# APPLIED THERMODYNAMICS

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# INTRODUCTION TO FUELS

The various types of fuels like liquid, solid and gaseous fuels are available for firing in boilers, furnaces and other combustion equipments. The selection of right type of fuel depends on various factors such as availability, storage, handling, pollution and landed cost of fuel. The knowledge of the fuel properties helps in selecting the right fuel for the right purpose and efficient use of the fuel. The following characteristics, determined by laboratory tests, are generally used for assessing the nature and quality of fuels.

# PROPERTIES OF LIQUID FUELS

Liquid fuels like furnace oil and LSHS are predominantly used in industrial application. The various properties of liquid fuels are given below.

**Density**

This is defined as the ratio of the mass of the fuel to the volume of the fuel at a reference temperature of 15°C. Density is measured by an instrument called hydrometer. The knowledge of density is useful for quantity calculations and assessing ignition quality. The unit of density is kg/m3.

**Specific gravity**

This is defined as the ratio of the weight of a given volume of oil to the weight of the same volume of water at a given temperature. The density of fuel, relative to water, is called specific gravity. The specific gravity of water is defined as 1. Since specific gravity is a ratio, it has no units. The measurement of specific gravity is generally made by a hydrometer. Specific gravity is used in calculations involving weights and volumes. The specific gravity of various fuel oils are given in Table

# PROPERTIES OF LIQUID FUELS

| TABLE 1 SPECIFIC GRAVITY OF VARIOUS FUEL OILS | | | |
|---|---|---|---|
| Fuel Oil | L.D.O<br>Light Diesel Oil | Furnace oil | L.S.H.S<br>Low Sulphur<br>Heavy Stock |
| Specific Gravity | 0.85-0.87 | 0.89-0.95 | 0.88-0.98 |

# PROPERTIES OF LIQUID FUELS (VISCOSITY)

The viscosity of a fluid is a measure of its internal resistance to flow. Viscosity depends on temperature and decreases as the temperature increases. Any numerical value for viscosity has no meaning unless the temperature is also specified. Viscosity is measured in Stokes / Centistokes. Sometimes viscosity is also quoted in Engler, Saybolt or Redwood. Each type of oil has its own temperature - viscosity relationship. The measurement of viscosity is made with an instrument called Viscometer. Viscosity is the most important characteristic in the storage and use of fuel oil. It influences the degree of pre-heat required for handling, storage and satisfactory atomization. If the oil is too viscous, it may become difficult to pump, hard to light the burner, and tough to operate. Poor atomization may result in the formation of carbon deposits on the burner tips or on the walls. Therefore pre-heating is necessary for proper atomization.

# PROPERTIES OF LIQUID FUELS

**Flash Point**

The flash point of a fuel is the lowest temperature at which the fuel can be heated so that the vapour gives off flashes momentarily when an open flame is passed over it. Flash point for furnace oil is 66°C.

**Pour Point**

The pour point of a fuel is the lowest temperature at which it will pour or flow when cooled under prescribed conditions. It is a very rough indication of the lowest temperature at which fuel oil is readily pumpable.

**Specific Heat**

Specific heat is the amount of kCals needed to raise the temperature of 1 kg of oil by 1°C. The unit of specific heat is kCal/kg°C. It varies from 0.22 to 0.28 depending on the oil specific gravity. The specific heat determines how much steam or electrical energy it takes to heat oil to a desired temperature. Light oils have a low specific heat, whereas heavier oils have a higher specific heat.

# PROPERTIES OF LIQUID FUELS

**Calorific Value**

The calorific value is the measurement of heat or energy produced, and is measured either as gross calorific value or net calorific value. The difference being the latent heat of condensation of the water vapour produced during the combustion process. Gross calorific value (GCV) assumes all vapour produced during the combustion process is fully condensed. Net calorific value (NCV) assumes the water leaves with the combustion products without fully being condensed. Fuels should be compared based on the net calorific value. The calorific value of coal varies considerably depending on the ash, moisture content and the type of coal while calorific value of fuel oils are much more consistent. The typical Gross Calorific Values of some of the commonly used liquid fuels are given below:

# TYPICAL SPECIFICATION OF FUEL OILS

**TABLE 2 TYPICAL SPECIFICATION OF FUEL OILS**

| Properties | Fuel Oils | | |
|---|---|---|---|
| | Furnace Oil | L.S.H.S. | L.D.O. |
| Density (Approx. g/cc at 15°C) | 0.89–0.95 | 0.88–0.98 | 0.85–0.87 |
| Flash Point (°C) | 66 | 93 | 66 |
| Pour Point (°C) | 20 | 72 | 18 |
| G.C.V. (kCal/kg) | 10,500 | 10,600 | 10,700 |
| Sediment, % Wt. Max. | 0.25 | 0.25 | 0.1 |
| Sulphur Total, % Wt. Max. | Upto 4.0 | Upto 0.5 | Upto 1.8 |
| Water Content, % Vol. Max. | 1.0 | 1.0 | 0.25 |
| Ash % Wt. Max. | 0.1 | 0.1 | 0.02 |

# STORAGE OF FUEL OIL

It can be potentially hazardous to store furnace oil in barrels. A better practice is to store it in cylindrical tanks, either above or below the ground. Furnace oil, that is delivered, may contain dust, water and other contaminants. The sizing of storage tank facility is very important. A recommended storage estimate is to provide for at least 10 days of normal consumption. Industrial heating fuel storage tanks are generally vertical mild steel tanks mounted above ground. It is prudent for safety and environmental reasons to build bund walls around tanks to contain accidental spillages. As a certain amount of settlement of solids and sludge will occur in tanks over time, cleaning should be carried out at regular intervals-annually for heavy fuels and every two years for light fuels. A little care should be taken when oil is decanted from the tanker to storage tank. All leaks from joints, flanges and pipelines must be attended at the earliest. Fuel oil should be free from possible contaminants such as dirt, sludge and water before it is fed to the combustion system.

# REMOVAL OF CONTAMINANTS

Furnace oil arrives at the factory site either in tank lorries by road or by rail. Oil is then decanted into the main storage tank. To prevent contaminants such as rags, cotton waste, loose nuts or bolts or screws entering the system and damaging the pump, coarse strainer of 10 mesh size (not more than 3 holes per linear inch) is positioned on the entry pipe to the storage tanks. Progressively finer strainers should be provided at various points in the oil supply system to filter away finer contaminants such as external dust and dirt, sludge or free carbon. It is advisable to provide these filters in duplicate to enable one filter to be cleaned while oil supply is maintained through the other.

# REMOVAL OF CONTAMINANTS

**Duplex Arrangement of Strainers in a Pipeline**

The Figure gives an illustration of the duplex system of arrangement of strainers.

The Table gives sizing of strainers at various locations.

# REMOVAL OF CONTAMINANTS

| SIZING OF STRAINERS | | |
|---|---|---|
| Location | Strainer Sizes | |
| | Mesh | Holes/Linear inch |
| Between rail/tank lorry decanting point and main storage tank | 10 | 3 |
| Between service tank and pre-heater | 40 | 6 |
| Between pre-heater and burner | 100 | 10 |

# PUMPING

Heavy fuel oils are best pumped using positive displacement pumps, as they are able to get fuel moving when it is cold. A circulation gear pump running on LDO should give between 7000- 10000 hours of service. Diaphragm pumps have a shorter service life, but are easier and less expensive to repair. A centrifugal pump is not recommended, because as the oil viscosity increases, the efficiency of the pump drops sharply and the horsepower required increases. Light fuels are best pumped with centrifugal or turbine pumps. When higher pressures are required, piston or diaphragm pumps should be used.

# STORAGE TEMPERATURE AND PUMPING TEMPERATURE

The viscosity of furnace oil and LSHS increases with decrease in temperature, which makes it difficult to pump the oil. At low ambient temperatures (below 25°C), furnace oil is not easily pumpable. To circumvent this, preheating of oil is accomplished in two ways: (a) the entire tank may be preheated. In this form of bulk heating, steam coils are placed at the bottom of the tank, which is fully insulated; (b) the oil can be heated as it flows out with an outflow heater. To reduce steam requirements, it is advisable to insulate tanks where bulk heating is used. Bulk heating may be necessary if flow rates are high enough to make outflow heaters of adequate capacity impractical, or when a fuel such as Low Sulphur Heavy Stock (LSHS) is used. In the case of outflow heating, only the oil, which leaves the tank, is heated to the pumping temperature. The outflow heater is essentially a heat exchanger with steam or electricity as the heating medium.

# PROPERTIES OF COAL

**Coal Classification**

Coal is classified into three major types namely anthracite, bituminous, and lignite. However there is no clear demarcation between them and coal is also further classified as semianthracite, semi-bituminous, and sub-bituminous. Anthracite is the oldest coal from geological perspective. It is a hard coal composed mainly of carbon with little volatile content and practically no moisture. Lignite is the youngest coal from geological perspective. It is a soft coal composed mainly of volatile matter and moisture content with low fixed carbon. Fixed carbon refers to carbon in its free state, not combined with other elements. Volatile matter refers to those combustible constituents of coal that vaporize when coal is heated. The common coals used in Indian industry are bituminous and sub-bituminous coal. The gradation of Indian coal based on its calorific value is as follows:

# PROPERTIES OF COAL

| Grade | Calorific Value Range ( in kCal/Kg) |
|---|---|
| A | Exceeding 6200 |
| B | 5600 – 6200 |
| C | 4940 – 5600 |
| D | 4200 – 4940 |
| E | 3360 – 4200 |
| F | 2400 – 3360 |
| G | 1300 – 2400 |

Normally D, E and F coal grades are available to Indian Industry.

# PROPERTIES OF COAL

The chemical composition of coal has a strong influence on its combustibility. The properties of coal are broadly classified as

- Physical properties
- Chemical properties

**Physical Properties**

**Heating Value:**
The heating value of coal varies from coal field to coal field. The typical GCVs for various coals are given in the Table

# PROPERTIES OF COAL

| GCV FOR VARIOUS COALS | | | | |
|---|---|---|---|---|
| Parameter | Lignite (Dry Basis) | Indian Coal | Indonesian Coal | South African Coal |
| GCV (kCal/kg) | 4,500* | 4,000 | 5,500 | 6,000 |

*GCV of lignite on '*as received basis*' is 2500 – 3000

# PROPERTIES OF COAL

**Analysis of Coal**

There are two methods: ultimate analysis and proximate analysis. The ultimate analysis determines all coal component elements, solid or gaseous and the proximate analysis determines only the fixed carbon, volatile matter, moisture and ash percentages. The ultimate analysis is determined in a properly equipped laboratory by a skilled chemist, while proximate analysis can be determined with a simple apparatus. It may be noted that proximate has no connection with the word “approximate”.

**Measurement of Moisture**

Determination of moisture is carried out by placing a sample of powdered raw coal of size 200- micron size in an uncovered crucible and it is placed in the oven kept at 108±2°C along with the lid. Then the sample is cooled to room temperature and weighed again. The loss in weight represents moisture.

**Measurement of Volatile Matter**

Fresh sample of crushed coal is weighed, placed in a covered crucible, and heated in a furnace at 900 ± 15°C. For the methodologies including that for carbon and ash, refer to IS 1350 part I:1984, part III, IV. The sample is cooled and weighed. Loss of weight represents moisture and volatile matter. The remainder is coke (fixed carbon and ash).

# PROPERTIES OF COAL

**Measurement of Carbon and Ash**

The cover from the crucible used in the last test is removed and the crucible is heated over the Bunsen burner until all the carbon is burned. The residue is weighed, which is the incombustible ash. The difference in weight from the previous weighing is the fixed carbon. In actual practice Fixed Carbon or FC derived by subtracting from 100 the value of moisture, volatile matter and ash.

