Dimensional Analysis

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What is Dimensional Analysis

- Dimensional analysis is a means of simplifying a physical problem by appealing to dimensional homogeneity to reduce the number of relevant variables.

It is particularly useful for:

- presenting and interpreting experimental data;
- attacking problems not amenable to a direct theoretical solution;
- checking equations;
- establishing the relative importance of particular physical phenomena;
- physical modeling.

A dimension is the type of physical quantity.

A unit is a means of assigning a numerical value to that quantity.

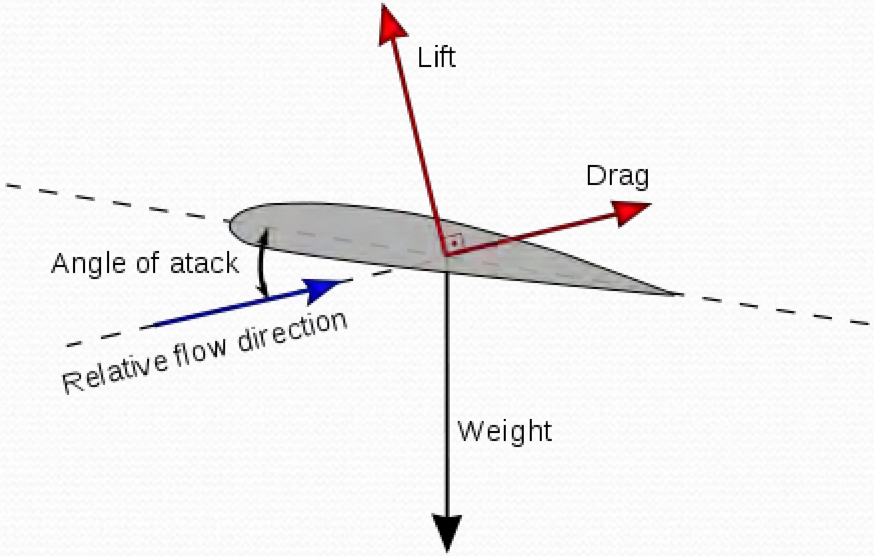
Example

The drag force F per unit length on a long smooth cylinder is a function of air speed U, density ρ, diameter D and viscosity μ. However, instead of having to draw hundreds of graphs portraying its variation with all combinations of these parameters, dimensional analysis tells us that the problem can be reduced to a single dimensionless relationship

$$C_D = f\,(Re)$$

where C_D is the drag coefficient and Re is the Reynolds number.

In this instance dimensional analysis has reduced the number of relevant variables from 5 to 2 and the experimental data to a single graph of C_D against Re.



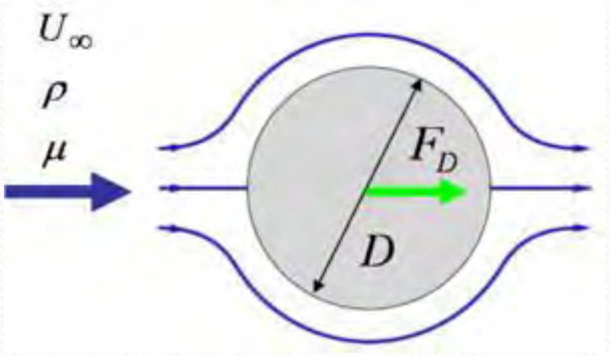
Primary Dimensions

In fluid mechanics the primary or fundamental dimensions, together with their SI units are:

- mass M (kilogram, kg)
- length L (metre, m)
- time T (second, s)
- temperature Θ (kelvin, K)

In other areas of physics additional dimensions may be necessary. The complete set specified by the SI system consists of the above plus

- electric current I (ampere, A)
- luminous intensity C (candela, cd)
- amount of substance n (mole, mol)



Some dimensions of derived quantities related to Fluid Mechanics

	Quantity	Common Symbol(s)	Dimensions
Geometry	Area	A	L^2
	Volume	V	L^3
	Second moment of area	I	L^4
Kinematics	Velocity	U	LT^{-1}
	Acceleration	a	LT^{-2}
	Angle	θ	1 (i.e. dimensionless)
	Angular velocity	ω	T^{-1}
	Quantity of flow	Q	$L^3 T^{-1}$
	Mass flow rate	m	MT^{-1}

Example

Use the definition $\tau = \mu\, dU/dy$ to determine the dimensions of viscosity.

Solution: From the definition,

$$\mu = \tau/(dU/dy)$$

$$=(\text{force}/\text{area})\,/\,(\text{Velocity}/\text{Length})$$

Hence

$$[\mu] = \frac{MLT^{-2}/L^2}{LT^{-1}/L} = ML^{-1}T^{-1}$$

Note: [ ] means "dimensions of"

Do it Yourself

- Since Re = ρ UL/ μ is known to be dimensionless, What will be the dimensions of ρ.

  where U is the velocity

  L is the length

  μ is dynamic viscosity

  Re is Reynold number which is dimensionless

Dimensional Homogeneity

- The Principle of Dimensional Homogeneity

  "All additive terms in a physical equation must have the same dimensions."

For example in equation $s = ut + 1/2at^2$ All terms have the dimension of length.

