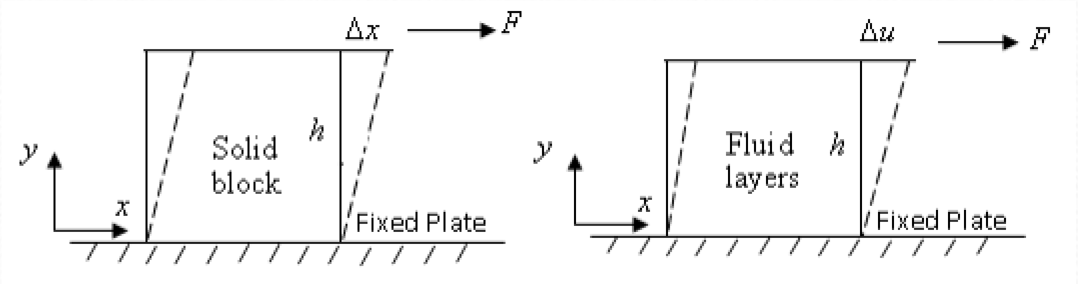
Introduction to Fluid Mechanics

Raman Gahlaut
(Assistant Professor)
Mechanical Engineering Department
Shobhit Institute of Engineering & Technology
(Deemed to be University) MEERUT - 250110

Fluid Definition

- Mechanics is the oldest physical science that deals with both stationery and moving boundaries under the influence of forces. The branch of the mechanics that deals with bodies at rest is called statics while the branch that deals with bodies in motion is called dynamics.
- Fluid Mechanics is the science that deals with behavior of fluids at rest (fluid statics) or in motion (fluid dynamics) and the interaction of fluids with solids or other fluids at the boundaries.
- A substance in liquid / gas phase is referred as 'fluid'. Distinction between a solid & a fluid is made on the basis of substance's ability to resist an applied shear (tangential) stress that tends to change its shape. A solid can resist an applied shear by deforming its shape whereas a fluid deforms continuously under the influence of shear stress, no matter how small is its shape. In solids, stress is proportional to strain, but in fluids, stress is proportional to 'strain rate'.



Fluid as Continuum

- Fluids are aggregations of molecules; widely spaced for a gas and closely spaced for liquids. Distance between the molecules is very large compared to the molecular diameter. The number of molecules involved is immense and the separation between them is normally negligible. Under these conditions, fluid can be treated as continuum and the properties at any point can be treated as bulk behavior of the fluids.
- For the continuum model to be valid, the smallest sample of matter of practical interest must contain a large number of molecules so that meaningful averages can be calculated. In the case of air at sea-level conditions, a volume of 10^{-9} mm^3 contains 3×10^7 molecules. In engineering sense, this volume is quite small, so the continuum hypothesis is valid.
- In certain cases, such as, very-high-altitude flight, the molecular spacing becomes so large that a small volume contains only few molecules and the continuum model fails. For all situations in these lectures, the continuum model will be valid.

Properties of Fluid

Any characteristic of a system is called property. It may either be intensive (mass independent) or extensive (that depends on size of system). The state of a system is described by its properties. The number of properties required to fix the state of the system is given by state postulates. Most common properties of the fluid are:

Pressure (p) : It is the normal force exerted by a fluid per unit area. More details will be available in the subsequent lecture. In SI system the unit and dimension of pressure can be written as, N/m2 and $ML^{-1}T^{-2}$, respectively.

Density (ρ) : It is defined as mass per unit volume of the substance.

$$\left(\rho = \frac{\text{mass}}{\text{volume}}\right)$$

specific weight(γ): It is the weight of the substance per unit volume.

specific weight $\gamma = \rho g$

Specific Gravity(sg): Specific gravity of a fluid is defined as the density of the fluid with respect to the density of standard fluid.

(sg)= Density of fluid/Density of standard fluid

Properties of Fluid

Relative Density (RD) : It is defined as the density of one substance with re to the density of the other substance.

$$RD=\rho_1/\rho_2$$

Temperature (T): It is the measure of hotness and coldness of a system. In thermodynamic sense, it is the measure of internal energy of a system. Many a times, the temperature is expressed in centigrade scale (°C) where the freezing and boiling point of water is taken as 0°C and 100°C, respectively. In SI system, the temperature is expressed in terms of absolute value in Kelvin scale (K = °C+ 273).

Viscosity(μ): When two solid bodies in contact, move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. The property that represents the internal resistance of a fluid to motion (i.e. fluidity) is called as viscosity.

$$\tau = \mu \frac{du}{dy}$$

Cont…

Coefficient of compressibility/Bulk modulus(Ev): It is the property of that f
that represents the variation of density with pressure at constant temperature. Mathematically, it is represented as,

$$E_v = -\forall\left(\frac{\partial p}{\partial \forall}\right)_T = \rho\left(\frac{\partial \rho}{\partial T}\right)_T$$

In terms of finite changes, it is approximated as,

$$E_v = \frac{(\Delta\forall/\forall)}{\Delta T} = -\frac{(\Delta\rho/\rho)}{\Delta T}$$

It can be shown easily that Ev for an ideal gas at a temperature p is equal to its absolute pressure (N/m-2).

Coefficient of volume expansion(β): It is the property of that fluid that represents the variation of density with temperature at constant pressure. Mathematically, it is represented as,

$$\beta = \frac{1}{\forall}\left(\frac{\partial \forall}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

In terms of finite changes, it is approximated as,

$$\beta = \frac{(\Delta\forall/\forall)}{\Delta T} = -\frac{(\Delta\rho/\rho)}{\Delta T}$$

Specific Heat

Specific heats: It is the amount of energy required for a unit mass of a fluid for unit rise in temperature. Since the pressure, temperature and density of a gas are interrelated, the amount of heat required to raise the temperature from 1 2 T T to depends on whether the gas is allowed to expand during the process so that the energy supplied is used in doing the work instead of raising the temperature. For a given gas, two specific heats are defined corresponding to the two extreme conditions of constant volume and constant pressure.

(a) Specific heat at constant volume (Cv)

(b) Specific heat at constant pressure (cp)

The following relation holds good for the specific heat at constant volume and constant pressure. For air ; $c_p = 1.005$ KJ/kg.K $c_v = 0.718$ KJ/kg. K

$$c_p - c_v = R; \quad c_p = \frac{\gamma R}{\gamma - 1}; \quad c_v = \frac{R}{\gamma - 1}$$

Sonic Speed

Speed of sound (c): An important consequence of compressibility of the fluid is that the disturbances introduced at some point in the fluid propagate at finite velocity. The velocity at which these disturbances propagate is known as "acoustic velocity/speed of sound". Mathematically, it is represented as below,

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{E_v}{\rho}}$$

In an isothermal process,

$$E_v = p \Rightarrow c = \sqrt{\frac{p}{\rho}}$$

$$c = \sqrt{RT} \quad \text{(for an ideal gas medium)}$$

In isentropic process,

$$E_v = \gamma p \Rightarrow c = \sqrt{\frac{\gamma p}{\rho}}$$

$$c = \sqrt{\gamma RT} \quad \text{(for an ideal gas medium)}$$

Vapour Pressure & Surface Tension

Vapour pressure (pv) : It is defined as the pressure exerted by its vapour in phase equilibrium with its liquid at a given temperature. For a pure substance, it is same as the saturation pressure. In a fluid motion, if the pressure at some location is lower than the vapour pressure, bubbles start forming. This phenomenon is called as cavitation because they form cavities in the liquid.

Surface Tension (σ): When a liquid and gas or two immiscible liquids are in contact, an unbalanced force is developed at the interface stretched over the entire fluid mass. The intensity of molecular attraction per unit length along any line in the surface is called as surface tension. For example, in a spherical liquid droplet of radius (r), the pressure difference (Δp) between the inside and outside surface of the droplet is given by,

$$\Delta p = \frac{2\sigma}{r}$$

In SI system the unit and dimension of pressure can be written as, N/m and 2 MT– , respectively.

Relations for Gases and Liquids

All gases at high temperatures and low pressures are in good agreements with 'perfect gas law' given by,

$$p = \rho R T = \rho \left(\frac{\bar{R}}{M} \right) T$$

where, R is the characteristic gas constant, $\bar{R}$ is the universal gas constant and M is the molecular weight.

Liquids are nearly incompressible and have a single reasonable constant specific heat. Density of a liquid decreases slightly with temperature and increases moderately with pressure. Neglecting the temperature effect, an empirical pressure- density relation is expressed as.

$$\frac{p}{p_a} = (B+1)\left(\frac{\rho}{\rho_a} \right)^n - B$$

Here, B and n are the non-dimensional parameters that depend on the fluid type and vary slightly with the temperature. For water at 1 atm, the density is 1000 kg/m^3 and the constants are taken as, B = 3000 and n= 7

Law of Viscosity

- It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient of viscosity.
- It is expressed by equation $\tau = \mu \frac{du}{dy}$.
- Fluid which obey the above relation are known as Newtonian fluids and the fluids which do not obey the above relation are called Non-Newtonian fluids.