**Proximate Analysis**

Proximate analysis indicates the percentage by weight of the Fixed Carbon, Volatiles, Ash, and Moisture Content in coal. The amounts of fixed carbon and volatile combustible matter directly contribute to the heating value of coal. Fixed carbon acts as a main heat generator during burning. High volatile matter content indicates easy ignition of fuel. The ash content is important in the design of the furnace grate, combustion volume, pollution control equipment and ash handling systems of a furnace. A typical proximate analysis of various coal is given in the Table

# PROPERTIES OF COAL

**TYPICAL PROXIMATE ANALYSIS OF VARIOUS COALS (IN PERCENTAGE)**

| Parameter | Indian Coal | Indonesian Coal | South African Coal |
|---|---|---|---|
| Moisture | 5.98 | 9.43 | 8.5 |
| Ash | 38.63 | 13.99 | 17 |
| Volatile matter | 20.70 | 29.79 | 23.28 |
| Fixed Carbon | 34.69 | 46.79 | 51.22 |

# SIGNIFICANCE OF VARIOUS PARAMETERS IN PROXIMATE ANALYSIS

**(a) Fixed carbon:**

Fixed carbon is the solid fuel left in the furnace after volatile matter is distilled off. It consists mostly of carbon but also contains some hydrogen, oxygen, sulphur and nitrogen not driven off with the gases. Fixed carbon gives a rough estimate of heating value of coal.

**(b) Volatile Matter:**

Volatile matters are the methane, hydrocarbons, hydrogen and carbon monoxide, and incombustible gases like carbon dioxide and nitrogen found in coal. Thus the volatile matter is an index of the gaseous fuels present. Typical range of volatile matter is 20 to 35%. Volatile Matter

- Proportionately increases flame length, and helps in easier ignition of coal.
- Sets minimum limit on the furnace height and volume.
- Influences secondary air requirement and distribution aspects.
- Influences secondary oil support

# SIGNIFICANCE OF VARIOUS PARAMETERS IN PROXIMATE ANALYSIS

**(c) Ash Content:**

Ash is an impurity that will not burn. Typical range is 5 to 40% Ash

- Reduces handling and burning capacity.
- Increases handling costs.
- Affects combustion efficiency and boiler efficiency
- Causes clinkering and slagging.

**(d) Moisture Content:**

Moisture in coal must be transported, handled and stored. Since it replaces combustible matter, it decreases the heat content per kg of coal. Typical range is 0.5 to 10% Moisture

- Increases heat loss, due to evaporation and superheating of vapour
- Helps, to a limit, in binding fines.
- Aids radiation heat transfer.

# SIGNIFICANCE OF VARIOUS PARAMETERS IN PROXIMATE ANALYSIS

**(e) Sulphur Content:**

Typical range is 0.5 to 0.8% normally.
Sulphur

- Affects clinkering and slagging tendencies
- Corrodes chimney and other equipment such as air heaters and economisers
- Limits exit flue gas temperature.

**Chemical Properties**

**Ultimate Analysis:**

The ultimate analysis indicates the various elemental chemical constituents such as Carbon, Hydrogen, Oxygen, Sulphur, etc. It is useful in determining the quantity of air required for combustion and the volume and composition of the combustion gases. This information is required for the calculation of flame temperature and the flue duct design etc. Typical ultimate analyses of various coals are given in the Table

# SIGNIFICANCE OF VARIOUS PARAMETERS IN PROXIMATE ANALYSIS

**TYPICAL ULTIMATE ANALYSIS OF COALS**

| Parameter | Indian Coal, % | Indonesian Coal, % |
|---|---|---|
| Moisture | 5.98 | 9.43 |
| Mineral Matter (1.1 × Ash) | 38.63 | 13.99 |
| Carbon | 41.11 | 58.96 |
| Hydrogen | 2.76 | 4.16 |
| Nitrogen | 1.22 | 1.02 |
| Sulphur | 0.41 | 0.56 |
| Oxygen | 9.89 | 11.88 |

**RELATIONSHIP BETWEEN ULTIMATE ANALYSIS AND PROXIMATE ANALYSIS**

| | | | |
|---|---|---|---|
| | %C | = | $0.97C + 0.7(VM - 0.1A) - M(0.6 - 0.01M)$ |
| | %H | = | $0.036C + 0.086(VM - 0.1xA) - 0.0035M^2(1 - 0.02M)$ |
| | $\%N_2$ | = | $2.10 - 0.020\ VM$ |
| where | C | = | % of fixed carbon |
| | A | = | % of ash |
| | VM | = | % of volatile matter |
| | M | = | % of moisture |

**Note: The above equation is valid for coal containing greater than 15% Moisture content.**

# PREPARATION OF COAL

Preparation of coal prior to feeding into the boiler is an important step for achieving good combustion. Large and irregular lumps of coal may cause the following problems:

1. Poor combustion conditions and inadequate furnace temperature.
2. Higher excess air resulting in higher stack loss.
3. Increase of un burnts in the ash.
4. Low thermal efficiency.

**(a) Sizing of Coal**

Proper coal sizing is one of the key measures to ensure efficient combustion. Proper coal sizing, with specific relevance to the type of firing system, helps towards even burning, reduced ash losses and better combustion efficiency. Coal is reduced in size by crushing and pulverizing. Pre-crushed coal can be economical for smaller units, especially those which are stoker fired. In a coal handling system, crushing is limited to a top size of 6 or 4 mm. The devices most commonly used for crushing are the rotary breaker, the roll crusher and the hammer mill. It is necessary to screen the coal before crushing, so that only oversized coal is fed to the crusher. This helps to reduce power consumption in the crusher. Recommended practices in coal crushing are:

1. Incorporation of a screen to separate fines and small particles to avoid extra fine generation in crushing.
2. Incorporation of a magnetic separator to separate iron pieces in coal, which may damage the crusher.

# PREPARATION OF COAL

**PROPER SIZE OF COAL FOR VARIOUS TYPES OF FIRING SYSTEM**

| S. No. | Types of Firing System | Size (in mm) |
|---|---|---|
| 1. | Hand Firing | |
| | (a) Natural draft | 25–75 |
| | (b) Forced draft | 25–40 |
| 2. | Stoker Firing | |
| | (a) Chain grate | |
| | i) Natural draft | 25–40 |
| | ii) Forced draft | 15–25 |
| | (b) Spreader Stoker | 15–25 |
| 3. | Pulverized Fuel Fired | 75% below 75 micron* |
| 4. | Fluidized bed boiler | < 10 mm |

*1 Micron = 1/1000 mm

# PREPARATION OF COAL

**(b) Conditioning of Coal**

The fines in coal present problems in combustion on account of segregation effects. Segregation of fines from larger coal pieces can be reduced to a great extent by conditioning coal with water. Water helps fine particles to stick to the bigger lumps due to surface tension of the moisture, thus stopping fines from falling through grate bars or being carried away by the furnace draft. While tempering the coal, care should be taken to ensure that moisture addition is uniform and preferably done in a moving or falling stream of coal. If the percentage of fines in the coal is very high, wetting of coal can decrease the percentage of unburnt carbon and the excess air level required to be supplied for combustion.

# PREPARATION OF COAL

**EXTENT OF WETTING: FINES VS SURFACE MOISTURE IN COAL**

| Fines (%) | Surface Moisture (%) |
|---|---|
| 10 – 15 | 4 – 5 |
| 15 – 20 | 5 – 6 |
| 20 – 25 | 6 – 7 |
| 25 – 30 | 7 – 8 |

# PREPARATION OF COAL

**(c) Blending of Coal**

In case of coal lots having excessive fines, it is advisable to blend the predominantly lumped coal with lots containing excessive fines. Coal blending may thus help to limit the extent of fines in coal being fired to not more than 25%. Blending of different qualities of coal may also help to supply a uniform coal feed to the boiler.

**PROXIMATE ANALYSIS OF TYPICAL COAL**

| | Lignite | Bituminous coal (Sample I) | Bituminous Coal (Sample II) | Indonesian Coal |
|---|---|---|---|---|
| Moisture (%) | 50 | 5.98 | 4.39 | 9.43 |
| Ash (%) | 10.41* | 38.65 | 47.86 | 13.99 |
| Volatile matter (%) | 47.76* | 20.70 | 17.97 | 29.79 |
| Fixed carbon (%) | 41.83* | 34.69 | 29.78 | 46.79 |

*Dry Basis

# PREPARATION OF COAL

**ULTIMATE ANALYSIS OF VARIOUS COALS**

| | Bituminous Coal (Sample I) | Bituminous Coal (Sample II) | Indonesian Coal |
|---|---|---|---|
| Moisture (%) | 5.98 | 4.39 | 9.43 |
| Mineral matter (%) | 38.63 | 47.86 | 13.99 |
| Carbon (%) | 42.11 | 36.22 | 58.96 |
| Hydrogen (%) | 2.76 | 2.64 | 4.16 |
| Nitrogen (%) | 1.22 | 1.09 | 1.02 |
| Sulphur (%) | 0.41 | 0.55 | 0.56 |
| Oxygen (%) | 9.89 | 7.25 | 11.88 |
| GCV (kCal/kg) | 4000 | 3500 | 5500 |

# PROPERTIES OF GASEOUS FUELS

Gaseous fuels in common use are liquefied petroleum gases (LPG), Natural gas, producer gas, blast furnace gas, coke oven gas etc. The calorific value of gaseous fuel is expressed in Kilocalories per normal cubic meter (kCal/Nm3) i.e. at normal temperature (20°C) and pressure (760 mm Hg).

**Calorific Value**

Since most gas combustion appliances cannot utilize the heat content of the water vapour, gross calorific value is of little interest. Fuel should be compared based on the net calorific value. This is especially true for natural gas, since increased hydrogen content results in high water formation during combustion.

# PROPERTIES OF GASEOUS FUELS

**Typical physical and chemical properties of various gaseous fuels are given in Table**

| Fuel Gas | Relative Density | Higher Heating Value kCal/Nm$^3$ | Air/Fuel ratio- m$^3$ of air to m$^3$ of Fuel | Flame Temp. °C | Flame Speed m/s |
|---|---|---|---|---|---|
| Natural Gas | 0.6 | 9350 | 10 | 1954 | 0.290 |
| Propane | 1.52 | 22200 | 25 | 1967 | 0.460 |
| Butane | 1.96 | 28500 | 32 | 1973 | 0.870 |

# NATURAL GAS

Methane is the main constituent of Natural gas and accounting for about 95% of the total volume. Other components are: Ethane, Propane, Butane, Pentane, Nitrogen, Carbon Dioxide, and traces of other gases. Very small amounts of sulphur compounds are also present. Since methane is the largest component of natural gas, generally properties of methane are used when comparing the properties of natural gas to other fuels. Natural gas is a high calorific value fuel requiring no storage facilities. It mixes with air readily and does not produce smoke or soot. It has no sulphur content. It is lighter than air and disperses into air easily in case of leak. A typical comparison of carbon contents in oil, coal and gas is given in the table

# COMPARISON OF CHEMICAL COMPOSITION OF VARIOUS FUELS

| | Fuel Oil | Coal | Natural Gas |
|---|---|---|---|
| Carbon | 84 | 41.11 | 74 |
| Hydrogen | 12 | 2.76 | 25 |
| Sulphur | 3 | 0.41 | - |
| Oxygen | 1 | 9.89 | Trace |
| Nitrogen | Trace | 1.22 | 0.75 |
| Ash | Trace | 38.63 | - |
| Water | Trace | 5.98 | - |

# PROPERTIES OF AGRO RESIDUES

The use of locally available agro residues is on the rise. This includes rice husk, coconut shells, groundnut shells, Coffee husk, Wheat stalk etc. The properties of a few of them are given in the table

**PROXIMATE ANALYSIS OF TYPICAL AGRO RESIDUES**

| | Deoiled Bran | Paddy Husk | Saw Dust | Coconut Shell |
|---|---|---|---|---|
| Moisture | 7.11 | 10.79 | 37.98 | 13.95 |
| Ash | 18.46 | 16.73 | 1.63 | 3.52 |
| Volatile Matter | 59.81 | 56.46 | 81.22 | 61.91 |
| Fixed Carbon | 14.62 | 16.02 | 17.15 | 20.62 |

# PROPERTIES OF AGRO RESIDUES

## ULTIMATE ANALYSIS OF TYPICAL AGRO RESIDUES

| | Deoiled Bran | Paddy Husk | Saw Dust | Coconut Shell |
|---|---|---|---|---|
| Moisture | 7.11 | 10.79 | 37.98 | 13.95 |
| Mineral Matter | 19.77 | 16.73 | 1.63 | 3.52 |
| Carbon | 36.59 | 33.95 | 48.55 | 44.95 |
| Hydrogen | 4.15 | 5.01 | 6.99 | 4.99 |
| Nitrogen | 0.82 | 0.91 | 0.80 | 0.56 |
| Sulphur | 0.54 | 0.09 | 0.10 | 0.08 |
| Oxygen | 31.02 | 32.52 | 41.93 | 31.94 |
| GCV (kCal/kg) | 3151 | 3568 | 4801 | 4565 |

# COMBUSTION

## PRINCIPLE OF COMBUSTION

**Combustion** refers to the rapid oxidation of fuel accompanied by the production of heat, or heat and light. Complete combustion of a fuel is possible only in the presence of an adequate supply of oxygen. Oxygen ($O_2$) is one of the most common elements on earth making up 20.9% of our air. Rapid fuel oxidation results in large amounts of heat. Solid or liquid fuels must be changed to a gas before they will burn. Usually heat is required to change liquids or solids into gases. Fuel gases will burn in their normal state if enough air is present. Most of the 79% of air (that is not oxygen) is nitrogen, with traces of other elements. Nitrogen is considered to be a temperature reducing dilutant that must be present to obtain the oxygen required for combustion. Nitrogen reduces combustion efficiency by absorbing heat from the combustion of fuels and diluting the flue gases. This reduces the heat available for transfer through the heat exchange surfaces. It also increases the volume of combustion by-products, which then have to travel through the heat exchanger and up the stack faster to allow the introduction of additional fuel air mixture.