Dimensional homogeneity is a useful tool for checking formulae. For this reason it is useful when analyzing a physical problem to retain algebraic symbols for as long as possible, only substituting numbers right at the end. However, dimensional analysis cannot determine numerical factors; e.g. it cannot distinguish between $\frac{1}{2}at^2$ and at^2 in the first formula above

Method of Dimensional Analysis

Buckingham's π Method:

- Buckingham Pi theorem relies on the identification of variables involved in a process. Further, a few of these variables have to be marked as "Repeating Variables".
- The repeating variables among themselves should not form a non-dimensional number.
- If a physical process has "n" variables and from these "j" are "Repeating Variables", then there are "n-j" independent non-dimensional numbers that can describe the process.

Buckingham's π Method

- Step 1: Identify the relevant variables and function
- Step 2: Write down dimensions.
- Step3: Establishment of the number of independent dimensions and non-dimensional groups.
- Step 4: Choose j (=3) dimensionally independent repeating variables Generally:
  - The first representing the fluid property.
  - The second representing the flow characteristics.
  - The third representing the geometric characteristics.

One should not choose density, gravity and specific weight as repeating variables.

Buckingham's π Method

- Step 5: Creating the π_s by non-dimensioning the remaining variables and by solving the coefficients.
- Step 6: Setting the non-dimensional relationship.
- Step7: Rearrange (if required) for convenience. In this case, you are free to replace any of the π_s by a power of that π, or by a product with the other π_s , provided retaining the same number of independent dimensionless groups.

Example

Let us consider a frictional resistance (F) when a liquid is flowing through a pipe depends on the viscosity, density of the fluid, velocity of the flow, diameter of the pipe and surface roughness. Derive a rational equation for the pipe flow in terms of dimensionless groups by Buckingham's pi method.

Solution:

- Step 1: Relevant variables and function: $f(F,\mu,\rho,v, D,\varepsilon)=0$
- Step 2: Dimensions:

  $F:MLT^{-2}$; $\mu: ML^{-1}T^{-1}$: $\rho: ML^{-3}$; $v:LT^{-1}$; $D:L$; $\varepsilon:L$
- Step3: Number of relevant variables: $n=6$

  Number of independent dimensions: $j=3$(M,L and T)

  Number of non-dimensional group (πs): $n-j=3$

Cont…

- Step 4: Choosing j(=3) dimensionally-independent repeating variables as:

  Fluid property: ρ

  Flow characteristics: v

  Geometric characteristics: D

- Step 5: Creating the πs as:

$$\pi_1 = \rho^{a1} v^{b1} D^{c1} F$$

$$\pi_2 = \rho^{a2} v^{b2} D^{c2} \mu$$

$$\pi_3 = \rho^{a3} v^{b3} D^{c3} \varepsilon$$

Remember if n=j, you have to consider j=j-1. Example: If n=3, but =3, then n-j=0, i.e, No dimensionless group will be formed. Therefore in this case, j should be 3-1=2. In this case, you have to pick 2 repeating variables.

Cont…

Solving coefficient by considering the dimensions of both sides

For $\pi_1 = \rho^{a1} v^{b1} D^{c1} F$

$$M^0 L^0 T^0 = (ML^{-3})^{a1} (LT^{-1})^{b1} (L)^{c1} MLT^{-2}$$

$$0 = a_1 + 1$$

$$0 = -3a_1 + b_1 + c_1 + 1$$

$$0 = -b_1 - 2$$

Implies that: $\Rightarrow$ $a_1 = -1$

$b_1 = -2$

$c_1 = -2$

So , $\boldsymbol{\pi_1 = F/\rho v^2 D^2}$

Cont…

Similarly

For $$\pi_2 = \rho^{a2} v^{b2} D^{c2} \mu$$

$$M^0 L^0 T^0 = (ML^{-3})^{a2} (LT^{-1})^{b2} (L)^{c2} ML^{-1} T^{-1}$$

$$0 = a_2 + 1$$

$$0 = -3a_2 + b_2 + c_2 - 1$$

$$0 = -b_{12} - 1$$

Implies that: $\Rightarrow$ $a_2 = -1$

$b_2 = -1$

$c_2 = -1$

So , $\boldsymbol{\pi_2 = \mu / \rho v D}$

Cont…

Similarly

For $\pi_3 = \rho^{a3}\, v^{b3}\, D^{c3}\, \varepsilon$

$$M^0 L^0 T^0 = (ML^{-3})^{a3}\, (LT^{-1})^{b3}\, (L)^{c3}\, L$$

$$0 = a_3$$

$$0 = -3a_3 + b_3 + c_3 + 1$$

$$0 = -b_3$$

Implies that: $\Rightarrow$ $a_3 = 0$

$b_3 = 0$

$c_3 = -1$

So , $\pi_3 = \varepsilon / D$

Cont…

- Step 6: Setting the non-dimensional relationship

$$f(\pi_{1,}\ \pi_{2,}\ \pi_3)=0$$

Or $\pi_1 = f(\pi_{2,}\ \pi_3) \longrightarrow F/\ \rho\, v^2\ D^2\ = f(\mu/\ \rho\, v\, D,\ \varepsilon/D)$

- Step 7: Rearrangement

$$F/\ \rho\, v^2\ D^2\ = f(\rho\, v\, D\ /\ \mu,\ \varepsilon/D)$$

Do it Yourself

Question:

The drag D of a sphere is influenced by, sphere diameter d, flow velocity U, fluid density ρ and fluid viscosity μ. Obtain $\pi_{1,}$ π_2 by Buckingham's pi method, with density, velocity and diameter as repeating variables.