Types of Fluids

1. **Ideal Fluid.** A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. **Real Fluid.** A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. **Newtonian Fluid.** A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. **Non-Newtonian Fluid.** A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

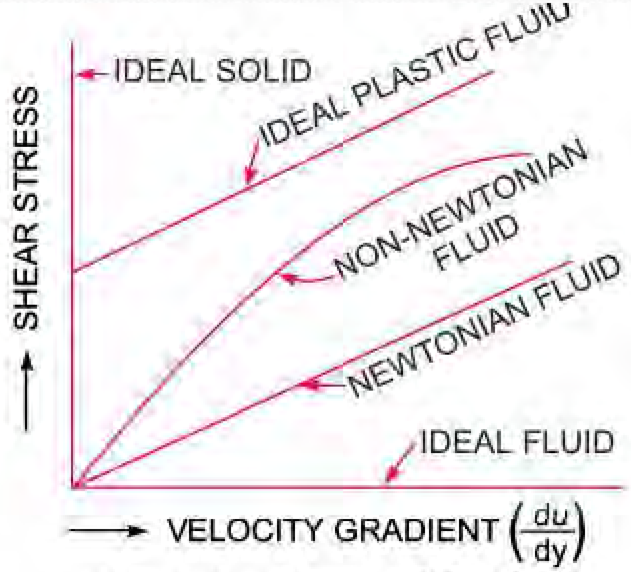
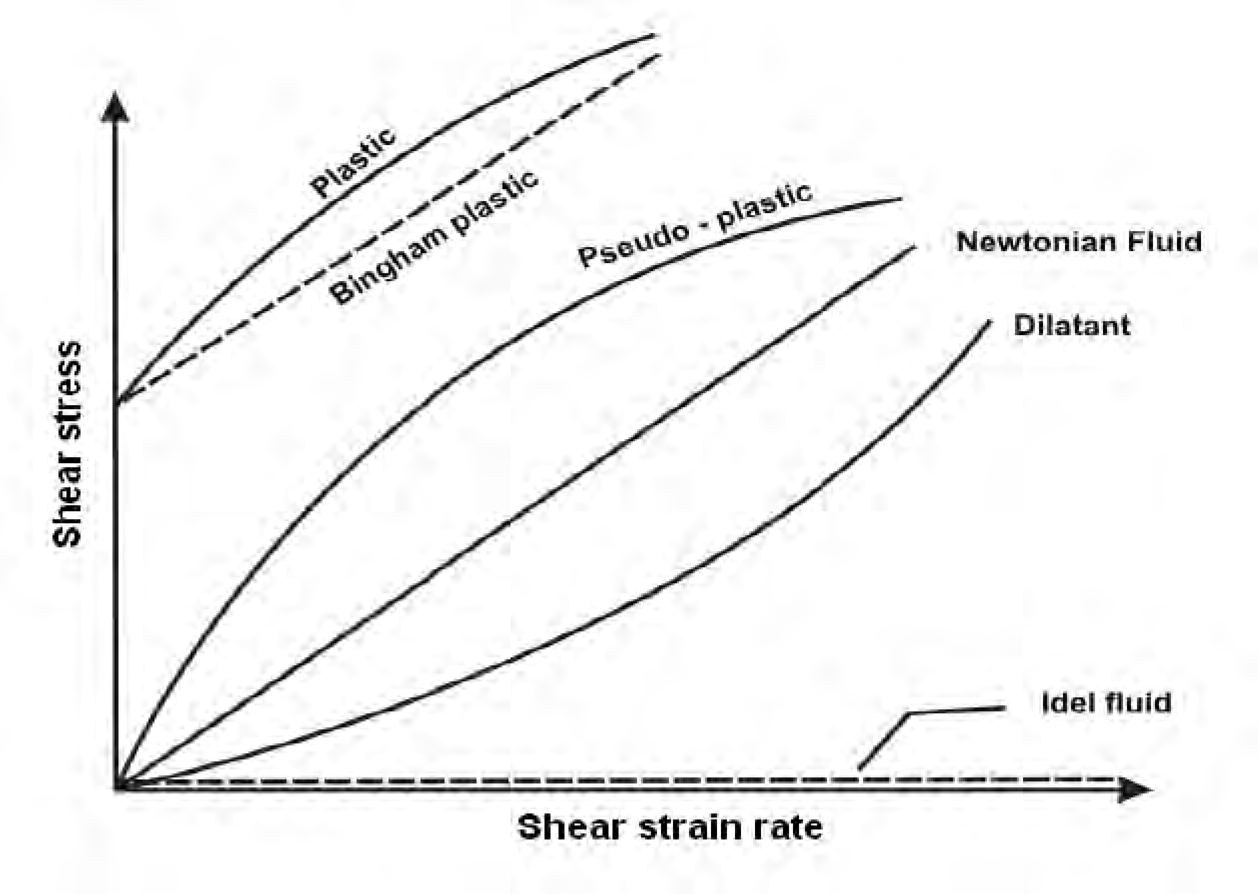


Fig. 1.2 *Types of fluids.*

5. **Ideal Plastic Fluid.** A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

Rheology



Plastic
Bingham plastic
Pseudo - plastic
Newtonian Fluid
Dilatant
Idel fluid
Shear stress
Shear strain rate

Example

A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.

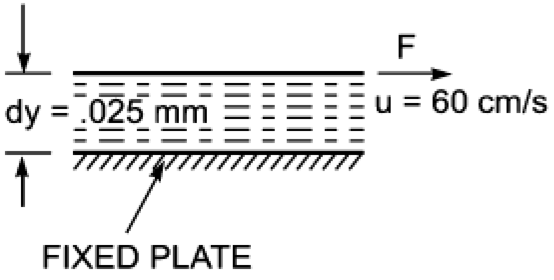


Fig. 1.3

Solution. Given :

Distance between plates, $dy = .025$ mm

$= .025 \times 10^{-3}$ m

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0 \dfrac{\text{N}}{\text{m}^2}$.

This is the value of shear stress *i.e.*, τ

Let the fluid viscosity between the plates is μ.

Using the equation (1.2), we have $\tau = \mu \dfrac{du}{dy}$.

where du = Change of velocity $= u - 0 = u = 0.60$ m/s

dy = Change of distance $= .025 \times 10^{-3}$ m

τ = Force per unit area $= 2.0 \dfrac{\text{N}}{\text{m}^2}$

$$\therefore \quad 2.0 = \mu \frac{0.60}{.025 \times 10^{-3}} \quad \therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$= 8.33 \times 10^{-5} \times 10$ poise $=$ **8.33×10^{-4} poise. Ans.**

Example

Determine the intensity of shear of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.

Solution. Given : $\mu = 1 \text{ poise} = \frac{1}{10} \frac{\text{N s}}{\text{m}^2}$

Dia. of shaft, $D = 10 \text{ cm} = 0.1 \text{ m}$

Distance between shaft and journal bearing,

$$dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

Speed of shaft, $N = 150$ r.p.m.

Tangential speed of shaft is given by

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.1 \times 150}{60} = 0.785 \text{ m/s}$$

Using equation (1.2), $\tau = \mu \frac{du}{dy}$,

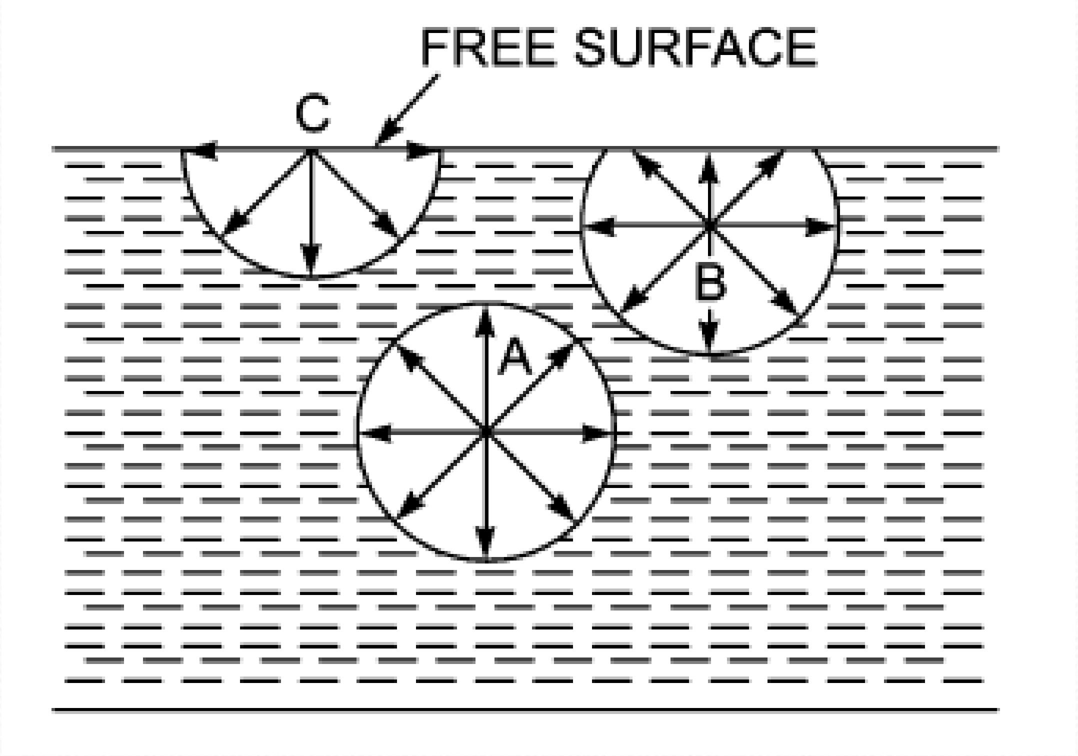
where du = change of velocity between shaft and bearing = $u - 0 = u$

$$= \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}} = \mathbf{52.33 \text{ N/m}^2. \text{ Ans.}}$$

Surface Tension

Liquids are having very important property by virtue of which it tries to minimize its surface area up to the maximum extent such a property of liquid is called 'Surface Tension'.

Basic cause of surface tension: Cohesive forces between the molecules which is known as cohesion.