# COMBUSTION

This nitrogen also can combine with oxygen (particularly at high flame temperatures) to produce oxides of nitrogen (NOx), which are toxic pollutants. Carbon, hydrogen and sulphur in the fuel combine with oxygen in the air to form carbon dioxide, water vapour and sulphur dioxide, releasing 8084 kCals, 28922 kCals & 2224 kCals of heat respectively. Under certain conditions, Carbon may also combine with Oxygen to form Carbon Monoxide, which results in the release of a smaller quantity of heat (2430 kCals/kg of carbon) Carbon burned to $CO_2$ will produce more heat per pound of fuel than when CO or smoke are produced.

# COMBUSTION

| | | | | | |
|---|---|---|---|---|---|
| $C$ | $+ O_2$ | $\rightarrow$ | $CO_2$ | $+$ | 8084 kCals/kg of Carbon |
| $2C$ | $+ O_2$ | $\rightarrow$ | $2\ CO$ | $+$ | 2430 kCals/kg of Carbon |
| $2H_2$ | $+ O_2$ | $\rightarrow$ | $2H_2O$ | $+$ | 28,922 kCals/kg of Hydrogen |
| $S$ | $+ O_2$ | $\rightarrow$ | $SO_2$ | $+$ | 2,224 kCals/kg of Sulphur |

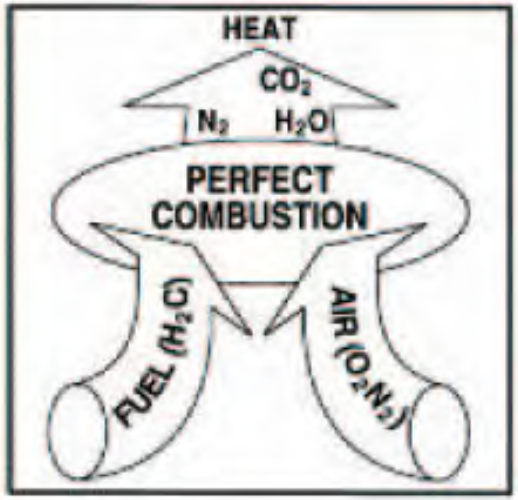
**Each kilogram of CO formed means a loss of 5654 kCal of heat.(8084-2430).**

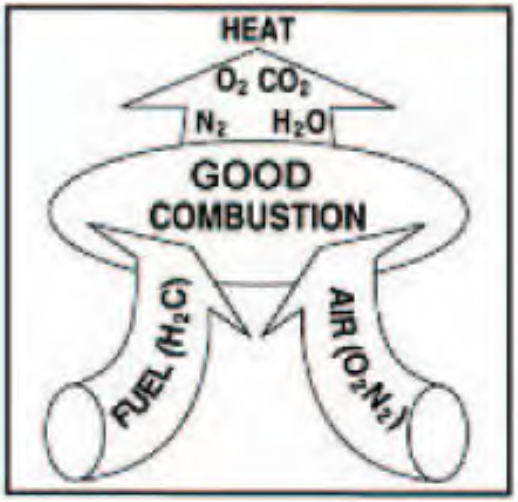
# 3 T'S OF COMBUSTION

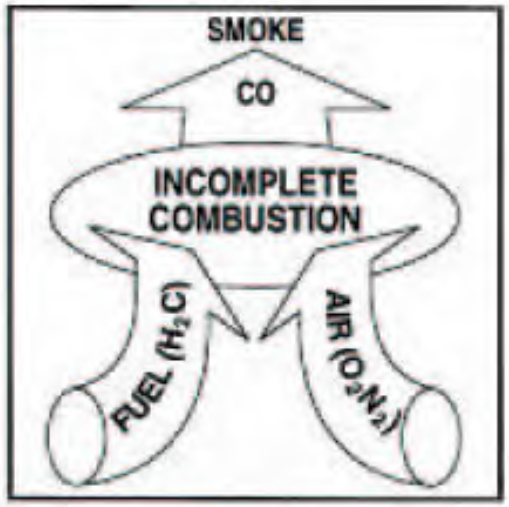
The objective of good combustion is to release all of the heat in the fuel. This is accomplished by controlling the "three T's" of combustion which are (1) Temperature high enough to ignite and maintain ignition of the fuel, (2) Turbulence or intimate mixing of the fuel and oxygen, and (3) Time sufficient for complete combustion. Commonly used fuels like natural gas and propane generally consist of carbon and hydrogen. Water vapor is a by-product of burning hydrogen. This robs heat from the flue gases, which would otherwise be available for more heat transfer. Natural gas contains more hydrogen and less carbon per kg than fuel oils and as such produces more water vapor. Consequently, more heat will be carried away by exhaust while firing natural gas.

# 3 T'S OF COMBUSTION

Too much, or too little fuel with the available combustion air may potentially result in unburned fuel and carbon monoxide generation. A very specific amount of O2 is needed for perfect combustion and some additional (excess) air is required for ensuring complete combustion. However, too much excess air will result in heat and efficiency losses.



# COMBUSTION OF OIL

**Heating Oil to Correct Viscosity**

When atomizing oil, it is necessary to heat it enough to get the desired viscosity. This temperature varies slightly for each grade of oil. The lighter oils do not usually require pre-heating. Typical viscosity at the burner tip ( for LAP, MAP & HAP burners) for furnace oil should be 100 Redwood seconds-1 which would require heating the oil to about 105°C.

**Rules for combustion of oil**

1. Atomize the oil completely to produce a fine uniform spray
2. Mix the air and fuel thoroughly
3. Introduce enough air for combustion, but limit the excess air to a maximum of 15%
4. Keep the burners in good condition

# STOICHIOMETRIC COMBUSTION

The efficiency of a boiler or furnace depends on efficiency of the combustion system. The amount of air required for complete combustion of the fuel depends on the elemental constituents of the fuel that is Carbon, Hydrogen, and Sulphur etc. This amount of air is called stoichiometric air. For ideal combustion process for burning one kg of a typical fuel oil containing 86% Carbon, 12% Hydrogen, 2% Sulphur, theoretically required quantity of air is 14.1 kg. This is the minimum air that would be required if mixing of fuel and air by the burner and combustion is perfect. The combustion products are primarily Carbon Dioxide ($CO_2$), water vapor ($H_2O$) and Sulphur Dioxide ($SO_2$), which pass through the chimney along with the Nitrogen ($N_2$) in the air,. After surrendering useful heat in the heat absorption area of a furnace or boiler, the combustion products or fuel gases leave the system through the chimney, carrying away a significant quantity of heat with them.

# CALCULATION OF STOICHIOMETRIC AIR

**The specifications of furnace oil from lab analysis is given below:**

| Constituents | % By weight |
|---|---|
| Carbon | 85.9 |
| Hydrogen | 12 |
| Oxygen | 0.7 |
| Nitrogen | 0.5 |
| Sulphur | 0.5 |
| $H_2O$` | 0.35 |
| Ash | 0.05 |

**GCV of fuel: 10880 kCal/kg**

# CALCULATION FOR REQUIREMENT OF THEORETICAL AMOUNT OF AIR

Considering a sample of 100 kg of furnace oil. The chemical reactions are:

| Element | Molecular Weight kg / kg mole |
|---|---|
| C | 12 |
| $O_2$ | 32 |
| $H_2$ | 2 |
| S | 32 |
| $N_2$ | 28 |
| $CO_2$ | 44 |
| $SO_2$ | 64 |
| $H_2O$ | 18 |

$C + O_2 \rightarrow CO_2$

$H_2 + 1/2O_2 \rightarrow H_2O$

$S + O_2 \rightarrow SO_2$

Constituents of fuel

$C + O_2 \rightarrow CO_2$

$12 + 32 \rightarrow 44$

12 kg of carbon requires 32 kg of oxygen to form 44 kg of carbon dioxide therefore 1 kg of carbon requires 32/12 kg i.e 2.67 kg of oxygen

$(85.9)\,C + (85.9 \times 2.67)\,O2 \rightarrow 315.25\,CO_2$

$2H_2 + O_2 \rightarrow 2H_2O$

$4 + 32 \rightarrow 36$

# CALCULATION FOR REQUIREMENT OF THEORETICAL AMOUNT OF AIR

4 kg of hydrogen requires 32 kg of oxygen to form 36 kg of water, therefore 1 kg of hydrogen requires 32/4 kg i.e 8 kg of oxygen

$(12)\ H_2 + (12 \times 8)\ O2 \rightarrow (12 \times 9)\ H_2O$

$S + O_2 \rightarrow SO_2$
$32 + 32 \rightarrow 64$

32 kg of sulphur requires 32 kg of oxygen to form 64 kg of sulphur dioxide, therefore 1 kg of sulphur requires 32/32 kg i.e 1 kg of oxygen

$(0.5)\ S + (0.5 \times 1)\ O_2 \rightarrow 1.0\ SO_2$

| | | |
|---|---|---|
| Total Oxygen required (229.07+96+0.5) | = | 325.57 kg |
| Oxygen already present in 100 kg fuel (given) | = | 0.7 kg |
| Additional Oxygen Required | = | 325.57 – 0.7 |
| | = | 324.87 kg |
| Therefore quantity of dry air reqd. (air contains 23% oxygen by wt.) | = | (324.87) / 0.23 |

# CALCULATION FOR REQUIREMENT OF THEORETICAL AMOUNT OF AIR

| | | |
|---|---|---|
| | = | 1412.45 kg of air |
| Theoretical Air required | = | (1412.45) / 100 |
| | = | 14.12 kg of air / kg of fuel |

# CALCULATION OF THEORETICAL CO2 CONTENT IN FLUE GASES

Nitrogen in flue gas = 1412.45 - 324.87
= 1087.58 kg

Theoretical $CO_2$% in dry flue gas by volume is calculated as below :

| | | | | |
|---|---|---|---|---|
| Moles of $CO_2$ in flue gas | = | (314.97) / 44 | = | 7.16 |
| Moles of $N_2$ in flue gas | = | (1087.58) / 28 | = | 38.84 |
| Moles of $SO_2$ in flue gas | = | 1/64 | = | 0.016 |

$$Theoritical\ CO_2\%\ by\ volume = \frac{Moles\ of\ CO_2}{Total\ moles\ (dry)} \times 100$$

$$= \frac{7.16}{7.16 + 38.84 + 0.016} \times 100$$

$$= 15.5\ \%$$

# CALCULATION OF CONSTITUENTS OF FLUE GAS WITH EXCESS AIR

% $CO_2$ measured in flue gas = 10% (measured)

$$\% \, Excess \; air = \left( \frac{Theoritical \; CO_2 \%}{Actual \, CO_2 \%} - 1 \right) \times 100$$

$$\% \, Excess \; air = \left( \frac{15.5}{10} - 1 \right) \times 100 = 55\%$$

| | | |
|---|---|---|
| Theoretical air required for 100 kg of fuel burnt | = | 1412.45 kg |
| Total quantity. of air supply required with 55% excess air | = | 1412.45 X 1.55 |
| | = | 2189.30 kg |
| Excess air quantity | = | 2189.30 – 1412.45 |
| | = | 776.85 kg. |

| | | |
|---|---|---|
| $O_2$ | = | 776.85 X 0.23 |
| | = | 178.68 |
| $N_2$ | = | 776.85 – 178.68 |
| | = | 598.17 kg |