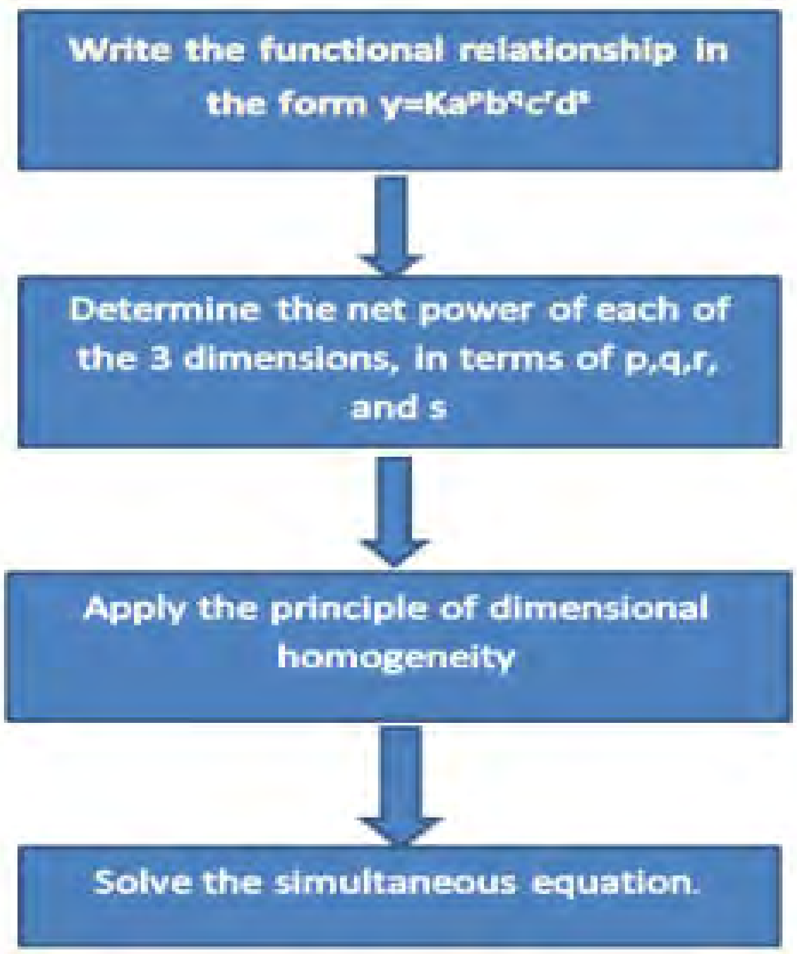
Method of Dimensional Analysis

Rayleigh's Method:

- As early as 1899, Lord Rayleigh made an ingenious application of dimensional analysis to the problem of the effect of temperature on the viscosity of gas. Rayleigh's method is outwardly different from Buckingham's method.
- In this method first we assume some functional relationships and then we equate the dimensions at both sides and developing the relationship between the dependent & independent variables.
- A basic method to dimensional analysis method and can be simplified to yield dimensionless groups controlling the phenomenon. Flow chart below shows the procedures.

Rayleigh's Method

Flow chart below shows the procedures,



$$X = CX_1^a X_2^b X_3^c \cdots X_n^m$$

where C = dimensionless constant

$a, b, c, \ldots, m$ are arbitrary exponents.

X_n = independent variables

The Rayleigh Method has limitations because of the premise that an exponential relationship exists between the variables

Rayleigh's Method

- An elementary method for finding a functional relationship with respect to a parameter in interest is the Rayleigh Method, and will be illustrated with an example, using the MLT system.
- Say that we are interested in the drag, D, which is a force on a ship. What exactly is the drag a function of? These variables need to be chosen correctly, though selection of such variables depends largely on one's experience in the topic. It is known that drag depends on

Quantity	Symbol	Dimension
Size	l	L
Viscosity	μ	M/LT
Density	ρ	M/L^3
Velocity	V	L/T
Gravity	g	L/T^2

Cont…

This means that $D = f(l,\rho,\mu V,g)$ where f is some function.

- With the Rayleigh Method, we assume that $D=Cl^a\rho^b\mu^cV^dg^e$, where C is a dimensionless constant, and a,b,c,d, and e are exponents, whose values are not yet known.
- Note that the dimensions of the left side, force, must equal those on the right side. Here, we use only the three independent dimensions for the variables on the right side: M, L, and T.

Procedure

- Step1:Establish relationship between dependent and independent variable.
- Step2: Setup an equation between dependent and independent variable. The dependent variable is expressed as a product of all the independent variables raised to unknown integer exponents. (a,b,c,d….. are unknown integer exponents)
- Step3: Form a tabular column representing the variables in the equation, their units and dimensions.
- Step4: Use the dimension in step 3 to obtain the unknown value a,b,c,d,…in step 2 by using the principles of dimensional homogeneity of the variables.
- Step5:Substitute the unknown values of a,b,c,d… in the equation which was formed in step 2.

Example

The velocity of propagation of a pressure wave through a liquid can be expected to depend on the elasticity of the liquid represented by the bulk modulus K, and its mass density ρ. Establish by D. A. the form of the possible relationship.

