine the range of speed of the governor.

Solution: Given : BP = BD = 250 mm ; DH = 30 mm ; m = 5 kg ; M = 50 kg ; r_1 = 150 mm ; r_2 = 200 mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 6.8 (a) and (b) respectively.

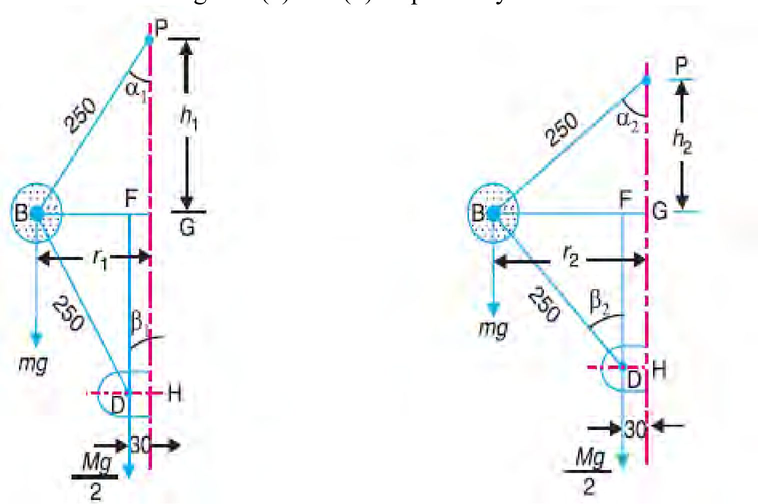


Fig. 6.8.

Let N1 = Minimum speed when r_1 = BG = 150 mm ; and

N_2 = Maximum speed when r_2 = BG = 200 mm.
From Fig. 18.8 (a), we find that height of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$BF = BG - FG = 150 - 30 = 120 \text{ mm} \qquad \ldots (\because FG = DH)$$

$$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (120)^2} = 219 \text{ mm}$$

$$\tan \alpha_1 = BG/PG = 150 / 200 = 0.75$$

$$\tan \beta_1 = BF/DF = 120/219 = 0.548$$

$$q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.548}{0.75} = 0.731$$

$$(N_1)^2 = \frac{m + \frac{M}{2}(1 + q_1)}{m} \times \frac{895}{h_1} = \frac{5 + \frac{50}{2}(1 + 0.731)}{5} \times \frac{895}{0.2} = 43\,206$$

$$N_1 = 208 \text{ r.p.m.}$$

From Fig. 6.8(b), we find that height of the governor,

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

$$BF = BG - FG = 200 - 30 = 170 \text{ mm}$$

$$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (170)^2} = 183 \text{ mm}$$

$$\tan \alpha_2 = BG/PG = 200/150 = 1.333$$

$$\tan \beta_2 = BF/DF = 170/183 = 0.93$$

$$q_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{0.93}{1.333} = 0.7$$

$$(N_2)^2 = \frac{m + \frac{M}{2}(1 + q_2)}{m} \times \frac{895}{h_2} = \frac{5 + \frac{50}{2}(1 + 0.7)}{5} \times \frac{895}{0.15} = 56\,683$$

$$N_2 = 238 \text{ r.p.m.}$$

We know that range of speed = $N_2 - N_1$ = 238 – 208 = 30 r.p.m.

Sensitiveness of Governors

Consider two governors A and B running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor A is greater than the lift of the sleeve of governor B. It is then said that the governor A is more sensitive than the governor B.

In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is the sensitiveness of the governor. It may also be stated in another way that for a given lift of the sleeve, the sensitiveness of the governor increases as the speed range decreases. This definition of sensitiveness may be quite satisfactory when the governor is considered as an independent mechanism. But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a fraction as possible of the mean equilibrium speed. The actual displacement of the sleeve is immaterial, provided that it is sufficient to change the energy supplied to the engine by the required amount.

For this reason, the sensitiveness is defined as the ***ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.***

Let

$N1$ = Minimum equilibrium speed,

$N2$ = Maximum equilibrium speed, and

$$N = \text{Mean equilibrium speed} = \frac{N_1 + N_2}{2}$$

Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

Hunting

A governor is said to be ***hunt*** if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place. For example, when the load on the engine increases, the engine speed decreases and, if the governor is very sensitive, the governor sleeve immediately falls to its lowest position. This will result in the opening of the control valve wide which will supply the fuel to the engine in excess of its requirement so that the engine speed rapidly increases again and the governor sleeve rises to its highest position. Due to this movement of the sleeve, the control valve will cut off the fuel supply to the engine and thus the engine speed begins to fall once again. This cycle is repeated indefinitely. Such a governor may admit either the maximum or the minimum amount of fuel. The effect of this will be to cause wide fluctuations in the engine speed or in other words, the engine will hunt.

Isochronous Governors

A governor is said to be ***isochronous*** when the equilibrium speed is constant (*i.e.* range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

Let us consider the case of a Porter governor running at speeds $N1$ and $N2$ r.p.m.

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1}$$

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2}$$

For isochronism, range of speed should be zero *i.e.* $N2 - N1 = 0$ or $N2 = N1$. Therefore from equations above, $h1 = h2$, which is impossible in case of a Porter governor. Hence a ***Porter governor cannot be isochronous.***

Note : The isochronous governor is not of practical use because the sleeve will move to one of its extreme positions immediately the speed deviates from the isochronous speed.

Stability of Governors

A governor is said to be ***stable*** when for every speed within the working range there is a definite configuration *i.e.* there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

Note : A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

Effort and Power of a Governor

The ***effort of a governor*** is the mean force exerted at the sleeve for a given percentage change of speed (or lift of the sleeve). It may be noted that when the governor is running steadily, there is no force at the sleeve. But, when the speed changes, there is a resistance at the sleeve which opposes its motion. It is assumed that this resistance which is equal to the effort, varies uniformly from a maximum value to zero while the governor moves into its new position of equilibrium.

The ***power of a governor*** is the work done at the sleeve for a given percentage change of speed.

It is the product of the mean value of the effort and the distance through which the sleeve moves.
Mathematically,
Power = Mean effort × lift of sleeve

The effort and power of a Porter governor may be determined as discussed below.
Let N = Equilibrium speed and
c = Percentage increase in speed.
Increase in speed = $c.N$
and increased speed = $N + c.N = N(1 + c)$

P = Mean force exerted on the sleeve during the increase in speed or the effort of the governor.

$$P = \frac{(M_1 - M)\,g}{2} = \frac{(m + M)\,[(1 + c)^2 - 1]\,g}{2}$$

$$= \frac{(m + M)\,[1 + c^2 + 2c - 1]\,g}{2} = c\,(m + M)\,g$$

If F is the frictional force (in newtons) at the sleeve, then

$$P = c\,(m.g + M.g \pm F)$$

We have already discussed that the power of a governor is the product of the governor effort and the lift of the sleeve.
Let
x = Lift of the sleeve.

Governor power = $P \times x$

$$\frac{4c^2}{1+2c}\left[m + \frac{M}{2}(1+q)\right]g.h$$