# CALCULATION OF CONSTITUENTS OF FLUE GAS WITH EXCESS AIR

The final constitution of flue gas with 55% excess air for every 100 kg fuel.

| | | |
|---|---|---|
| $CO_2$ | = | 314.97 kg |
| $H_2O$ | = | 108.00 kg |
| $SO_2$ | = | 1 kg |
| $O_2$ | = | 178.68 kg |
| $N_2$ | = | 1087.58 + 598.17 |
| | = | 1685.75 kg |

**Calculation of Theoretical $CO_2$% in Dry Flue Gas By Volume**

Moles of $CO_2$ in flue gas = 314.97/44 = 7.16

Moles of $SO_2$ in flue gas = 1/64 = 0.016

Moles of $O_2$ in flue gas = 178.68 / 32 = 5.58

Moles of $N_2$ in flue gas = 1685.75 / 28 = 60.20

$$Theoritical\ CO_2\%\ by\ volume = \frac{Moles\ of\ CO_2}{Total\ moles\ (dry)} \times 100$$

$$= \frac{7.16}{7.16 + 0.016 + 5.58 + 60.20} \times 100$$

# CALCULATION OF CONSTITUENTS OF FLUE GAS WITH EXCESS AIR

$$= \frac{7.16}{72.956} \times 100 = 10\%$$

$$\textit{Theoretical } O_2\% \textit{ by volume} = \frac{5.58\, x100}{72.956} \times 100 = 7.5\%$$

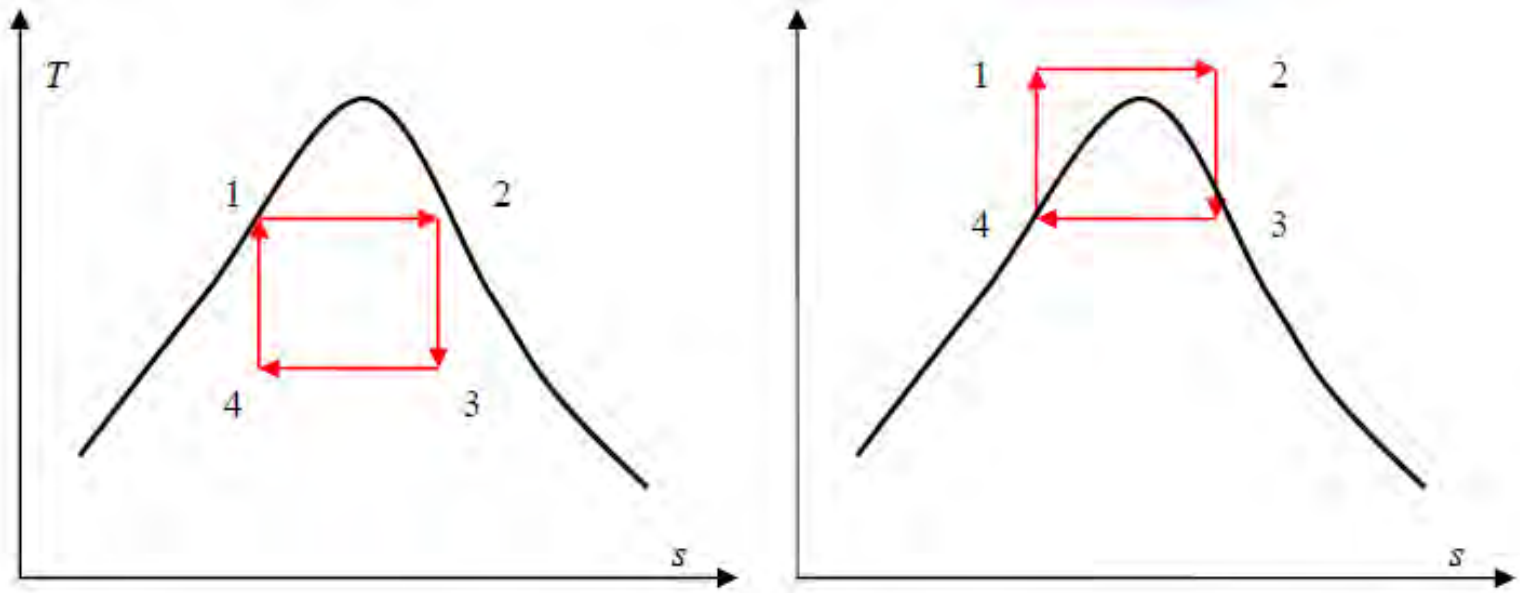
# VAPOR POWER CYCLES

We know that the Carnot cycle is most efficient cycle operating between two specified temperature limits. However; the Carnot cycle is not a suitable model for steam power cycle since:

- The turbine has to handle steam with low quality which will cause erosion and wear in turbine blades.
- It is impractical to design a compressor that handles two phase.
- It is difficult to control the condensation process that precisely as to end up with the desired at point 4.

# VAPOR POWER CYCLES

Other issues include: isentropic compression to extremely high pressure and isothermal heat transfer at variable pressures. Thus, the Carnot cycle cannot be approximated in actual devices and is not a realistic model for vapor power cycles.



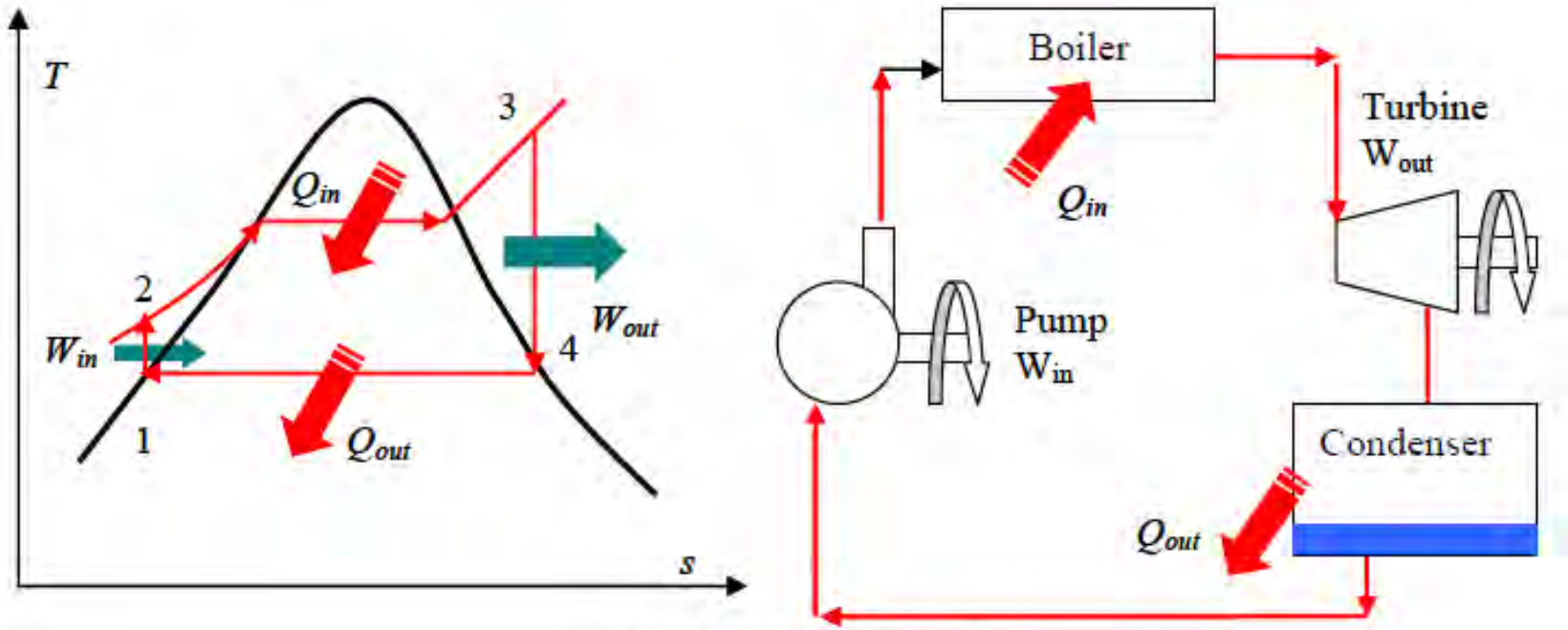

*T-s* diagram for two Carnot vapor cycle.

# IDEAL RANKINE CYCLE

The Rankine cycle is the ideal cycle for vapor power plants; it includes the following four reversible processes:

| | | |
|---|---|---|
| 1-2: | Isentropic compression | Water enters the pump as state 1 as saturated liquid and is compressed isentropically to the operating pressure of the boiler. |
| 2-3: | Const P heat addition | Saturated water enters the boiler and leaves it as superheated vapor at state 3 |
| 3-4: | Isentropic expansion | Superheated vapor expands isentropically in turbine and produces work. |
| 4-1: | Const P heat rejection | High quality steam is condensed in the condenser |

# IDEAL RANKINE CYCLE




The ideal Rankine cycle.

# ENERGY ANALYSIS FOR THE CYCLE

All four components of the Rankine cycle are steady-state steady-flow devices. The potential and kinetic energy effects can be neglected. The first law per unit mass of steam can be written as:

| | | |
|---|---|---|
| Pump | $q = 0$ | $w_{pump,in} = h_2 - h_1$ |
| Boiler | $w = 0$ | $q_{in} = h_3 - h_2$ |
| Turbine | $q = 0$ | $w_{turbine,out} = h_3 - h_4$ |
| Condenser | $w = 0$ | $q_{out} = h_4 - h_1$ |

The thermal efficiency of the cycle is determined from:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

where

$$w_{net} = q_{in} - q_{out} = w_{turbine,out} - w_{pump,in}$$

If we consider the fluid to be incompressible, the work input to the pump will be:

$$(h_2 - h_1) = v(P_2 - P_1)$$

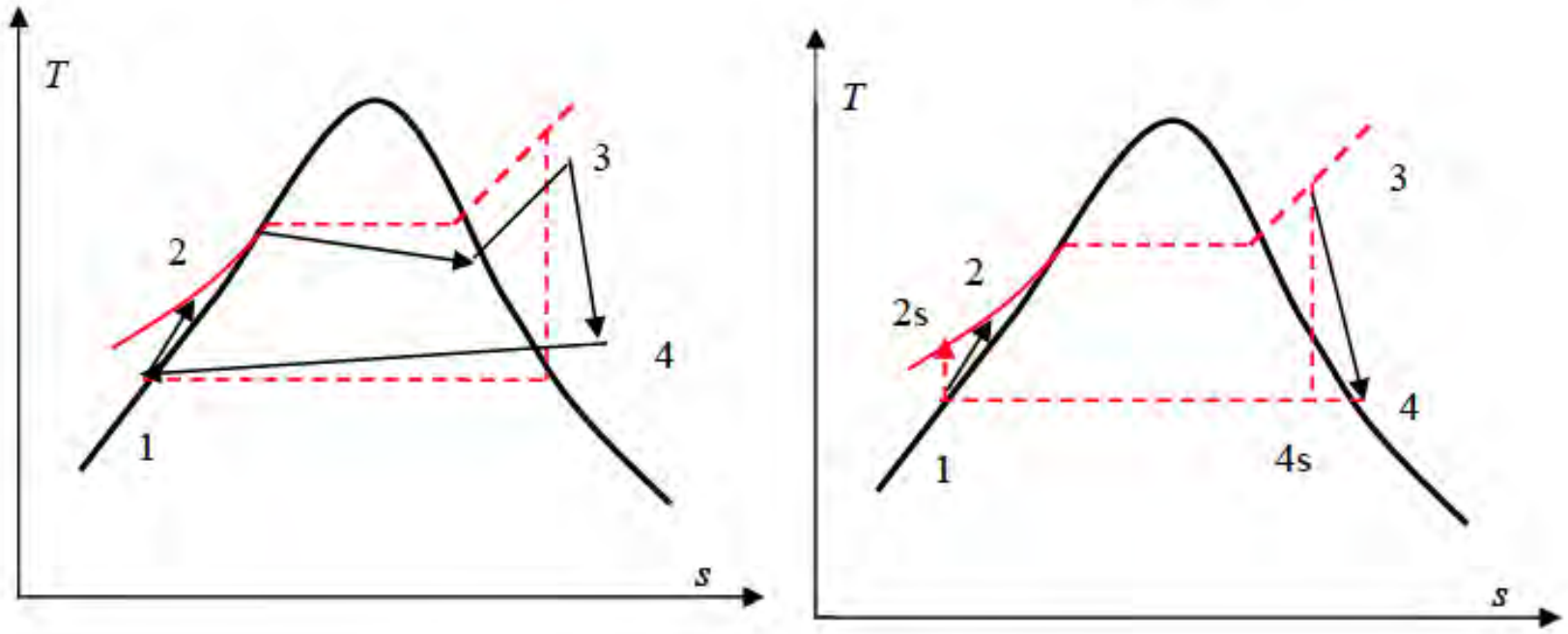
where $h_1 = h_{f@P1}$ & $v = v_1 = v_{f@P1}$

# DEVIATION OF ACTUAL VAPOR POWER CYCLE FROM IDEAL CYCLE

As a result of irreversibilities in various components such as fluid friction and heat loss to the surroundings, the actual cycle deviates from the ideal Rankine cycle. The deviations of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies defined as:

$$\eta_P = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \qquad \eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

# DEVIATION OF ACTUAL VAPOR POWER CYCLE FROM IDEAL CYCLE




Deviation from ideal Rankine cycle.