Solution: $U = \text{velocity} = \frac{L}{T}, \rho = \frac{M}{L^3}, K = \frac{M}{LT^2}$

$U = c\rho^a K^b$

$\frac{L}{T} = (\frac{M}{L^3})^a (\frac{M}{LT^2})^b$

M: $0 = a + b$

L: $1 = -a - 3b$

T: $-1 = -2a \quad 7$

Therefore: $a = ½$, $b = -½$, and a possible equation is:

$U = C\sqrt{\frac{K}{\rho}}$

Both Buckingham's method and Rayleigh's method of D.A determine only the relevant independent dimensionless parameters of a problem but not the exact relationship between them.

Do it Yourself

Question:

The time period of the simple pendulum depends upon the length of the simple pendulum and the acceleration due to gravity. Obtain the functional relationship for the time period of simple pendulum.

where, T-Time period

L-Length of pendulum

g- Acceleration due to gravity.

Definition

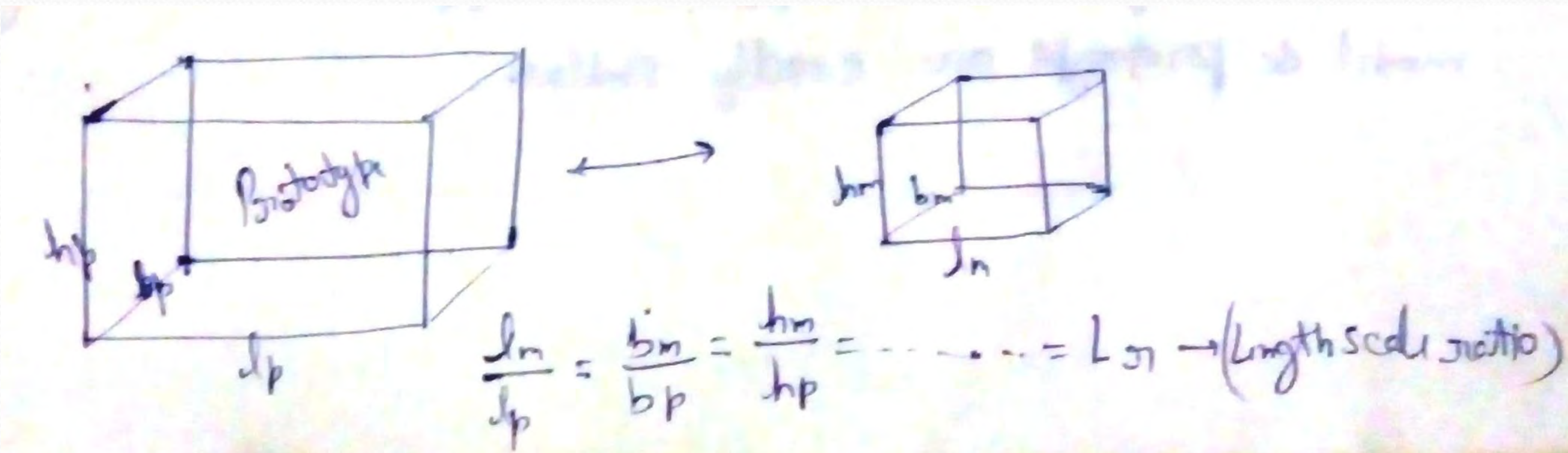
It is a branch of science in which we deal the experimental prototype studies just by doing the experiments on the modes which are comparatively smaller versions of the prototypes. The experimental results of the models will be in a position to predict the results for the prototype if the similarity exists between the prototype and its model.

Similarities between model and prototype:

1. Geometric similarity
2. Kinematic Similarity
3. Dynamic Similarity

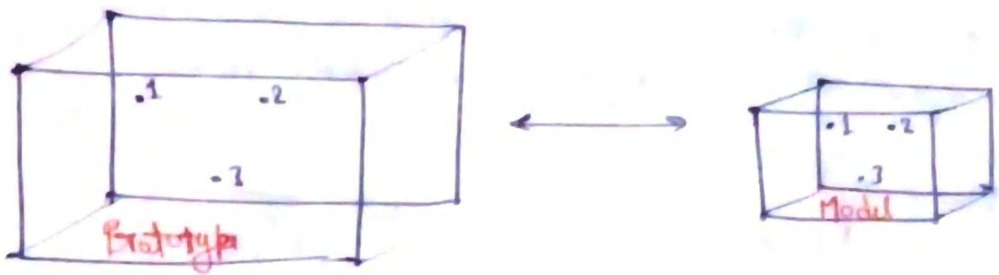
Geometric similarity

The model and its prototype will be geometrically similar if every dimension in the prototype is reduced to the same scale in the model.