Surface tension on liquid droplet

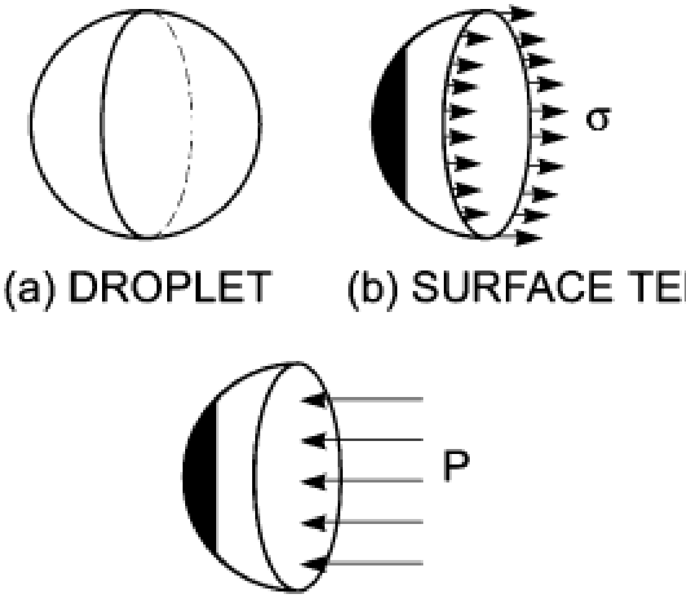
Let σ = Surface tension of the liquid

p = Pressure intensity inside the droplet (in excess of the outside pressure intensity)

d = Dia. of droplet.

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d}$$



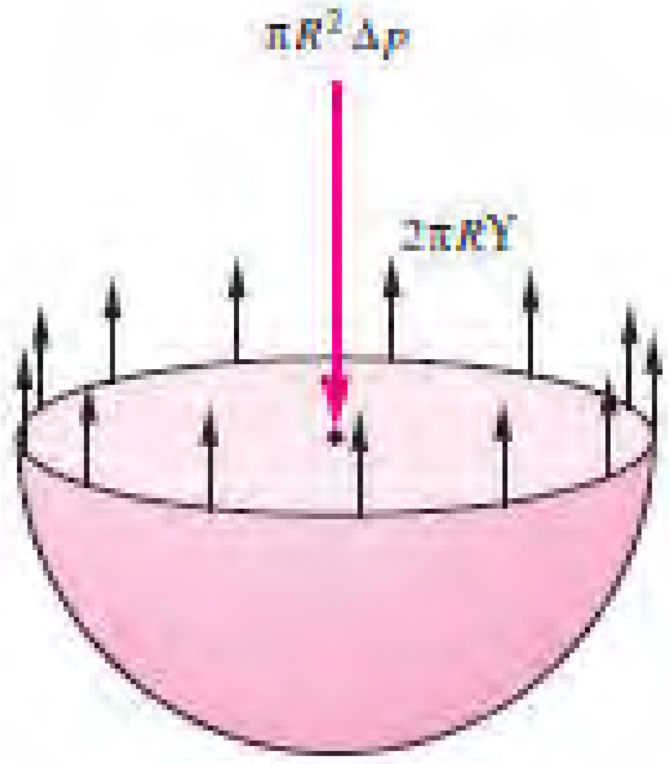
(a) DROPLET (b) SURFACE TENSION

(c) PRESSURE FORCES

Surface tension on liquid Bubble

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$p = \frac{2\sigma\pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$



Surface tension on liquid Jet

Force due to pressure $= p \times$ area of semi jet

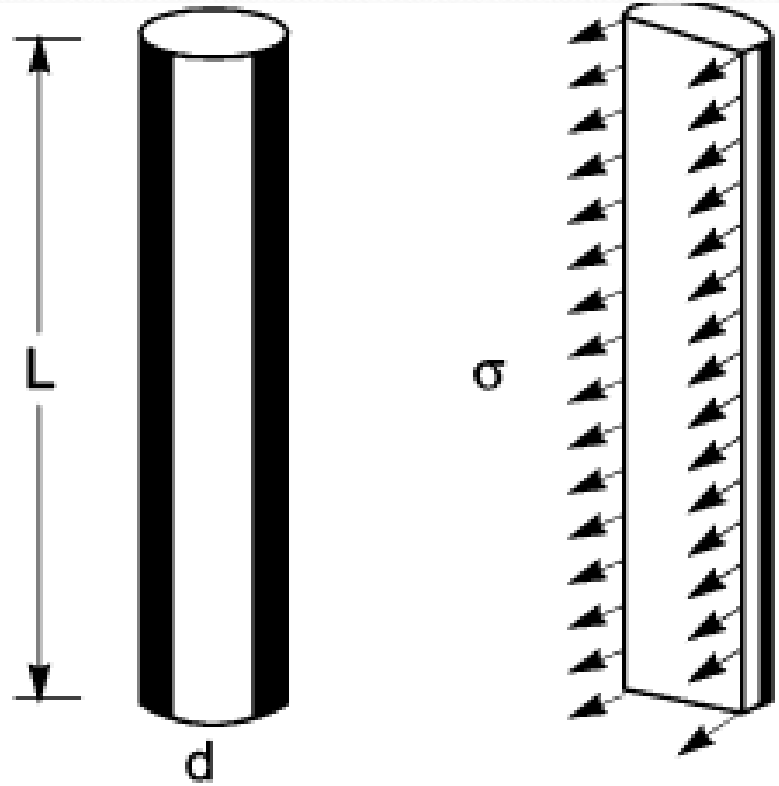
$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L.$

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$$\therefore \quad p = \frac{\sigma \times 2L}{L \times d}$$



Example on Surface Tension

The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

or $$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 =$ **11.045 N/cm^2. Ans.**

Do it Yourself

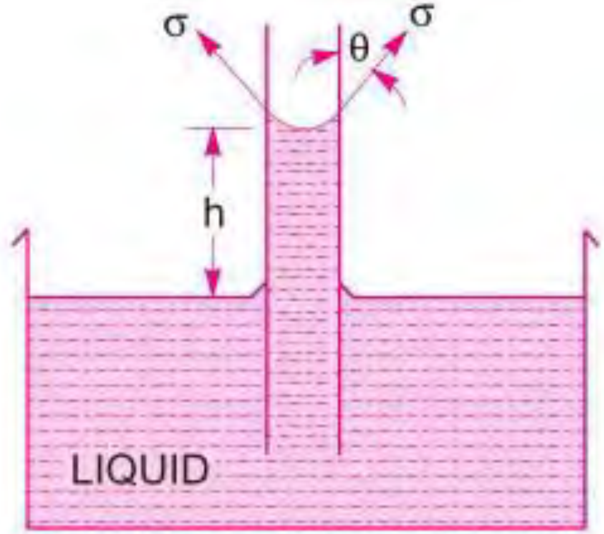
Questions:

1. Find the surface tension in soap bubble of 40mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.
2. Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes.

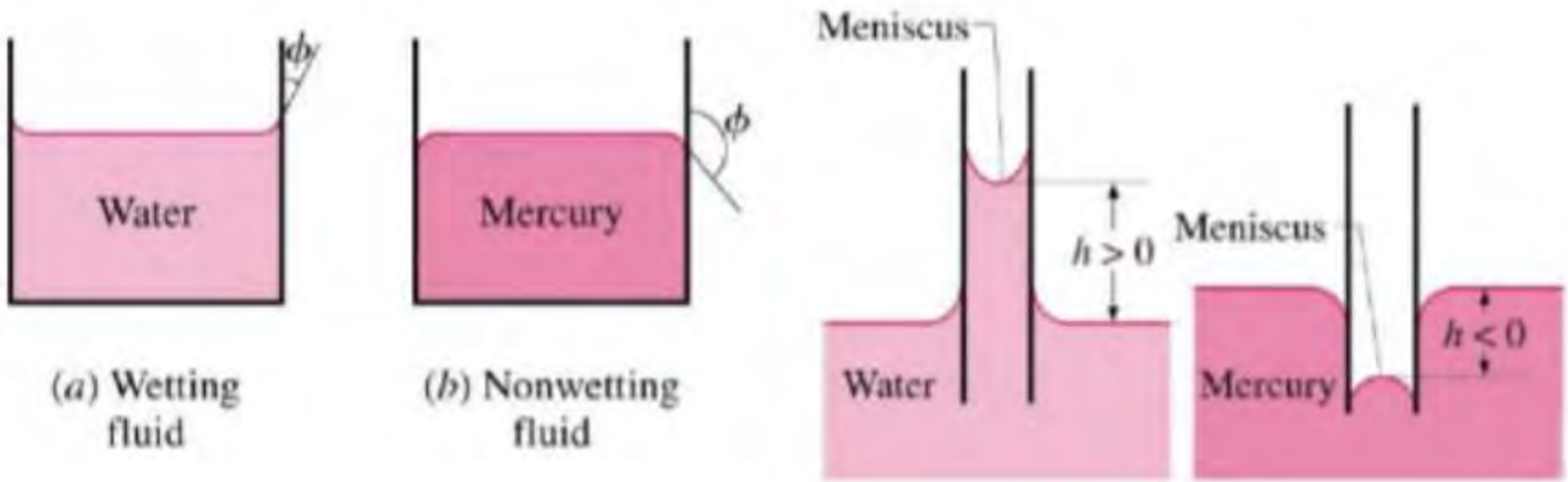
Capillarity

- Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.
- The rise of liquid surface is known as capillary rise while the fall of liquid surface is known as capillary depression.
- The attraction (adhesion) between the wall of the tube and liquid molecules is strong enough to overcome the mutual attraction(Cohesion) of the molecules and pull them up the wall. Hence the liquid is said to wet the solid surface.
- It is expressed in terms of cm or mm of liquid. Its values depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Capillarity



Capillary rise.