# INCREASING THE EFFICIENCY OF RANKINE CYCLE

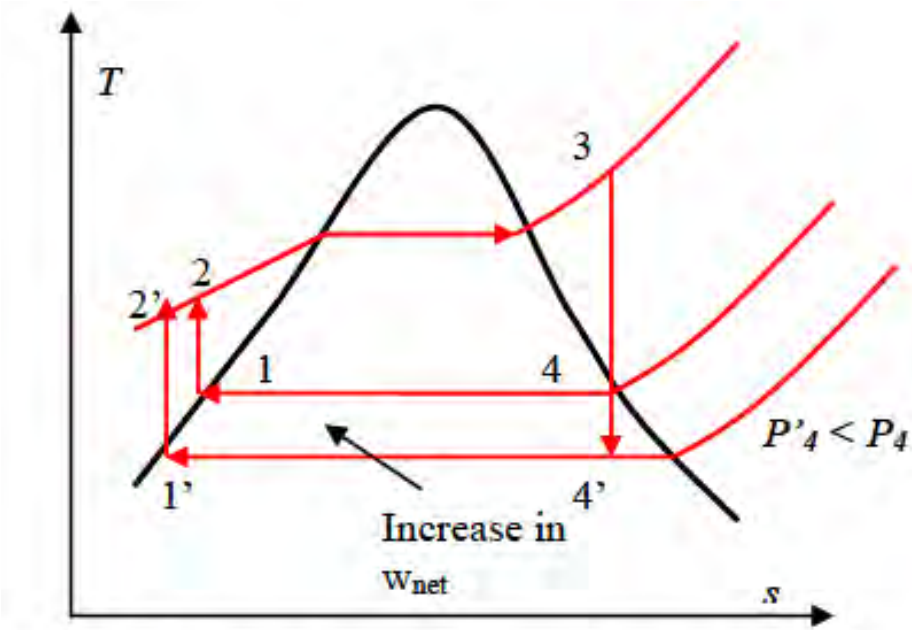
We know that the efficiency is proportional to:

$$\eta_{th} \propto 1 - \frac{T_L}{T_H}$$

That is, to increase the efficiency one should increase the average temperature at which heat is transferred to the working fluid in the boiler, and/or decrease the average temperature at which heat is rejected from the working fluid in the condenser.

# DECREASING THE OF CONDENSER PRESSURE (LOWER TL)

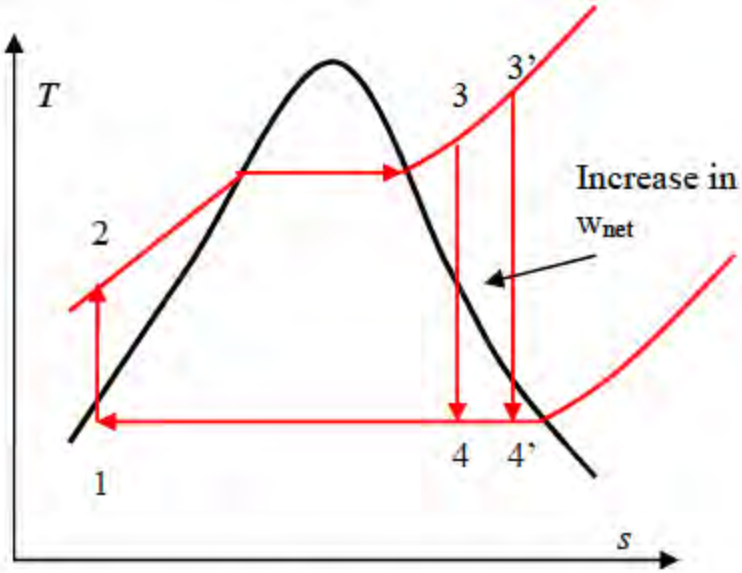
Lowering the condenser pressure will increase the area enclosed by the cycle on a *T-s* diagram which indicates that the net work will increase. Thus, the thermal efficiency of the cycle will be increased.




Effect of lowering the condenser pressure on ideal Rankine cycle.

# SUPERHEATING THE STEAM TO HIGH TEMPERATURES (INCREASE $T_h$)

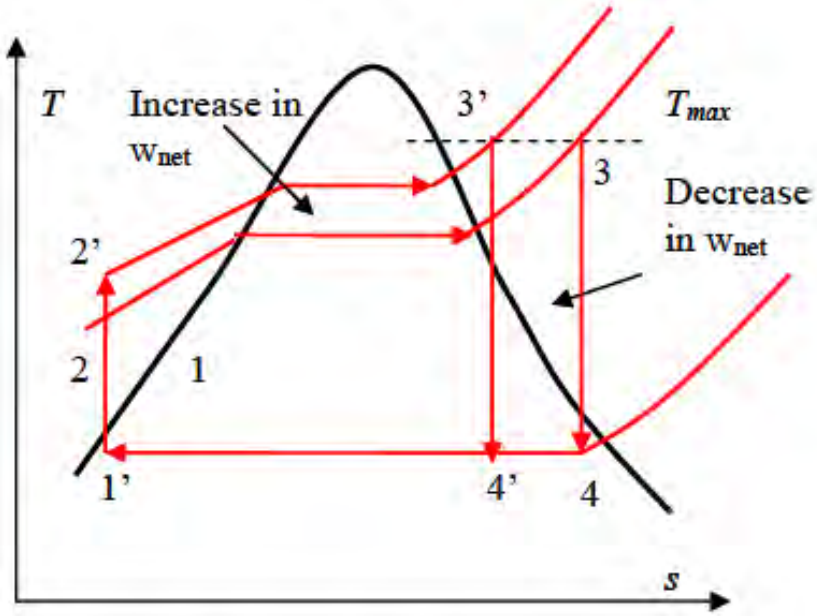
Superheating the steam will increase the net work output and the efficiency of the cycle. It also decreases the moisture contents of the steam at the turbine exit. The temperature to which steam can be superheated is limited by metallurgical considerations (~ 620°C).




The effect of increasing the boiler pressure on the ideal Rankine cycle.

# INCREASING THE BOILER PRESSURE (INCREASE $T_H$)

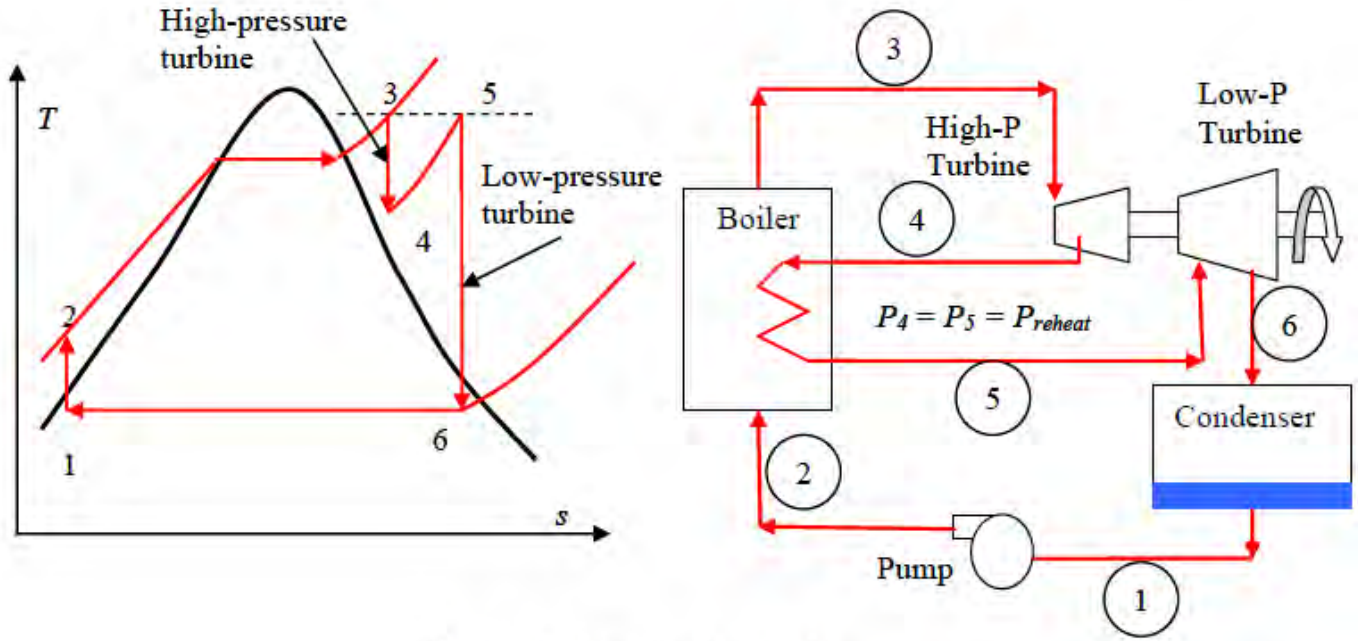
Increasing the operating pressure of the boiler leads to an increase in the temperature at which heat is transferred to the steam and thus raises the efficiency of the cycle.




The effect of increasing the boiler pressure on the ideal cycle.

# THE IDEAL REHEAT RANKINE CYCLE

To take advantage of the increased efficiencies at higher boiler pressure without facing the excessive moisture at the final stages of the turbine, reheating is used. In the ideal reheating cycle, the expansion process takes place in two stages, i.e., the high-pressure and low-pressure turbines.




The ideal reheat Rankine cycle.

# THE IDEAL REHEAT RANKINE CYCLE

The total heat input and total turbine work output for a reheat cycle become:

$$q_{in} = q_{primary} + q_{reheat} = (h_3 - h_2) + (h_5 - h_4)$$

$$w_{turbine,out} = w_{H-P\ turbine} + w_{L-P\ turbine} = (h_3 - h_4) + (h_5 - h_6)$$

The incorporation of the single reheat in a modern power plant improves the cycle efficiency by 4 to 5 percent by increasing the average temperature at which heat is transferred to the steam.