Kinematic Similarity

It is used for Velocities and Acceleration



$$\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = \frac{(V_3)_m}{(V_3)_p} = \dots \dots = V_r \Rightarrow \text{velocity scale ratio}$$

$$V_r = \frac{V_m}{V_p} = \frac{L_m/t_m}{L_p/t_p} = \frac{L_m/L_p}{t_m/t_p}$$

$L_m/L_p \rightarrow$ length scale ratio

$t_m/t_p \rightarrow$ time scale ratio

$$\boxed{V_r = \frac{L_r}{t_r}}$$

Accⁿ

$$\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} \dots \dots = a_r = \text{Acc}^n \text{ scale ratio}$$

$$\boxed{a_r = \frac{V_r}{t_r} = \frac{L_r}{t_r^2}}$$

Dynamic Similarity

It is the similarities of the forces between the model and prototype.

$$\frac{(Fv)_m}{(Fv)_p} = \frac{(F\sigma)_m}{(F\sigma)_p} = \frac{(F\ elastic)_m}{(F\ elastic)_p} = \frac{(Fp)_m}{(Fp)_p} = Fr(Force\ Scale\ Ratio)$$

NOTE: If all three similarities exists then we can say the model and prototype are exactly similar.

Method

If a dimensional analysis indicates that a problem is described by a functional relationship between non-dimensional parameters $\Pi_1, \Pi_2, \Pi_3, \ldots$ then full *similarity requires that these parameters be the same at both full ("prototype") scale and model scale i.e.*

$$(\Pi_1)_m = (\Pi_1)_p$$

$$(\Pi_2)_m = (\Pi_2)_p \text{ etc.}$$

Example

Example.

A prototype gate valve which will control the flow in a pipe system conveying paraffin is to be studied in a model. List the significant variables on which the pressure drop across the valve would depend. Perform dimensional analysis to obtain the relevant non-dimensional groups.

A 1/5 scale model is built to determine the pressure drop across the valve with water as the working fluid.

(a) For a particular opening, when the velocity of paraffin in the prototype is 3.0 m s^{-1} what should be the velocity of water in the model for dynamic similarity?

(b) What is the ratio of the quantities of flow in prototype and model?

(c) Find the pressure drop in the prototype if it is 60 kPa in the model.

(The density and viscosity of paraffin are 800 kg m^{-3} and 0.002 kg m^{-1} s^{-1} respectively. Take the kinematic viscosity of water as 1.0 x10^{-6} m^2 s^{-1})

Steps involved

Solution.

The pressure drop Δp *is expected to depend upon the gate opening h, the overall depth d, the velocity V, density ρ and viscosity μ.*

List the relevant variables:

$\Delta p, h, d, V, \rho, \mu$

Write down dimensions:

Δp	$ML^{-1}T^{-2}$
h	L
d	L
V	LT^{-1}
ρ	ML^{-3}
μ	$ML^{-1}T^{-1}$

Number of variables: $n = 6$

Number of independent dimensions: $m = 3$ *(M, L and T)*

Number of non-dimensional groups: $n - m = 3$

Cont…

Choose m (= 3) scaling variables:

geometric (d); kinematic/time-dependent (V); dynamic/mass-dependent (ρ).

Form dimensionless groups by non-dimensionalising the remaining variables: Δp, h and μ.

$$\Pi_1 = \Delta p d^a V^b \rho^c$$

$$M^0L^0T^0 = (ML^{-1}T^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c$$

$$= M^{1+c}L^{-1+a+b-3c}T^{-2-b}$$

M: $\quad 0 = 1 + c \quad \Rightarrow \quad c = -1$

T: $\quad 0 = -2 - b \quad \Rightarrow \quad b = -2$

L: $\quad 0 = -1 + a + b - 3c \quad \Rightarrow \quad a = 1 + 3c - b \quad = 0$

$$\Rightarrow \qquad \Pi_1 = \Delta p V^{-2}\rho^{-1} \quad = \frac{\Delta p}{\rho V^2}$$

$$\Pi_2 = \frac{h}{d}$$ (by inspection, since h is a length)

$$\Pi_3 = \mu d^a V^b \rho^c$$ (probably obvious by now, but here goes anyway …)

$$M^0L^0T^0 = (ML^{-1}T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c$$

Cont…

$$= \mathrm{M}^{1+c}\mathrm{L}^{-1+a+b-3c}\mathrm{T}^{-1-b}$$

$$\begin{array}{lll} \mathrm{M}: & 0 = 1 + c & \Rightarrow \; c = -1 \\ \mathrm{T}: & 0 = -1 - b + 0 & \Rightarrow \; b = -1 \\ \mathrm{L}: & 0 = -1 + a + b - 3c & \Rightarrow \; a = 1 + 3c - b \quad = -1 \end{array}$$

$$\Rightarrow \qquad \Pi_3 = \mu d^{-1} V^{-1} \rho^{-1} \quad = \frac{\mu}{\rho V d}$$

Recognition of the Reynolds number suggests that we replace Π_3 by

$$\Pi_3' = (\Pi_3)^{-1} = \frac{\rho V d}{\mu}$$

Hence, dimensional analysis yields

$$\Pi_1 = f(\Pi_2, \Pi_3')$$

i.e.

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{h}{d}, \frac{\rho V d}{\mu}\right)$$

Cont…

(a) Dynamic similarity requires that all non-dimensional groups be the same in model and prototype; i.e.

$$\Pi_1 = \left(\frac{\Delta p}{\rho V^2}\right)_p = \left(\frac{\Delta p}{\rho V^2}\right)_m$$

$$\Pi_2 = \left(\frac{h}{d}\right)_p = \left(\frac{h}{d}\right)_m$$ (automatic if similar shape; i.e. "geometric similarity")

$$\Pi_3' = \left(\frac{\rho V d}{\mu}\right)_p = \left(\frac{\rho V d}{\mu}\right)_m$$

From the last, we have a velocity ratio

$$\frac{V_p}{V_m} = \frac{(\mu/\rho)_p}{(\mu/\rho)_m}\frac{d_m}{d_p} = \frac{0.002/800}{1.0\times 10^{-6}} \times \frac{1}{5} = 0.5$$

Hence,

$$V_m = \frac{V_p}{0.5} = \frac{3.0}{0.5} = 6.0\ \mathrm{m\,s^{-1}}$$

Cont...