(a) Wetting fluid

(b) Nonwetting fluid

Expression for capillary Rise

h= height of the liquid in the tube

σ= Surface tension of liquid

θ=Angle of contact between liquid and tube glass

The weight of liquid of height h in the tube = (Area of tube × h) × ρ × g

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta$$

For equilibrium,

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d}$$

The value of θ between water and clean glass tube is approximately equal to zero and hence cos θ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d}$$

Expression for capillary Fall

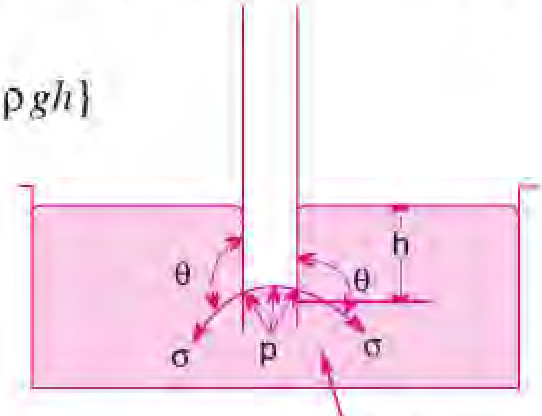
If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig.

Let h = Height of depression in tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × Area

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \, \{\because p = \rho g h\}$$



Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \qquad h = \frac{4\sigma\cos\theta}{\rho g d}$$

Value of θ for mercury and glass tube is 128°.

Example

Calculate the capillary effect in millimetres in a glass tube of 4 mm diameter, when immersed in (i) water, and (ii) mercury. The temperature of the liquid is 20°C and the values of the surface tension of water and mercury at 20°C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero and that for mercury is 130°. Take density of water at 20°C as equal to 998 kg/m³.

Solution. Given :

Dia. of tube, $d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

The capillary effect (*i.e.*, capillary rise or depression) is given by equation

$$h = \frac{4\sigma \cos\theta}{\rho \times g \times d}$$

where σ = surface tension in N/m

θ = angle of contact, and ρ = density

The negative sign indicates depression in tube

(*i*) **Capillary effect for water**

$$\sigma = 0.073575 \text{ N/m}, \theta = 0°$$

$$\rho = 998 \text{ kg/m}^3 \text{ at } 20°\text{C}$$

$$\therefore \quad h = \frac{4 \times 0.073575 \times \cos 0°}{998 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3} \text{ m} = \mathbf{7.51 \text{ mm. Ans.}}$$

(*ii*) **Capillary effect for mercury**

$$\sigma = 0.51 \text{ N/m}, \theta = 130° \text{ and}$$

$$\rho = \text{sp. gr.} \times 1000 = 13.6 \times 1000 = 13600 \text{ kg/m}^2$$

$$\therefore \quad h = \frac{4 \times 0.51 \times \cos 130°}{13600 \times 9.81 \times 4 \times 10^{-3}} = -2.46 \times 10^{-3} \text{ m} = \mathbf{-2.46 \text{ mm. Ans.}}$$

Pressure

- **Pressure** is defined as a normal force exerted by a fluid per unit area.
- We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is normal stress.
- Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m^2), which is called a **pascal (Pa).** That is,

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

- The pressure unit pascal is too small for pressures encountered in practice. Therefore, its multiples kilopascal ($1 \text{ kPa} = 10^3 \text{ Pa}$) and megapascal ($1 \text{ MPa} = 10^6 \text{ Pa}$) are commonly used.

Types of Pressure

Mathematically :

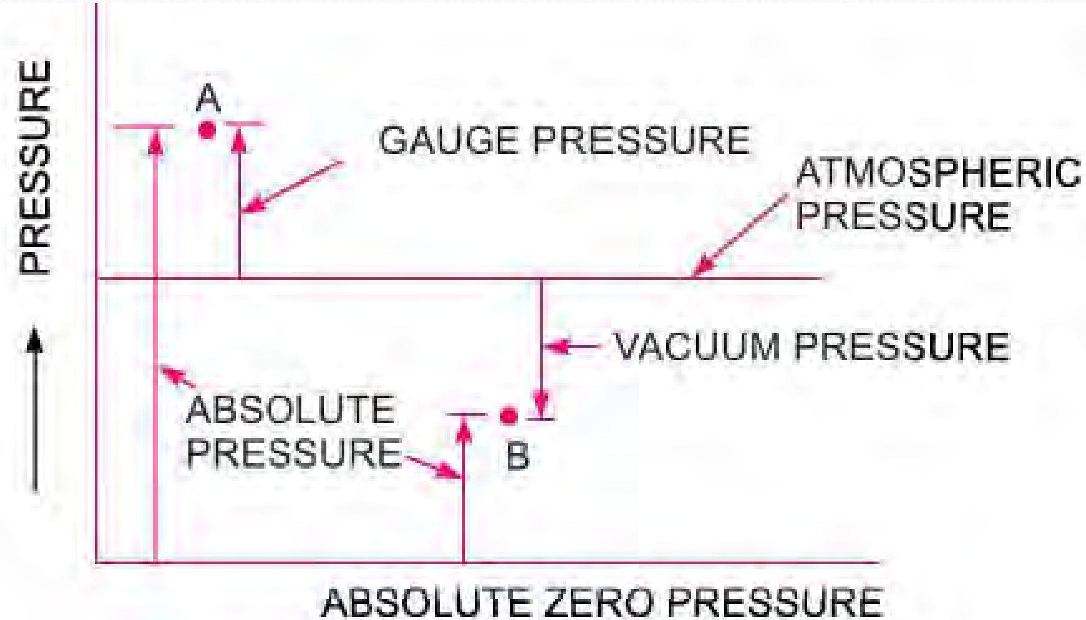
(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure – Absolute pressure.



Example

What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53×10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

$\therefore$ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6 × 1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

$$\therefore \quad p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \qquad (\because \; Z_0 = 0.75)$$

$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$

$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

$\therefore$ Gauge pressure, p = **45028 N/m^2. Ans.**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m^2. Ans.**

Variation of pressure with depth

- For fluid whose density changes significantly with elevation, a relation for the variation of pressure with elevation can be written as:

$$\boxed{\frac{dP}{dz} = -\rho g}$$

- The negative sign indicates that pressure decreases in an upward direction.
- When the variation of density with elevation is known the pressure difference between point 1 and 2 can be determine by integration to be:

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g \, dz$$

Measurement of pressure

Pressure of fluid is measured by the following device:

- Pressure measurement
  - Manometer
    - Simple manometers
      - Piezometer
      - U-Tube Manometer
      - Single column manometers
        - Vertical
        - Inclined
    - Differential Manometers
      - U-Tube differential
      - Inverted U-Tube Differential
  - Mechanical Gauges
    - Diaphragm pressure gauge
    - Bourdon tube
    - Dead weight pressure gauge
    - Bellows pressure gauge

Do it Yourself

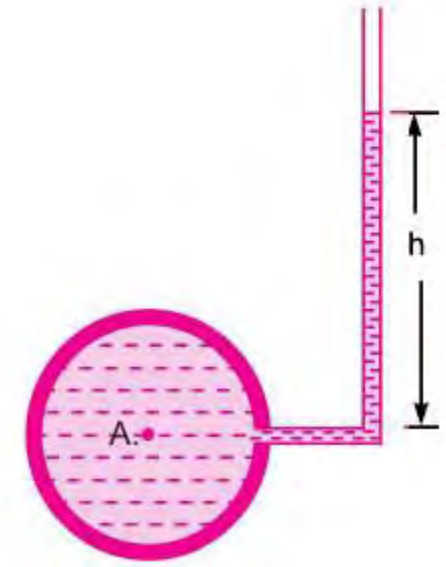
Questions:

1. The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension for water in contact with air=0.0725 N/m.
2. Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2mm. Consider surface tension of water in contact with air as 0.073575N/m

Piezometer

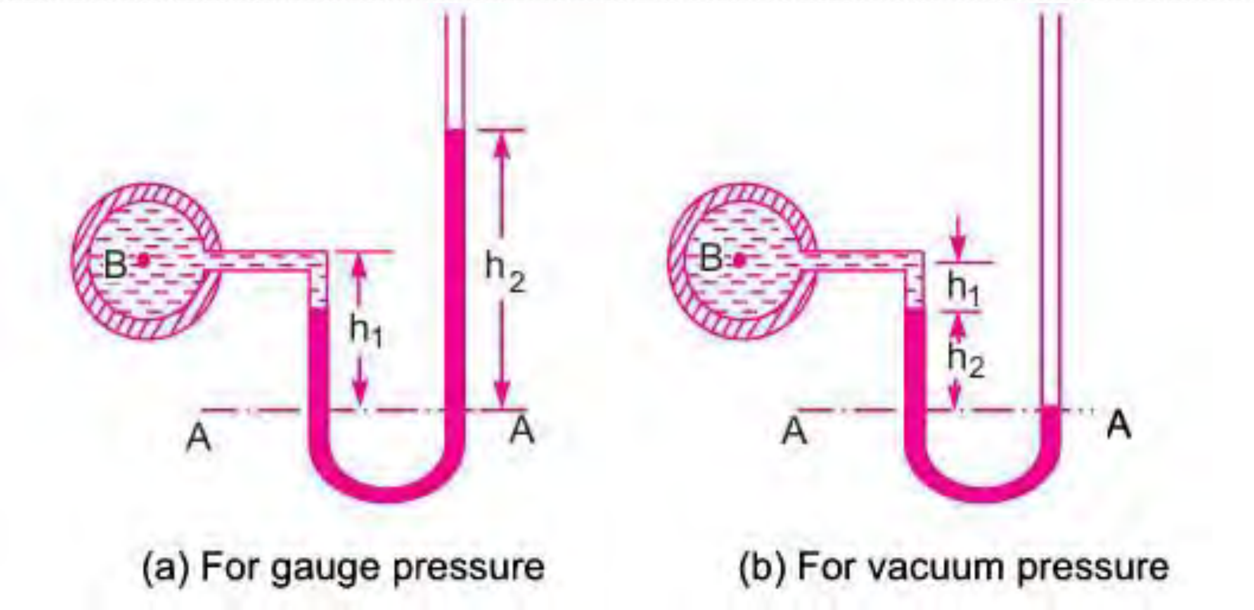
It is the simplest form of manometers used for measuring gauge pressure. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere. The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$= \rho \times g \times h \ \frac{\mathrm{N}}{\mathrm{m}^2}.$$



U-Tube Manometer

It consists of glass tube bent in U-Shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



For Gauge pressure

Let B is the point at which pressure is to be measured, whose value is p.
The datum line is A-A.