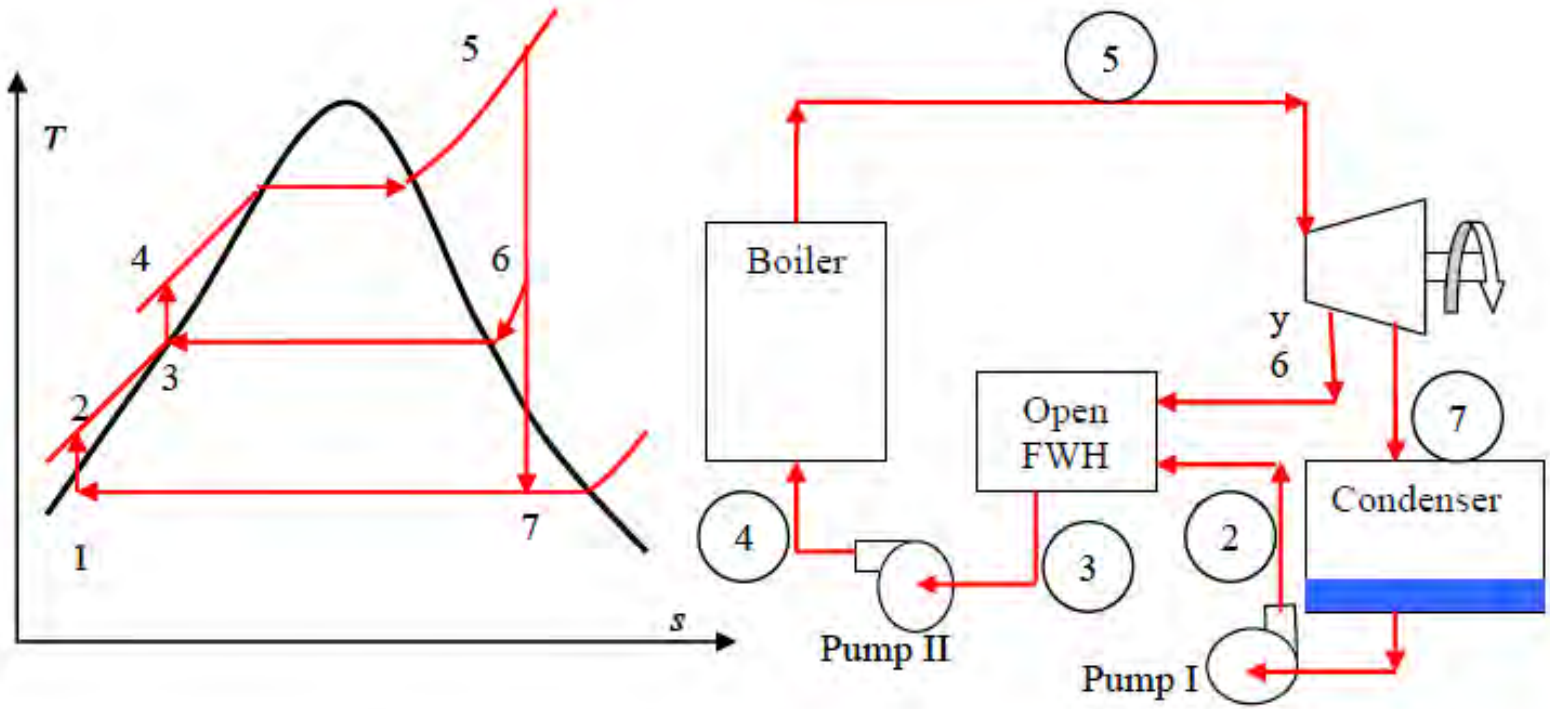
# THE IDEAL REGENERATIVE RANKINE CYCLE

The regeneration process in steam power plants is accomplished by extracting (or bleeding) steam from turbine at various stages and feed that steam in heat exchanger where the feedwater is heated. These heat exchangers are called regenerator or feedwater heater (FWH). FWH also help removing the air that leaks in at the condenser (deaerating the feedwater). There are two types of FWH's, open and closed.

**OPEN (DIRECT-CONTACT) FEEDWATER HEATERS**

An open FWH is basically a mixing chamber where the steam extracted from the turbine mixes with the feedwater exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure.

# OPEN (DIRECT-CONTACT) FEEDWATER HEATERS




The ideal regenerative Rankine cycle with an open FWH.

# OPEN (DIRECT-CONTACT) FEEDWATER HEATERS

Using Fig. , the heat and work interactions of a regenerative Rankine cycle with one FWH can be expressed per unit mass of steam flowing through the boiler as:

$$q_{in} = h_5 - h_4$$

$$q_{out} = (1-y)(h_7 - h_1)$$

$$w_{turbine,out} = (h_5 - h_6) + (1-y)(h_6 - h_7)$$

$$w_{pump,in} = (1-y)w_{PumpI} + w_{PumpII}$$

where

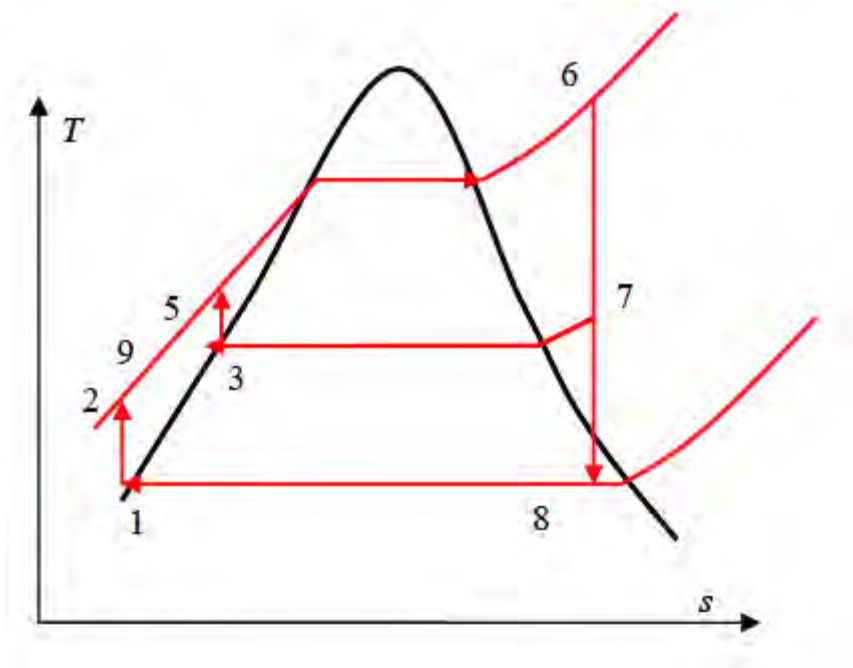
$$y = \dot{m}_6 / \dot{m}_5$$

$$w_{PumpI} = v_1(P_2 - P_1) \qquad w_{PumpII} = v_3(P_4 - P_3)$$

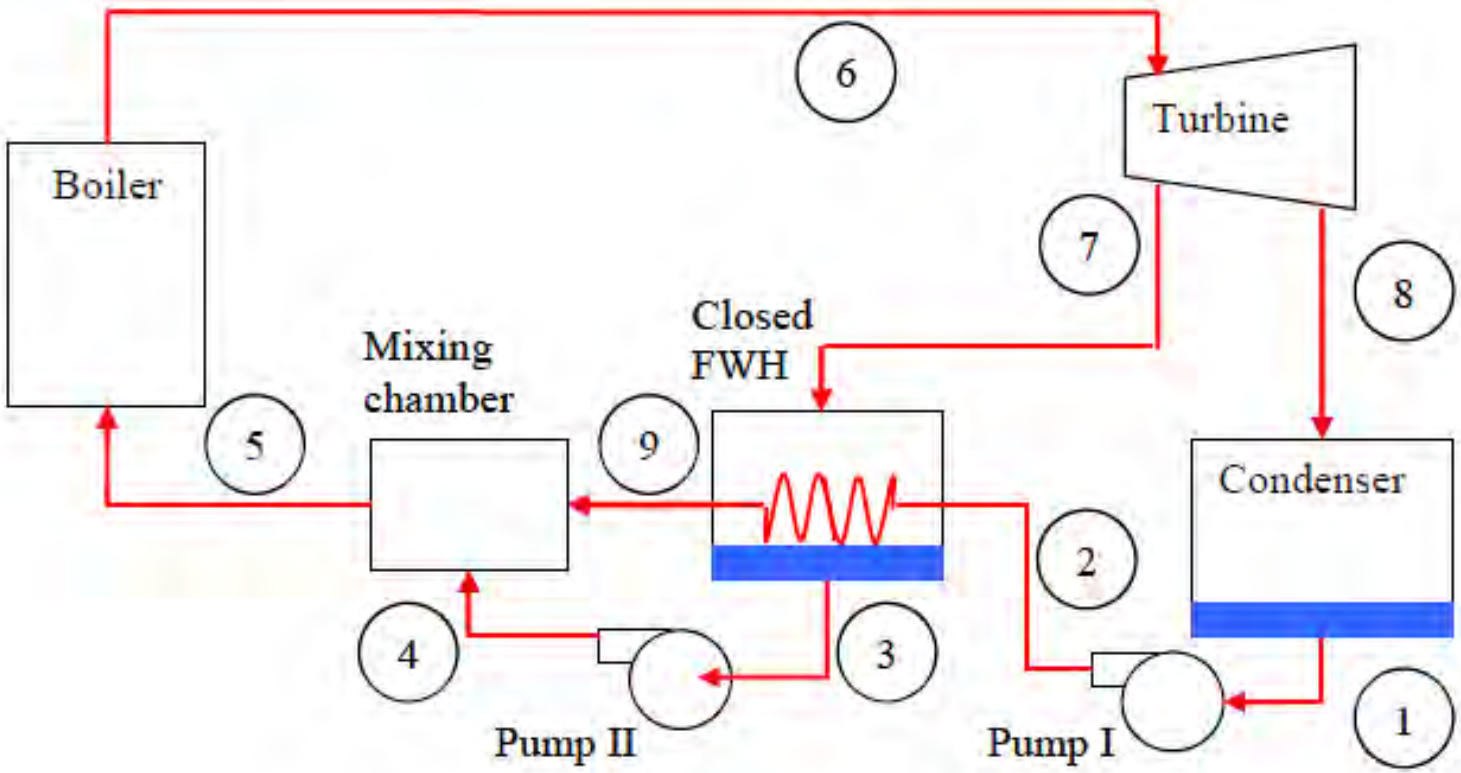
Thermal efficiency of the Rankine cycle increases as a result of regeneration since FWH raises the average temperature of the water before it enters the boiler. Many large power plants have as many as 8 FWH's.

# CLOSED FEEDWATER HEATERS

In closed FWH, heat is transferred from the extracted steam to the feedwater without any mixing taking place. Thus; two streams can be at different pressures, since they don't mix. In an ideal closed FWH, the feedwater is heated to the exit temperature of the extracted steam, which ideally leaves the heater as a saturated liquid at the extraction pressure.



# IDEAL REGENERATIVE RANKINE CYCLE WITH A CLOSED FWH.



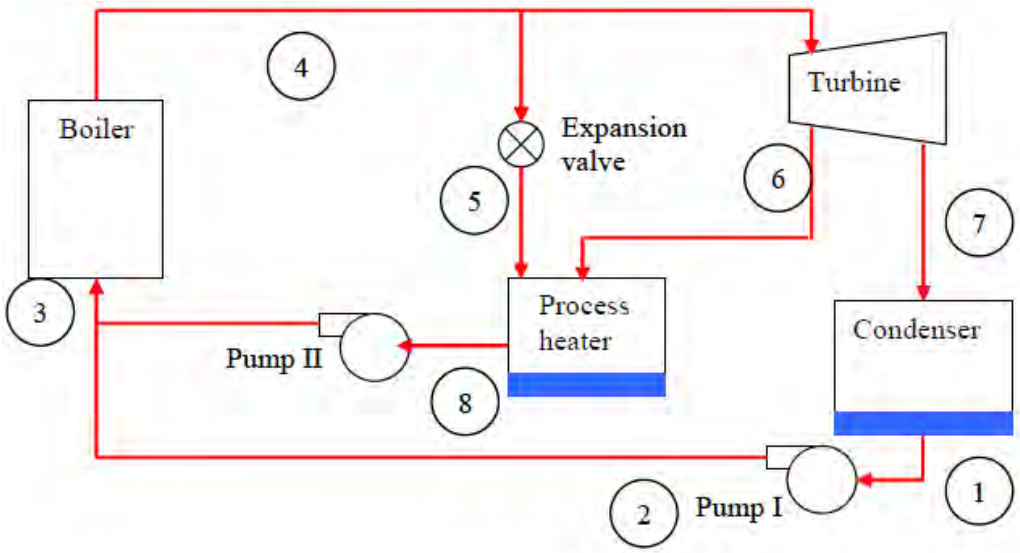

Ideal regenerative Rankine cycle with a closed FWH.