(b) The ratio of the quantities of flow is

$$\frac{Q_p}{Q_m} = \frac{(velocity \times area)_p}{(velocity \times area)_m} = \frac{V_p}{V_m}\left(\frac{d_p}{d_m}\right)^2 = 0.5 \times 5^2 = 12.5$$

(c) Finally, for the pressure drop,

$$\Pi_1 = \left(\frac{\Delta p}{\rho V^2}\right)_p = \left(\frac{\Delta p}{\rho V^2}\right)_m \Rightarrow \frac{(\Delta p)_p}{(\Delta p)_m} = \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2 = \frac{800}{1000} \times 0.5^2 = 0.2$$

Hence,

$$\Delta p_p = 0.2 \times \Delta p_m = 0.2 \times 60 = 12.0\,\text{kPa}$$

Do it Yourself

Question:

The force exerted on a bridge pier in a river is to be tested in a 1:10 scale model using water as the working fluid. In the prototype the depth of water is 2.0 m, the velocity of flow is 1.5 m s–1 and the width of the river is 20 m. What are the depth, velocity and quantity of flow in the model?

Submit Your answer by mail/whatsapp.

Incomplete Similarity

For a multi-parameter problem it is often not possible to achieve full similarity. In particular, it is rare to be able to achieve full Reynolds-number scaling when other dimensionless parameters are also involved. For hydraulic modelling of flows with a free surface the most important requirement is *Froude-number scaling* .

It is common to distinguish three levels of similarity.

similarities

- Geometric similarity – the ratio of all corresponding lengths in model and prototype are the same (i.e. they have the same shape).
- Kinematic similarity – the ratio of all corresponding lengths and times (and hence the ratios of all corresponding velocities) in model and prototype are the same.
- Dynamic similarity – the ratio of all forces in model and prototype are the same; e.g. Re = (inertial force) / (viscous force) is the same in both.

Cont…

Geometric similarity is almost always assumed. However, in some applications – notably river modelling – it is necessary to distort vertical scales to prevent undue influence of, for example, surface tension or bed roughness.

Achieving full similarity is particularly a problem with the Reynolds number Re = UL/v.

- Using the same working fluid would require a velocity ratio inversely proportional to the length-scale ratio and hence impractically large velocities in the scale model.
- A velocity scale fixed by, for example, the Froude number means that the only way to maintain the same Reynolds number is to adjust the kinematic viscosity (substantially).

Cont…

In practice, Reynolds-number similarity is unimportant if flows in both model and prototype are fully turbulent; then momentum transport by viscous stresses is much less than that by turbulent eddies and so the precise value of molecular viscosity μ is unimportant. In some cases this may mean deliberately triggering transition to turbulence in boundary layers (for example by the use of tripping wires or roughness strips).

Surface Effects

Full geometric similarity requires that not only the main dimensions of objects but also the surface roughness and, for mobile beds, the sediment size be in proportion. This would put impossible requirements on surface finish or grain size. In practice, it is sufficient that the surface be aerodynamically rough: $u_\tau k_s/\nu \geq 5$, where $u_\tau = \sqrt{\tau_w/\rho}$ is the friction velocity and ks a typical height of surface irregularities. This imposes a minimum velocity in model tests.

Froude-Number Scaling

The most important parameter to preserve in hydraulic modelling of free-surface flows driven by gravity is the Froude number, $\mathrm{Fr} = U/\sqrt{gL}$. Preserving this parameter between model (m) and prototype (p) dictates the scaling of other variables in terms of the length scale ratio.

<u>Velocity</u>

$$(\mathrm{Fr})_m = (\mathrm{Fr})_p$$

$$\left(\frac{U}{\sqrt{gL}}\right)_m = \left(\frac{U}{\sqrt{gL}}\right)_p \quad \Rightarrow \quad \frac{U_m}{U_p} = \left(\frac{L_m}{L_p}\right)^{1/2}$$

i.e. the velocity ratio is the square root of the length-scale ratio.

<u>Quantity of flow</u>

$$Q \sim \text{velocity} \times \text{area} \quad \Rightarrow \quad \frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)^{5/2}$$

<u>Force</u>

$$F \sim \text{pressure} \times \text{area} \quad \Rightarrow \quad \frac{F_m}{F_p} = \left(\frac{L_m}{L_p}\right)^{3}$$

Cont…

This arises since the pressure ratio is equal to the length-scale ratio – this can be seen from either hydrostatics (pressure α height) or from the dynamic pressure (proportional to (velocity)^2 which, from the Froude number, is proportional to length).

<u>Time</u>

$$t \sim \text{length} \div \text{velocity} \quad \Rightarrow \quad \frac{t_m}{t_p} = \left(\frac{L_m}{L_p}\right)^{1/2}$$

Hence the quantity of flow scales as the length-scale ratio to the 5/2 power, whilst the time-scale ratio is the square root of the length-scale ratio. For example, at 1:100 geometric scale, a full-scale tidal period of 12.4 hours becomes 1.24 hours.