Let h_1 = Height of light liquid above the datum line
h_2 = Height of heavy liquid above the datum line
S_1 = Sp. gr. of light liquid
ρ_1 = Density of light liquid = $1000 \times S_1$
S_2 = Sp. gr. of heavy liquid
ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1).$

For Vacuum Pressure

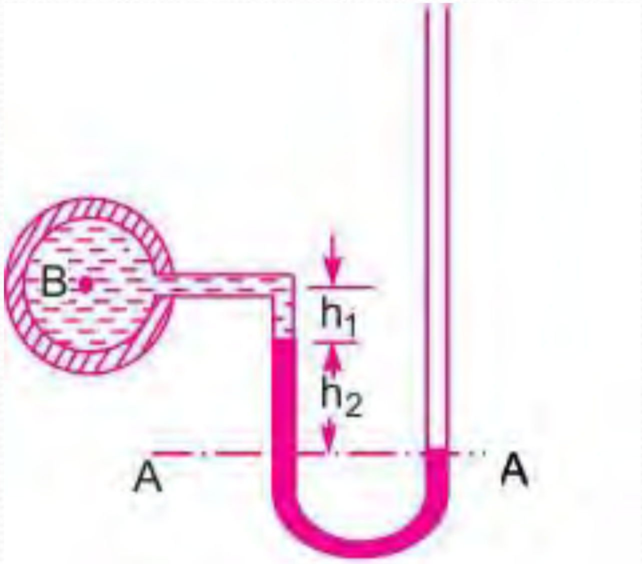
For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A $= 0$

$$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1).$$



Example on U-Tube Manometer

The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from A-A, $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

Let p = Pressure of fluid in pipe

Equating the pressure above A-A, we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or

$$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \textbf{ Ans.}$$

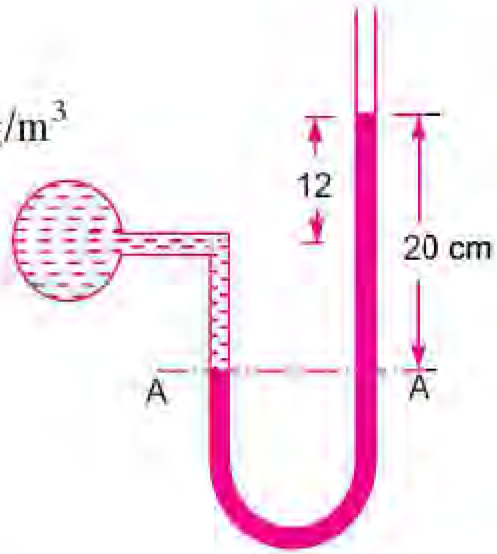
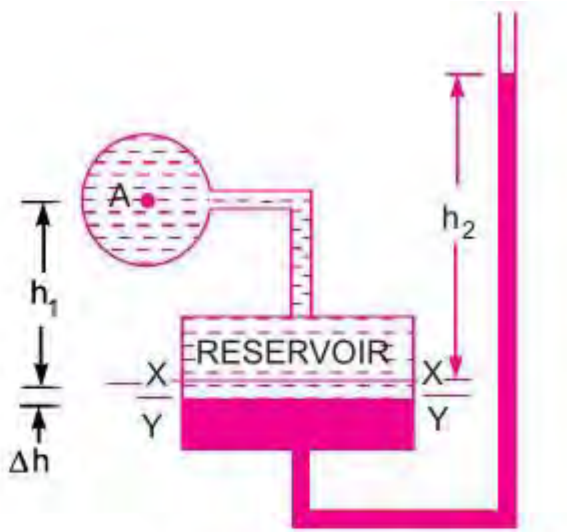


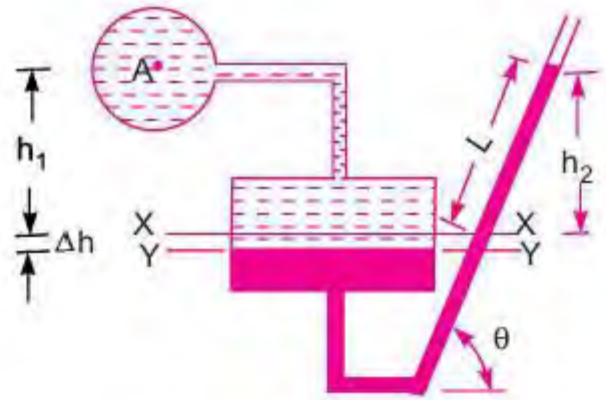
Fig. 2.10

Single Column Manometer

Single column manometer is a modified form of a U-Tube Manometer in which a reservoir, having a large cross sectional area(about 100times) as compared to the area of the tube is connected to one of the limb of the manometer. Due to large cross-sectional area of reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined.



Vertical single column manometer.

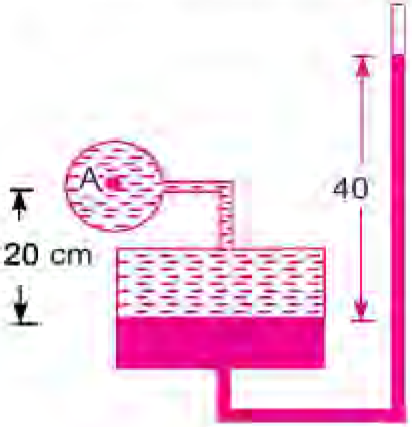


Inclined single column manometer.

Example

A single column manometer is connected to a pipe containing a liquid of sp. gr. 0.9 as shown in Fig. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. The specific gravity of mercury is 13.6.

Solution. Given :



Sp. gr. of liquid in pipe, $S_1 = 0.9$

$\therefore$ Density $\rho_1 = 900 \text{ kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20 \text{ cm} = 0.2 \text{ m}$

Rise of mercury in right limb,

$h_2 = 40 \text{ cm} = 0.4 \text{ m}$

Let p_A = Pressure in pipe

we get

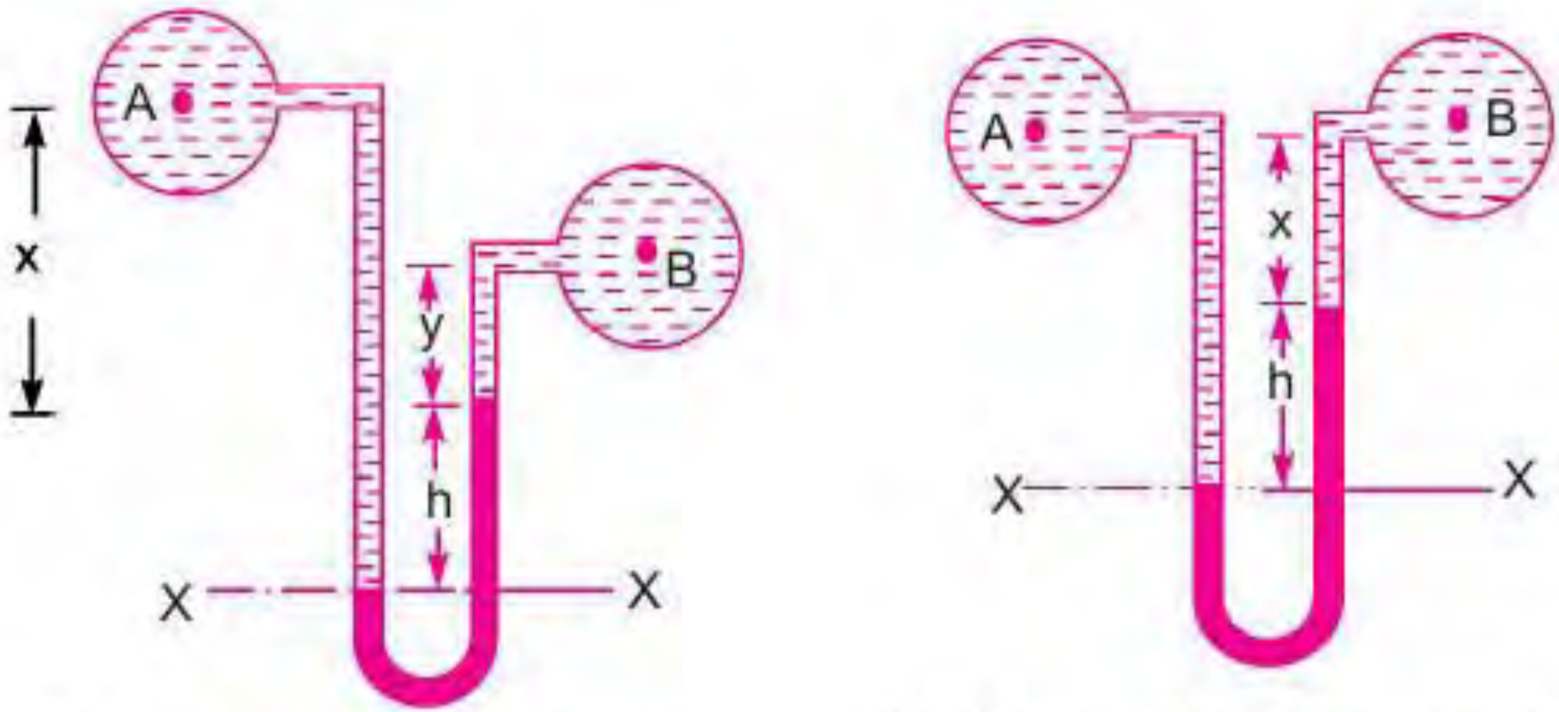
$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8$$

$= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 =$ **5.21 N/cm^2. Ans.**

U-Tube differential Manometer



(a)Two pipes at different levels

(b) A and B are at the same level

U-Tube Manometer Calculation

the two points A and B are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb.

x = Distance of the centre of A, from the mercury level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at A.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

Cont…

$\therefore$ Difference of pressure at A and $B = h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

In Fig. the two points A and B are at the same level and contains the same liquid of density ρ_1. Then

Pressure above X-X in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

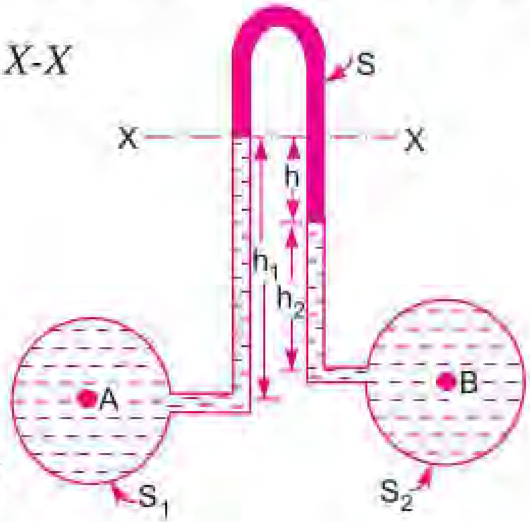
$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1).$$

Inverted U-Tube Manometer

It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. shows an inverted U-tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.



Let h_1 = Height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

p_A = Pressure at A

p_B = Pressure at B.

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

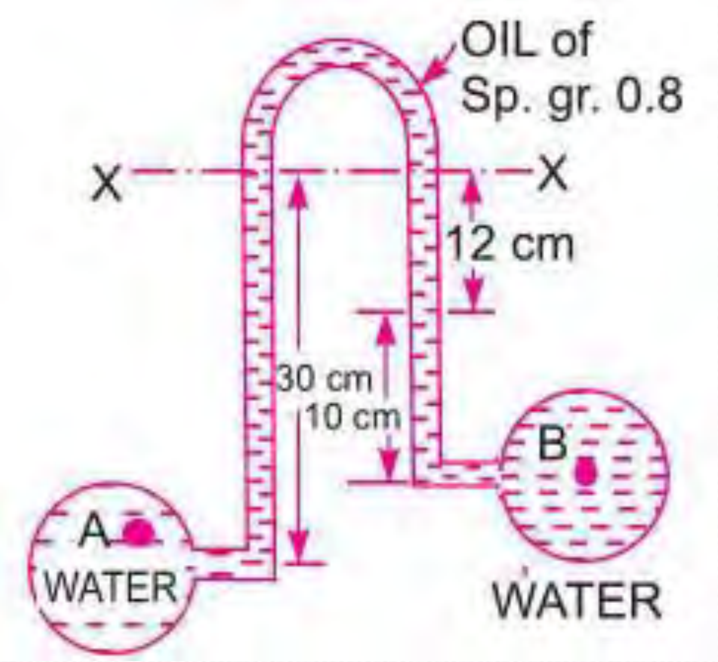
Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or
$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h.$$

Do it Yourself

Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig.



Introduction

- Fluid Kinematics deals with the motion of fluids without considering the forces and moments which create the motion.
- Flow visualization
- Plotting flow data
- Fundamental kinematic properties of fluid motion and deformation.

Lagrangian Description

- Lagrangian description of fluid flow tracks the position and velocity of individual particles.
- Based upon Newton's laws of motion.
- Fluids are composed of billions of molecules.
- Interaction between molecules hard to describe/model.
- However, useful for specialized applications

Eulerian Description

- Eulerian description of fluid flow a flow domain or control volume is defined by which fluid flows in and out.
- Field variables which are functions of space and time.
- Pressure field, PP(x,y,z,t)
- Velocity field, Acceleration field.
- Well suited for formulation of initial boundary-value problems.
- Named after Swiss mathematician Leonhard Euler (1707-1783).

Different types of Fluid flow

- Steady & Unsteady flows:

  If the properties in the fluid flow are not changing with respect to time at a particular place such kind of a flow is known as 'Steady flow' otherwise Unsteady.

- Uniform & Non-uniform Flows:

  If the properties in the fluid flow are not changing with respect to space at a particular time such kind of a flow is known as 'Uniform' otherwise Non-uniform.

Cont…

- Steady & Uniform Flows:

  If the properties are not changing with respect to time as well as space such type of flow are steady and uniform flow.

- Incompressible & Compressible Flow:

  If the density of fluid is not changing with respect to pressure in a flow, such a flow is known as Incompressible flow.

Different types of Fluid flow

- Laminar & Turbulent Flow:

When all the fluid particles lying in the same horizontal plane are having their velocity components in the same direction then these particles in a motion will form a layer, Such type of flow is known as Laminar Flow. If Their velocity components are in different direction that will be called Turbulent flow.

- Irrotational & Rotational Flow:

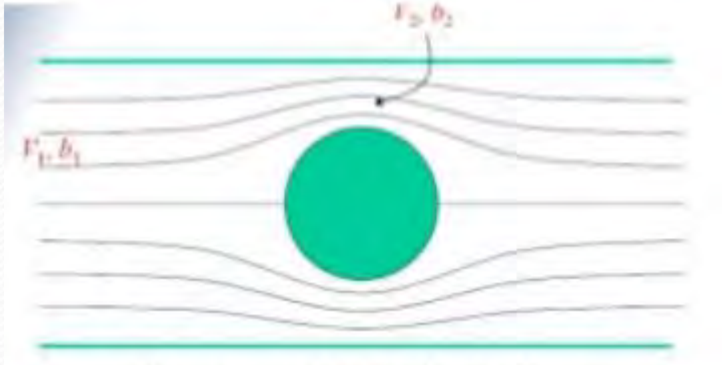
When the fluid particles are moving in a circular path along with stream line such kind of a flow is known as Vortex flow.

Along with the rotation of the fluid particles in a stream line if the particles are also rotating with respect to their own centre of masses such a flow is known as Rotational flow otherwise irrotational flow.