# DIFFERENCE BETWEEN OPEN FWH AND CLOSED FWH

| Open FWH | Closed FWH |
|---|---|
| • simple<br>• inexpensive<br>• good heat transfer characteristics<br>• bring feedwater to the saturation state<br>• a pump is required for each FWH | • more complex (internal tubing)<br>• less effective (no mixing)<br>• do not require a pump for each FWH |

# COGENERATION

Many system and industries require energy input in the form of heat, called *process heat*. Some industries such as chemical, pulp and paper rely heavily on process heat. The process heat is typically supplied by steam at 5 to 7 atm and 150 to 200 C. These plants also require large amount of electric power. Therefore, it makes economical and engineering sense to use the already-existing work potential (in the steam entering the condenser) to use as process heat. This is called cogeneration.

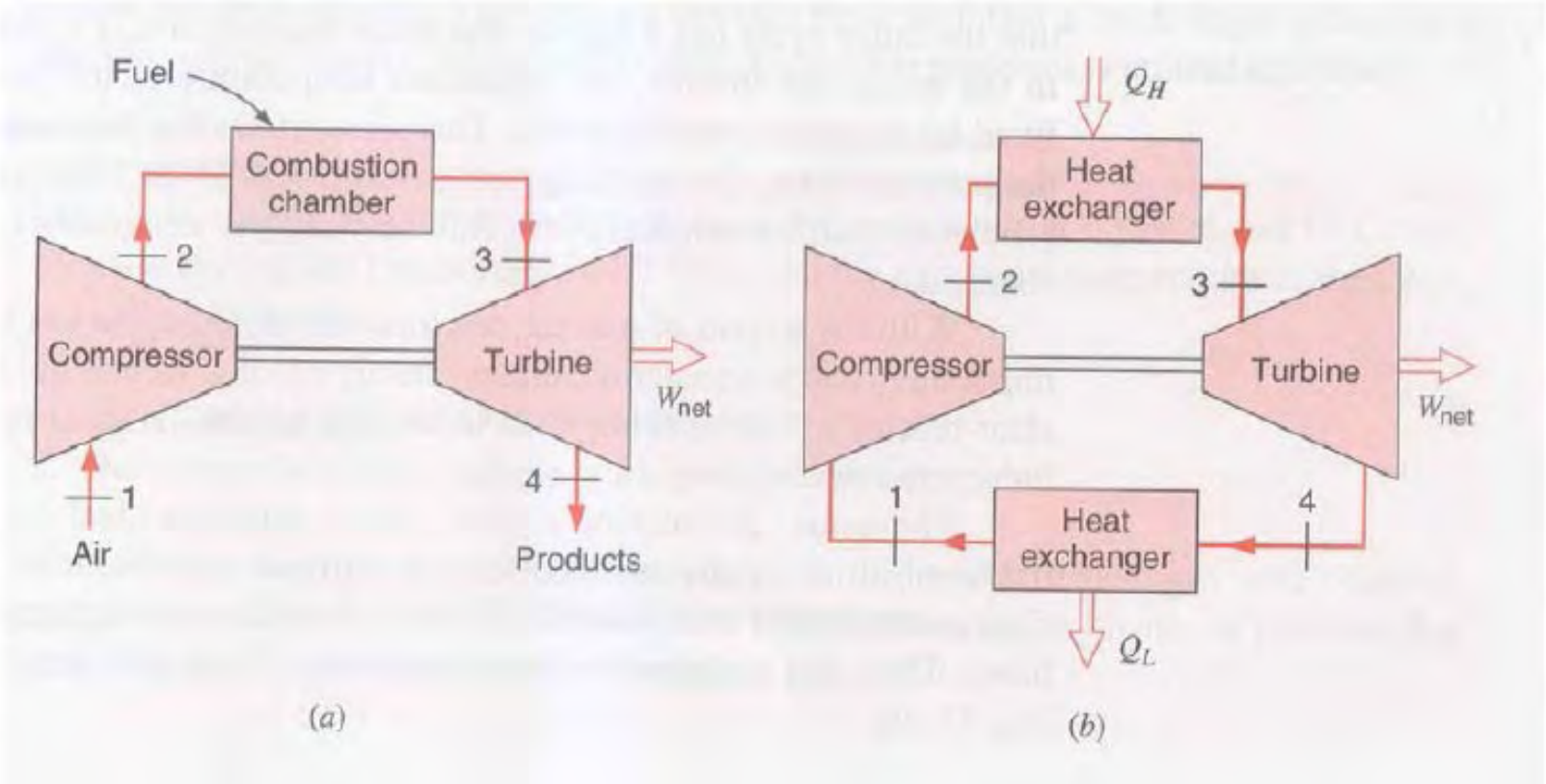



A cogeneration plant with adjustable loads.

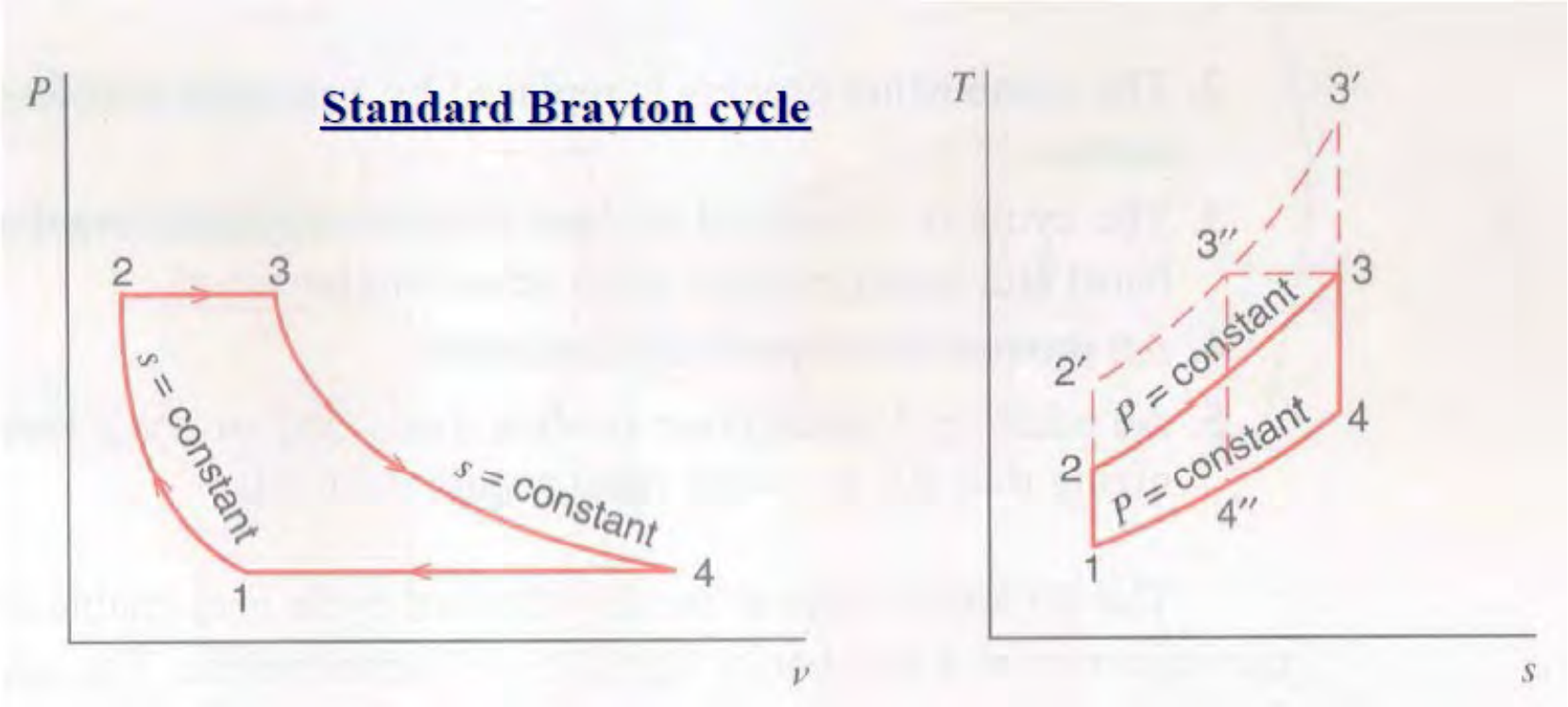
# COMBINED GAS-VAPOR POWER CYCLE

Gas-turbine cycles typically operate at considerably higher temperatures than steam cycles. The maximum fluid temperature at the turbine inlet is about 620C for modern steam power plants, but over 1425C for gas-turbine power plants. It is over 1500C at the burner exit of turbojet engines. It makes engineering sense to take advantage of the very desirable characteristics of the gas-turbine cycle at high-temperature and to use the high temperature exhaust gases as the energy source for the bottoming cycle as a steam power cycle. This is called *combined cycle*. Combined cycles can achieve high thermal efficiencies, some of recent ones have η about 60%.

# BRAYTON CYCLE



# BRAYTON CYCLE



# BRAYTON CYCLE

- Two const-P processes (combustor, and approximated condensed process) + two isentropic processes (compressor, turbine)
- Rankine cycle using a single phase, gaseous working fluid –Brayton cycle
- Ideal cycle for the simple gas turbine

$$\eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{C_P(T_4 - T_1)}{C_P(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$= 1 - \frac{T_1}{T_2} = 1 - \frac{1}{(P_2/P_1)^{k-1/k}}$$

*here, we note*

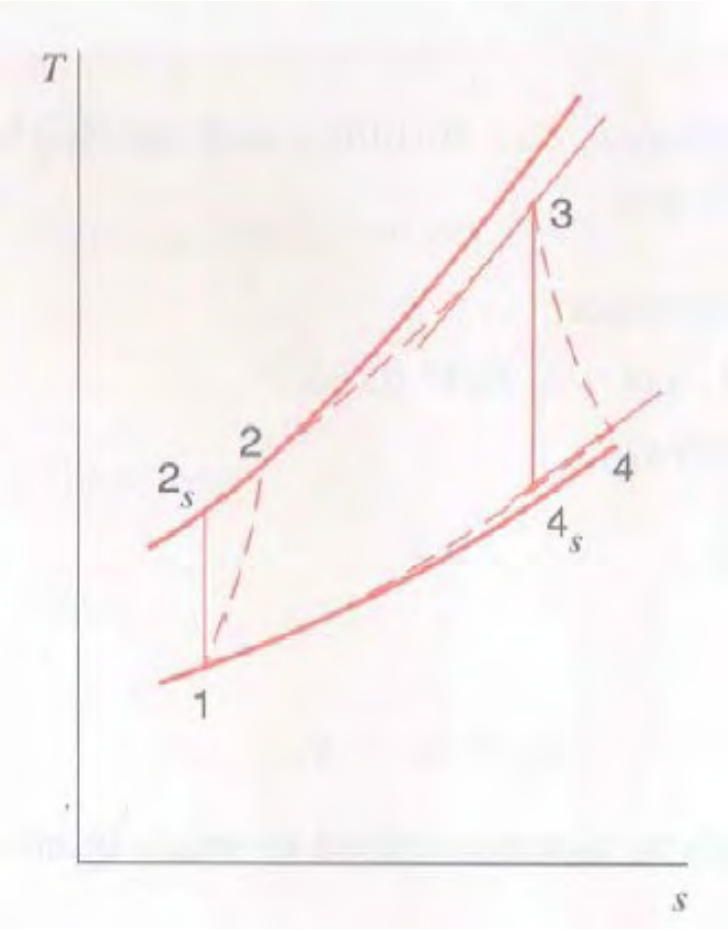
$$\frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$$

$$\because \frac{P_3}{P_4} = \frac{P_2}{P_1}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \frac{P_3}{P_4} = \left(\frac{T_3}{T_4}\right)^{k/(k-1)}$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