NON-DIMENSIONAL GROUPS IN FLUID MECHANICS

Dynamic similarity requires that the ratio of all forces be the same. The ratio of different forces produces many of the key non-dimensional parameters in fluid mechanics.

(Note that "inertial force" means "mass x acceleration" – since it is equal to the total applied force it is often one of the two "forces" in the ratio.)

Reynolds number	$\mathrm{Re} = \dfrac{\rho U L}{\mu}$	$= \dfrac{inertial\ \ force}{viscous\ \ force}$	(viscous flows)
Froude number	$\mathrm{Fr} = \dfrac{U}{\sqrt{gL}}$	$= \left(\dfrac{inertial\ \ force}{gravitational\ \ force}\right)^{1/2}$	(free-surface flows)
Weber number	$\mathrm{We} = \dfrac{\rho U^2 L}{\sigma}$	$= \dfrac{inertial\ \ force}{surface\ tension}$	(surface tension)
Rossby number	$\mathrm{Ro} = \dfrac{U}{\Omega L}$	$= \dfrac{inertial\ \ force}{Coriolis\ force}$	(rotating flows)
Mach number	$\mathrm{Ma} = \dfrac{U}{c}$	$= \left(\dfrac{inertial\ \ force}{compressibility\ force}\right)^{1/2}$	(compressible flows)

Note

These groups occur regularly when dimensional analysis is applied to fluid-dynamical problems. They can be derived by considering forces on a small volume of fluid. They can also be derived by non-dimensionalising the differential equations of fluid flow .

Inertia Force (F_I)

Inertial force, as the name implies is the force due to the momentum of the fluid. This is usually expressed in the momentum equation by the term (ρv)v. So, the denser a fluid is, and the higher its velocity, the more momentum (inertia) it has.

$$F_I = \rho\, L^2\, V^2$$

Where ρ= Density

L=Characteristic dimension of the system.

V= Flow Velocity

Viscous Force (F_V)

The **viscous force** is the force between a body and a fluid (liquid or gas) moving past it, in a direction so as to oppose the flow of the fluid past the object. In particular, the force acts on the object in the direction in which the *fluid* is moving relative to it (and hence, opposite to the direction in which *it* is moving relative to the fluid).

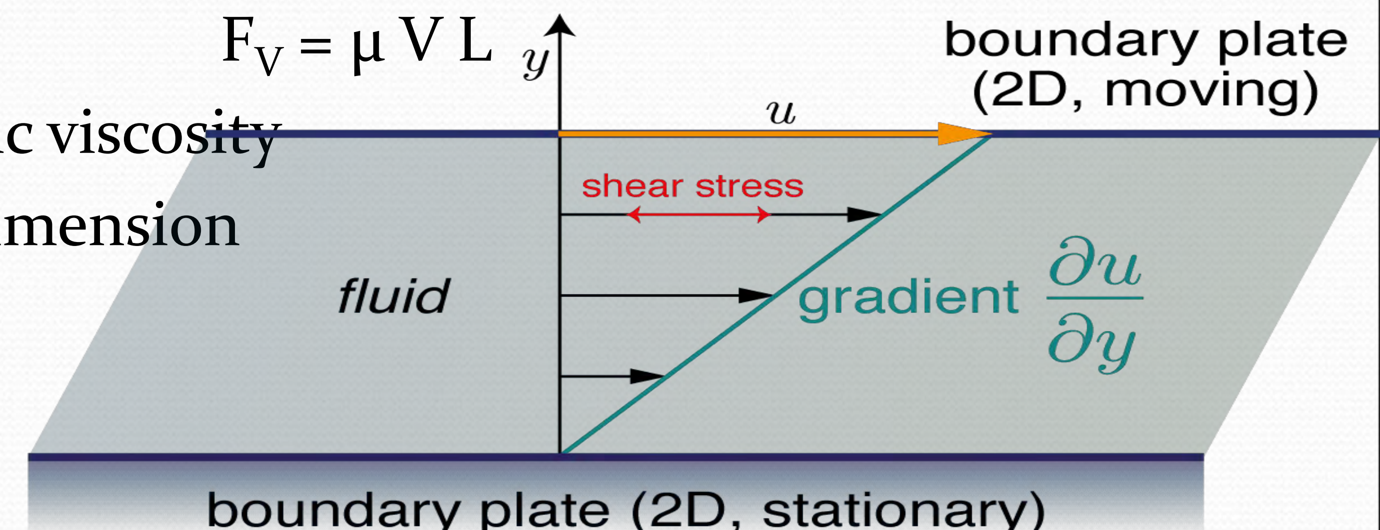
Viscous force is an analogue in fluids of the force of friction.

$$F_V = \mu\, V\, L$$

Where μ = Dynamic viscosity

L=Characteristic dimension of the system.

V= Flow Velocity



Pressure Force (Fp)

(Pressure force *Or* pressure-gradient force.)

The force due to differences of pressure within a fluid mass.

Pressure (symbol: *p* or *P*) is the force applied perpendicular to the surface of an object per unit area over which that force is distributed.