Some typical Flow

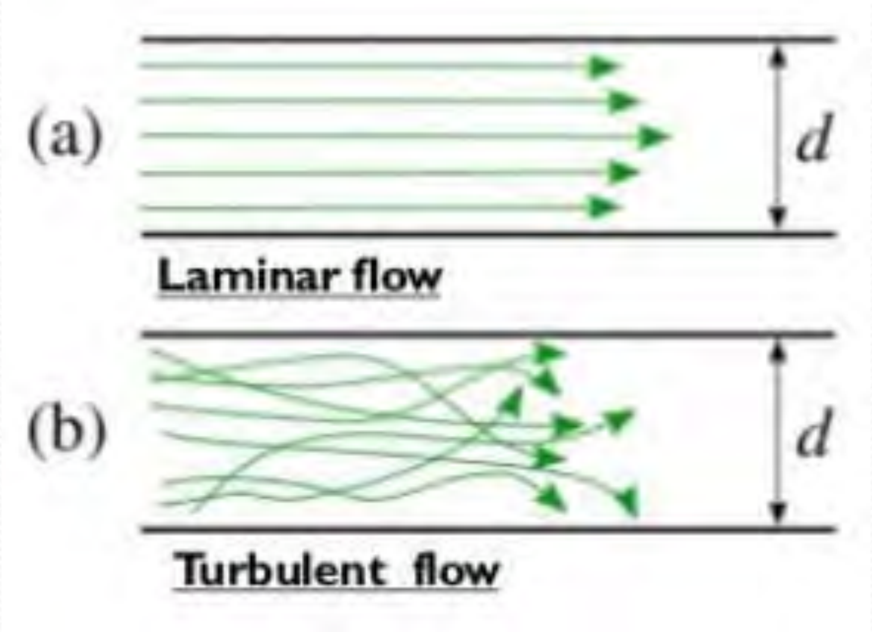
Fig. Laminar Flow

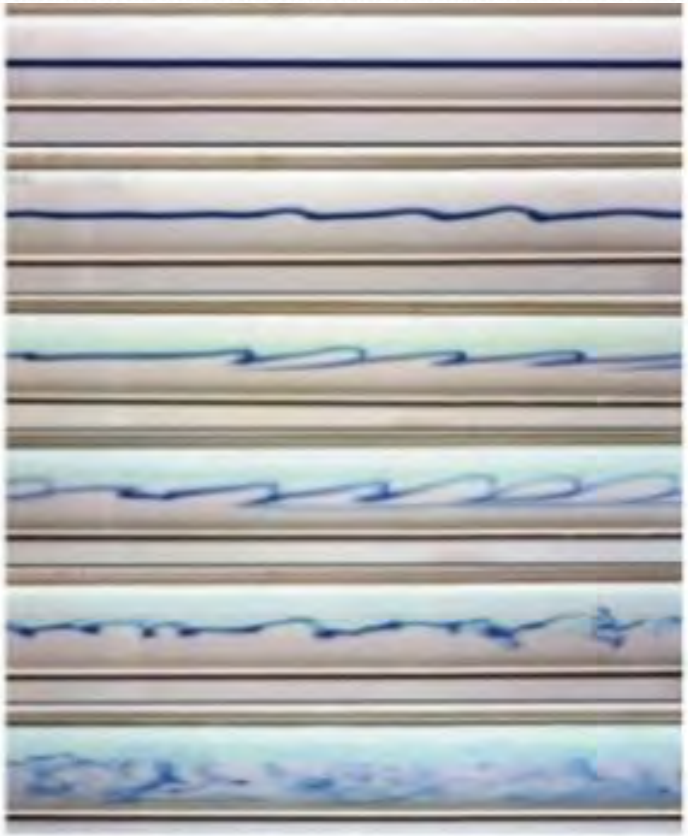
Fig. Turbulent Flow



Flow around cylindrical object

Transition flow from Laminar to Turbulent





Transition of flow from Laminar to turbulent

Continuity

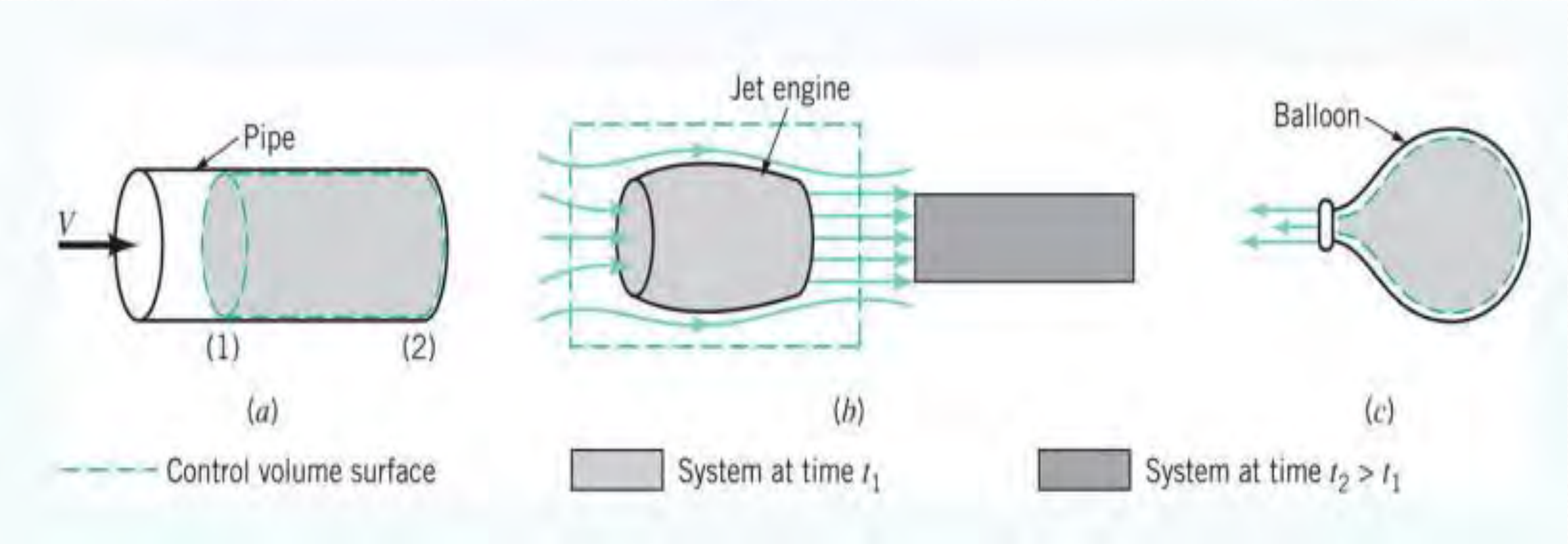
Control Volume:

This is a certain well defined extent of space. For the purpose of understanding the changes that take place in the fluid characteristics we may introduce a control volume so that we may compare the flow characteristics of a fluid just before it enters the control volume and just after it leaves the control volume.

Continuity:

This is an equation based on the principle of conservation of mass. Suppose we consider a stream tube. Since the stream tube is always full of the fluid, the quantity of the fluid entering the stream tube at one end per unit of time should be equal to the quantity of the fluid leaving the stream tube at the other end per unit of time.

Control Volume



Continuity Equation

Let V be the average velocity at any section and A the area of the section. If w be the specific weight of the fluid, the quantity of the fluid flowing per second across the section

$$=w\,A\,V$$

Specific weight at section 1-1 & 2-2 be w_1 and w_2, A_1 and A_2 be the sectional area at the section 1-1 and 2-2, and V_1 & V_2 be the velocities at these sections, then

$$w_1\,A_1\,V_1 = w_2\,A_2\,V_2$$

If the fluid is incompressible then $w_1 = w_2$ and the relation will be

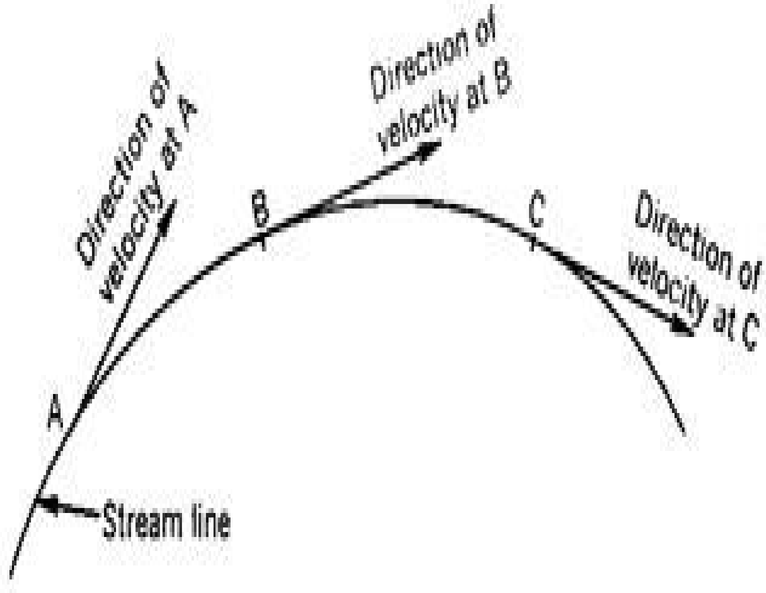
$$A_1\,V_1 = A_2\,V_2$$

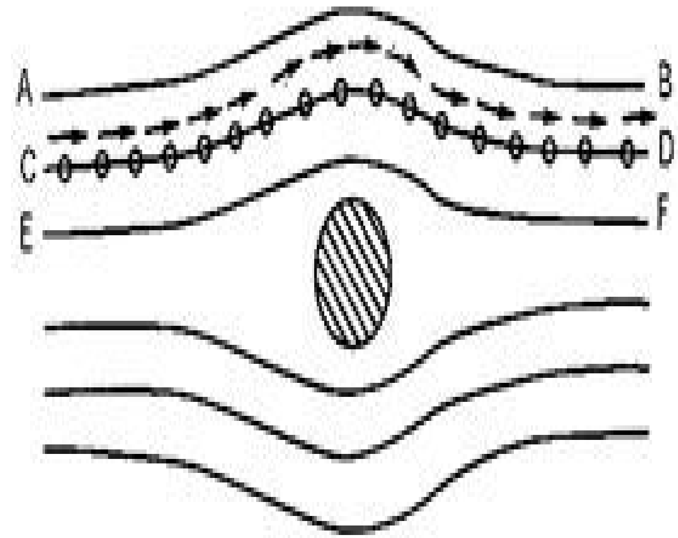
Note: Volume of the fluid flowing across a section per unit of time is constant.

Stream Line

A stream line is a continuous line in a fluid which shows the direction of the velocity of the fluid at each point along the line. The tangent to the stream line at any point on it is in the direction of the velocity at that point. Fluid particles lying on a stream line at an instant move along the stream line.