Gauge pressure (also spelled *gage* pressure) is the pressure relative to the ambient pressure

$$F_P = \Delta p\, L^2$$

Where Δp= Pressure difference

L=Characteristic dimension of the system.

Gravitational Force (Fg)

The **gravitational force** is a force that attracts any two objects with mass. We call the gravitational force *attractive* because it always tries to pull masses together, it never pushes them apart. In fact, every object, including you, is pulling on every other object in the entire universe! This is called Newton's **Universal Law of Gravitation**. Admittedly, you don't have a very large mass and so, you're not pulling on those other objects much. And objects that are really far apart from each other don't pull on each other noticeably either. But the force is there and we can calculate it

$$F_g = m\,g$$

$$F_g = \rho\, L^3\, g$$

Where ρ = Density, m= Mass, g= Acceleration due to gravity

L=Characteristic dimension of the system.

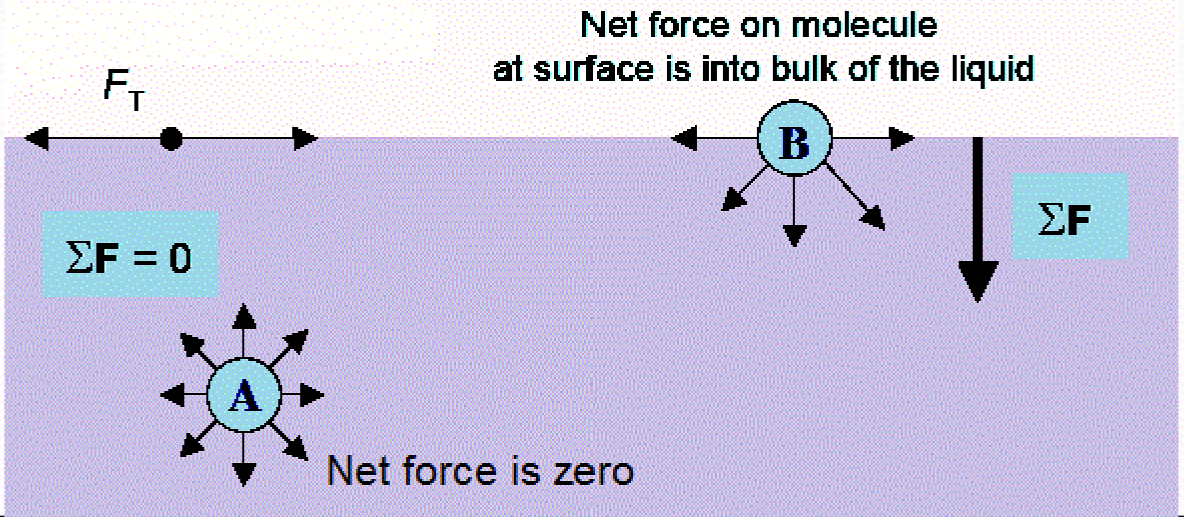
Surface Tension Force (F_σ)

Surface tension is the tendency of liquid surfaces to shrink into the minimum surface area possible. Surface tension allows insects (e.g. water striders), usually denser than water, to float and slide on a water surface.

$$F_\sigma = \sigma L$$

Where σ = Surface Tension

L=Characteristic dimension of the system.



Compressibility Force ($F_{elastic}$)

Compressibility of any substance is the measure of its change in volume under the action of external forces.

The normal compressive stress on any fluid element at rest is known as hydrostatic pressure p and arises as a result of innumerable molecular collisions in the entire fluid.

$$F_{Elastic} = K.\,A$$

$$F_{Elastic} = K.\,L^2$$

Where K = Bulk modulus of elasticity for medium

A= Area

L=Characteristic dimension of the system.

Different Model's Law

- Reynold's Model Law:

  $$(R_e)p = (R_e)m$$

- Euler's Model Law:

  $$(E_u)p = (E_u)m$$

- Froude's Model Law:

  $$(F_r)p = (F_r)m$$

- Weber's Model's Law:

  $$(W_b)p = (W_b)m$$

- Mach's Model Law:

  $$(M_a)p = (M_a)m$$

These models are used to find out various scale ratio for construction various prototypes and models of fluid system.

Example

A Model of a fluid flow system is constructed on the basis of reynold's model law. Find the velocity scale ratio and the discharge scale ratio.

Solution:

$$(R_e)p = (R_e)m$$

$$\left(\frac{\rho VL}{\mu}\right)_p = \left(\frac{\rho VL}{\mu}\right)_m$$

$$\frac{V_p L_p}{\nu_p} = \frac{V_m L_m}{\nu_M}$$

$$\frac{V_m}{V_p} = \frac{L_p}{L_m} \cdot \frac{\nu_m}{\nu_p} = \frac{(\nu_m/\nu_p)}{(L_m/L_p)}$$

velocity scale ratio $\boxed{V_r = \frac{\nu_r}{L_r}}$

Discharge scale ratio $Q_r = \frac{Q_m}{Q_p} = A_r \cdot V_r = L_r^2 \cdot \frac{\nu_r}{L_r}$

$$\boxed{Q_r = L_r \cdot \nu_r}.$$

Ans

Do It Yourself

Question:

A model is going to be constructed on the basis of reynold's model law as well as froude's model law. Calculate the velocity scale ratio and discharge scale ratio in terms of length scale ratio.