Stream Line


Direction of velocity at A
Direction of velocity at B
Direction of velocity at C
A
B
C
Stream line

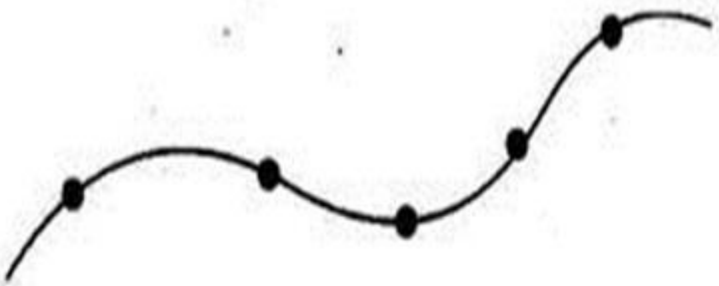

A
B
C
D
E
F

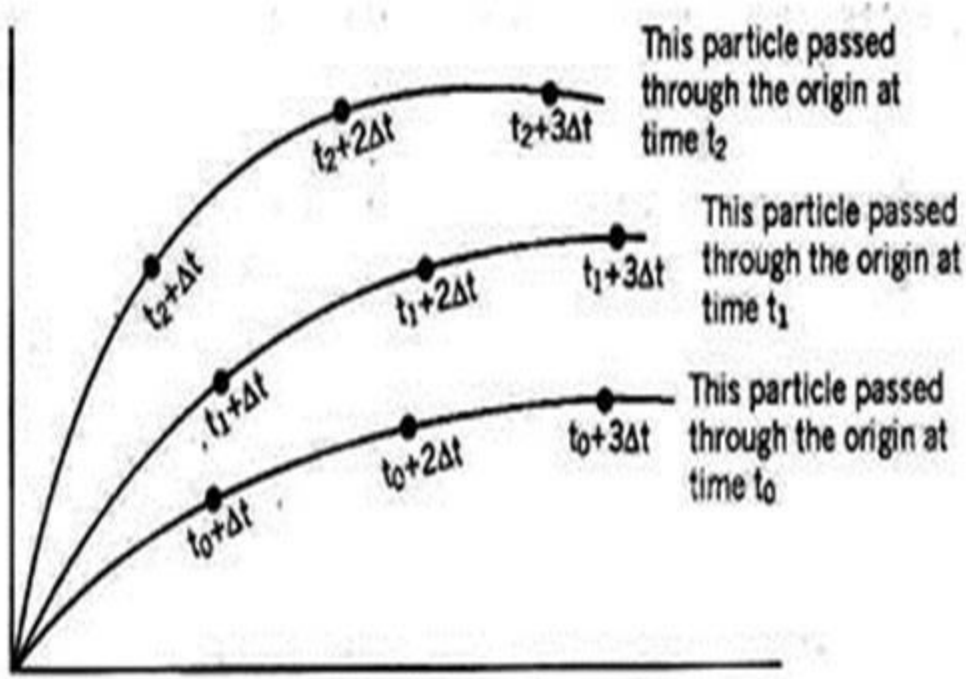
Path Line

A path line means the path or a line actually described by a single fluid particle as it moves during a period of time. The path line indicates the direction of the velocity of the same fluid particle at successive instants of time.

In the case of a steady flow since there are no fluctuations of the velocity, the path line coincides with the stream line. In the case of an unsteady flow the stream lines change their positions at every instant and thus the path line may fluctuate between different stream lines during an interval of time.

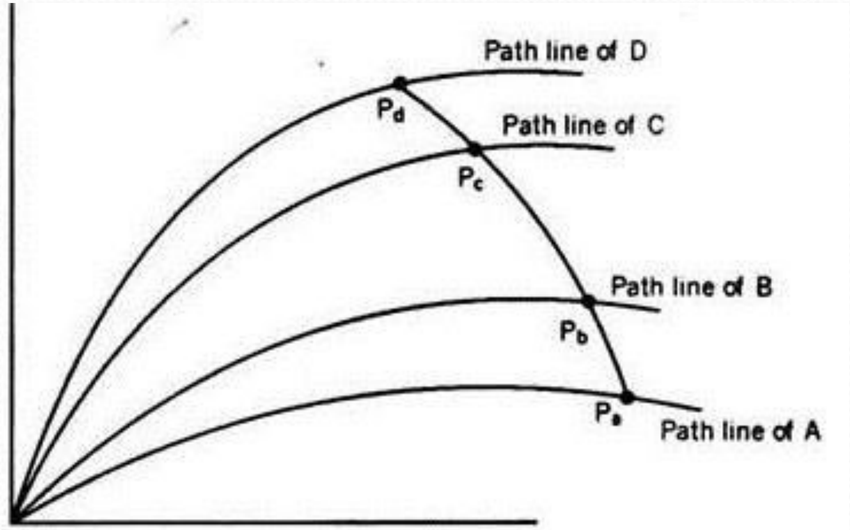
Path Line




This particle passed through the origin at time t_2
$t_2+\Delta t$
$t_2+2\Delta t$
$t_2+3\Delta t$
This particle passed through the origin at time t_1
$t_1+\Delta t$
$t_1+2\Delta t$
$t_1+3\Delta t$
This particle passed through the origin at time t_0
$t_0+\Delta t$
$t_0+2\Delta t$
$t_0+3\Delta t$

Streak line

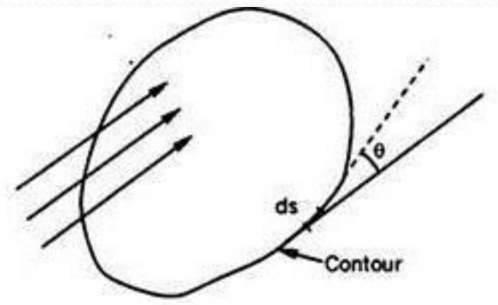
The streak line is the locus of the positions of fluid particles which have passed through a given point in succession. Suppose A, B, C, D… are fluid particles which passed through a reference point say the origin one after the other in succession. These particles have described their own path lines. Suppose at a time t, these particles A, B, C, D… are at P_a

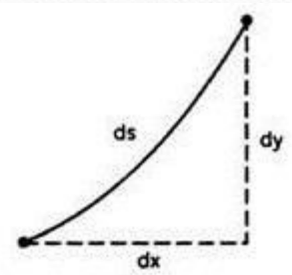


Circulation and Vorticity

Circulation is the line integral of velocity vector taken along a closed loop. Twice of the angular velocity is termed as Vorticity.

Consider a closed line or contour in a two-dimensional flow. Let V represent the resultant velocity at any point on the contour. Let θ be the angle between the velocity V and the elemental contour element ds. The integral of the product (contour length ds x the component of the velocity in the direction of ds) is called the line integral. The line integral of the velocity around a closed contour is called circulation (usually denoted by Γ).





Momentum Equation

Assumptions:

•Flow is laminar
•Flow is irrotational
•Flow is inviscid

We have all seen moving fluids exerting forces. The lift force on an aircraft is exerted by the air moving over the wing. A jet of water from a hose exerts a force on whatever it hits. In fluid mechanics the analysis of motion is performed in the same way as in solid mechanics - by use of Newton's laws of motion. Account is also taken for the special properties of fluids when in motion.

Momentum Equation

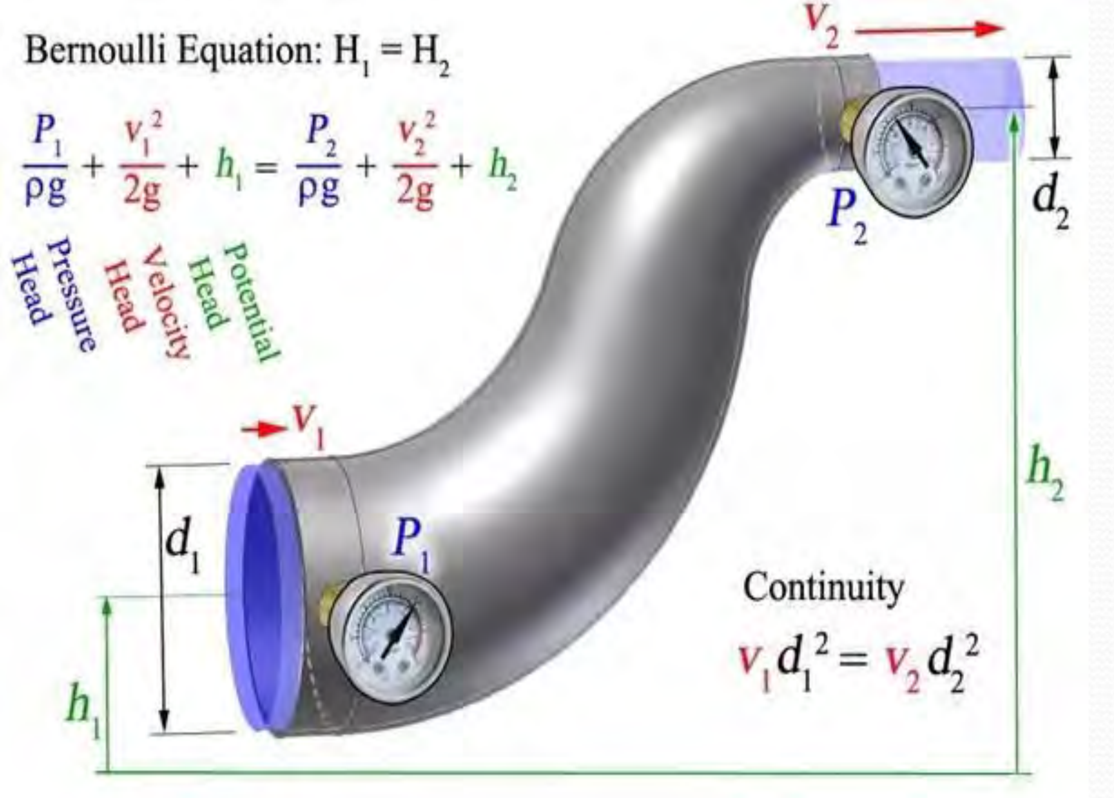
The momentum equation is a statement of Newton's Second Law and relates the sum of the forces acting on an element of fluid to its acceleration or rate of change of momentum. You will probably recognize the equation F = ma which is used in the analysis of solid mechanics to relate applied force to acceleration. In fluid mechanics it is not clear what mass of moving fluid we should use so we use a different form of the equation

Applications of the Momentum Equation:

- Force due to the flow of fluid round a pipe bend.
- Impact of a jet on a plane surface.
- Force due to flow round a curved vane.

Bernoulli's Theorem

Assumptions:
- Flow is laminar
- Flow is irrotational
- Flow is inviscid
- Flow is steady
- Flow is incompressible



Under the five assumptions in a flow stated above, the summation of all the energy that is

The pressure energy (P), the kinetic energy($1/2\ \rho\ V^2$) and the potential energy (ρgz) per unit volume will be constant between the any two points in a flow. This theorem is known as Bernoulli's theorem.