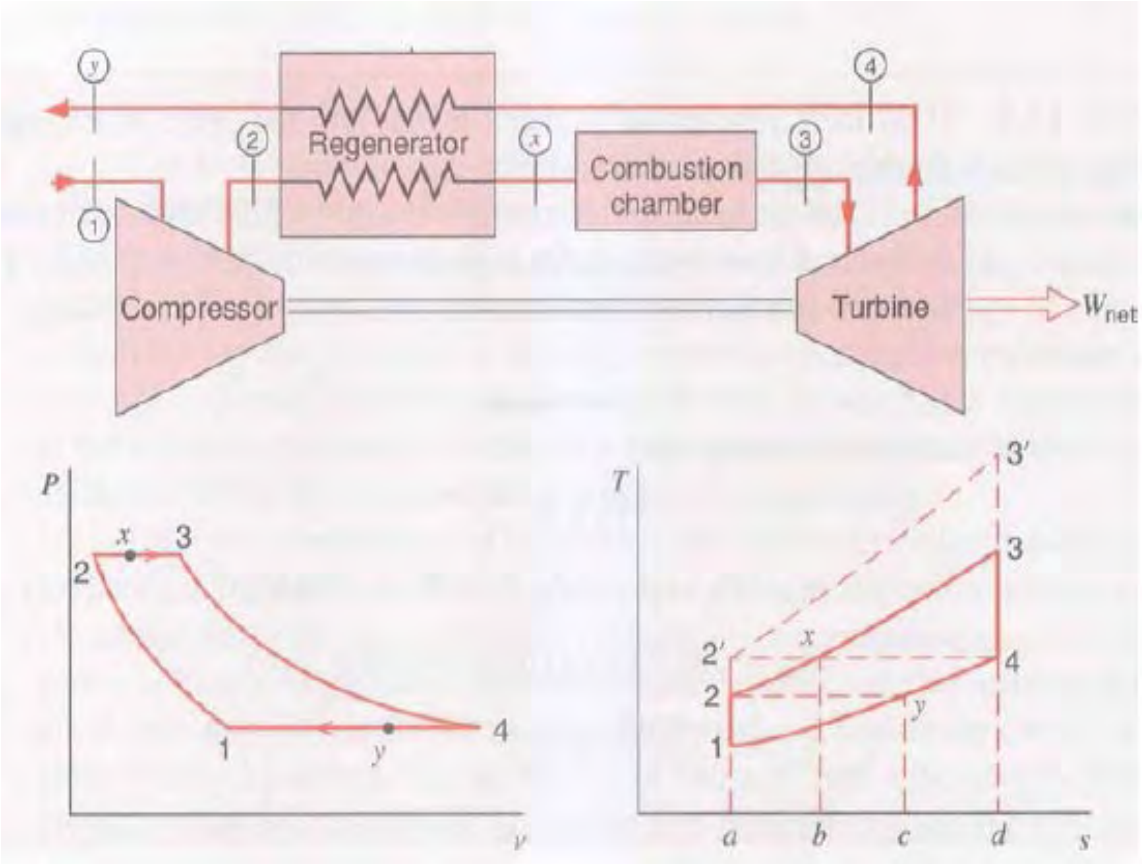
# BRAYTON CYCLE




$$\eta_{comp} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$\eta_{turb} = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

# GAS-TURBINE REGENERATORTURBINE CYCLE WITH A REGENERATOR




$$\eta_{th} = \frac{w_{net}}{q_H} = \frac{w_t - w_c}{q_H}$$

$$q_H = C_P\left(T_3 - T_x\right)$$

$$w_t = C_P\left(T_3 - T_4\right)$$

# GAS-TURBINE REGENERATORTURBINE CYCLE WITH A REGENERATOR

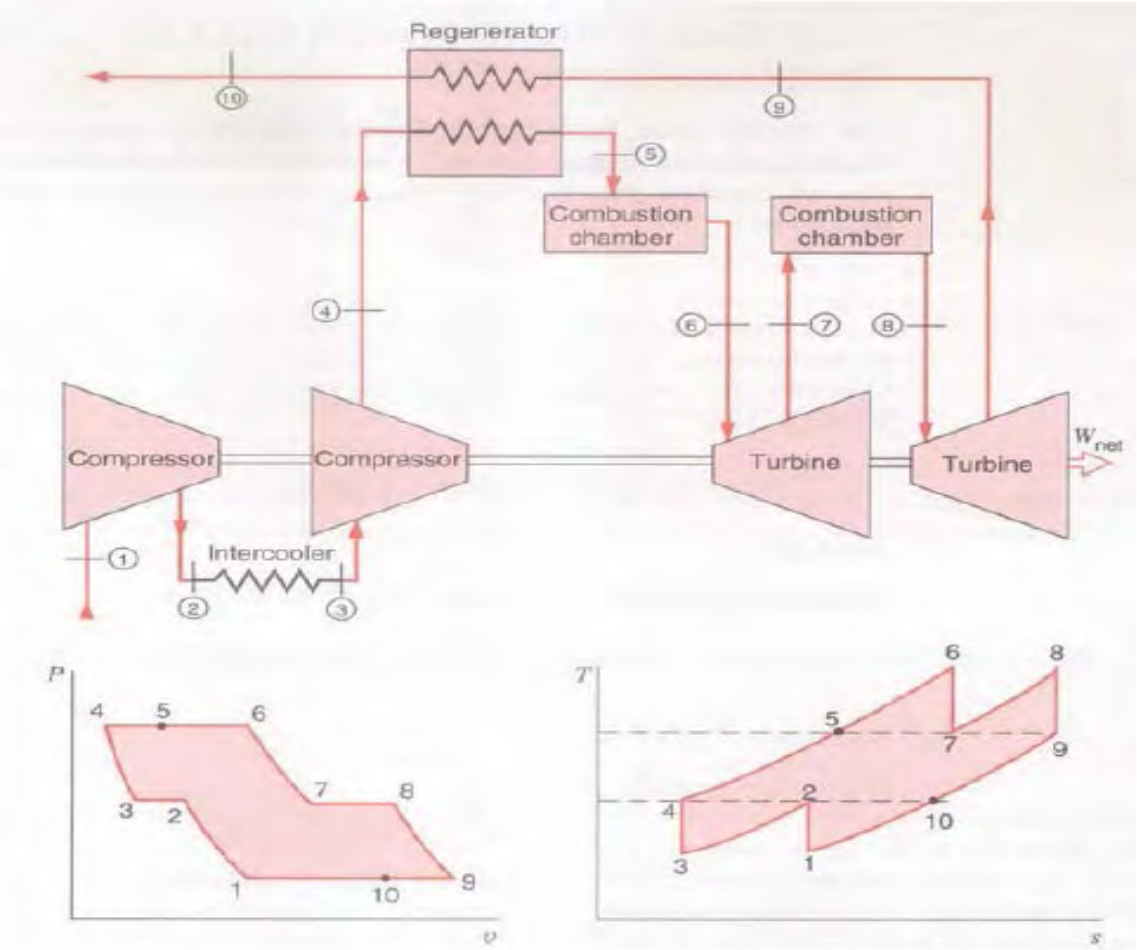
**For ideal regenerator** $w_t = q_H, \quad T_4 = T_x$

$$\eta_{th} = 1 - \frac{w_c}{q_H} = 1 - \frac{C_P\left(T_2 - T_1\right)}{C_P\left(T_3 - T_4\right)} = 1 - \frac{T_1\left(T_2/T_1 - 1\right)}{T_3\left(1 - T_4/T_3\right)}$$

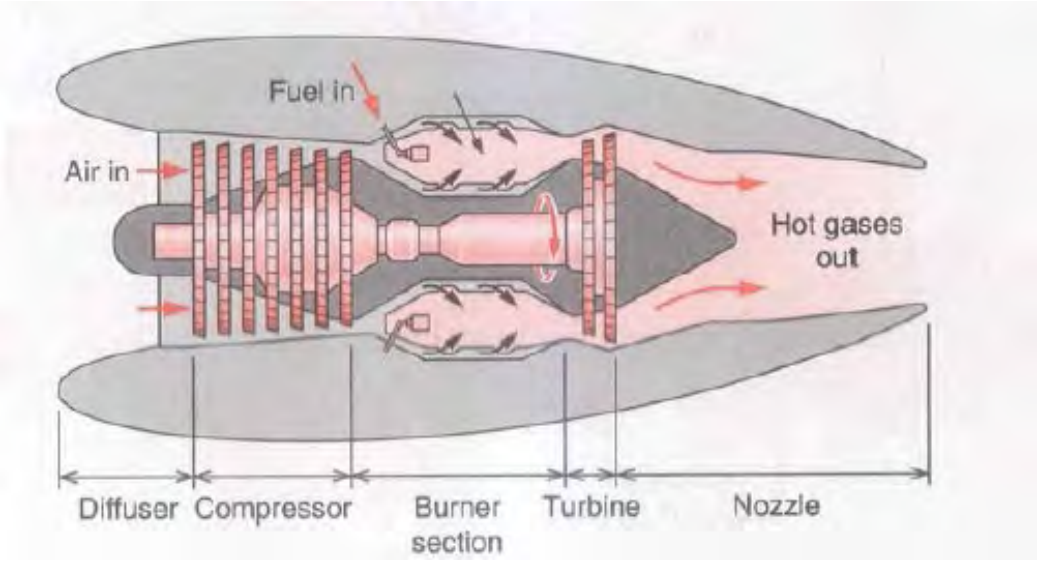
$$= 1 - \frac{T_1}{T_3}\frac{\left[\left(P_2/P_1\right)^{(k-1)/k} - 1\right]}{\left[1 - \left(P_1/P_2\right)^{(k-1)/k}\right]}$$

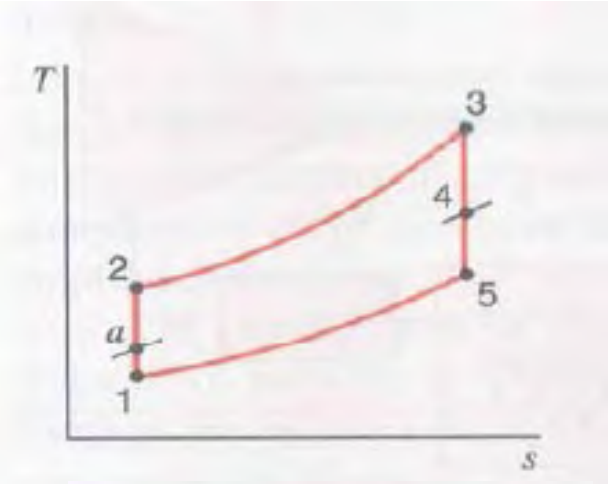
$$\eta_{th} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

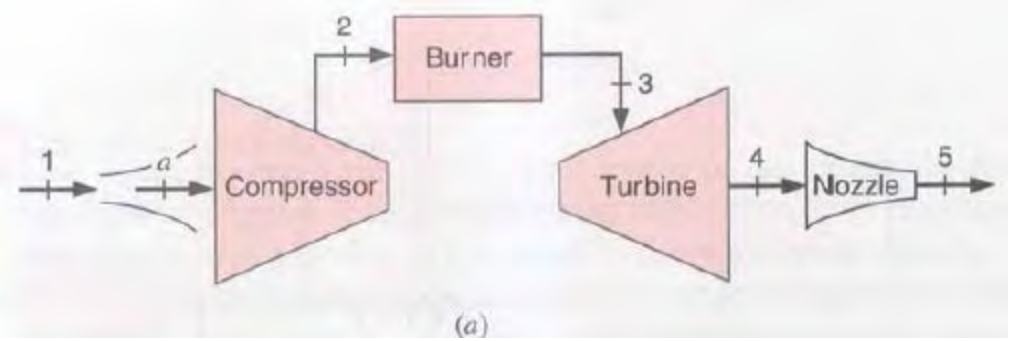
# ERICSSON CYCLE

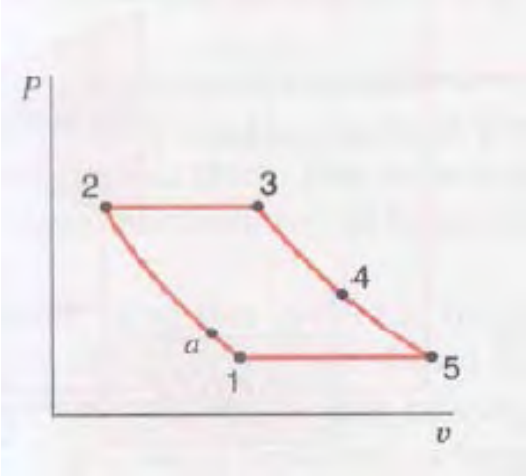


# JET PROPULSION



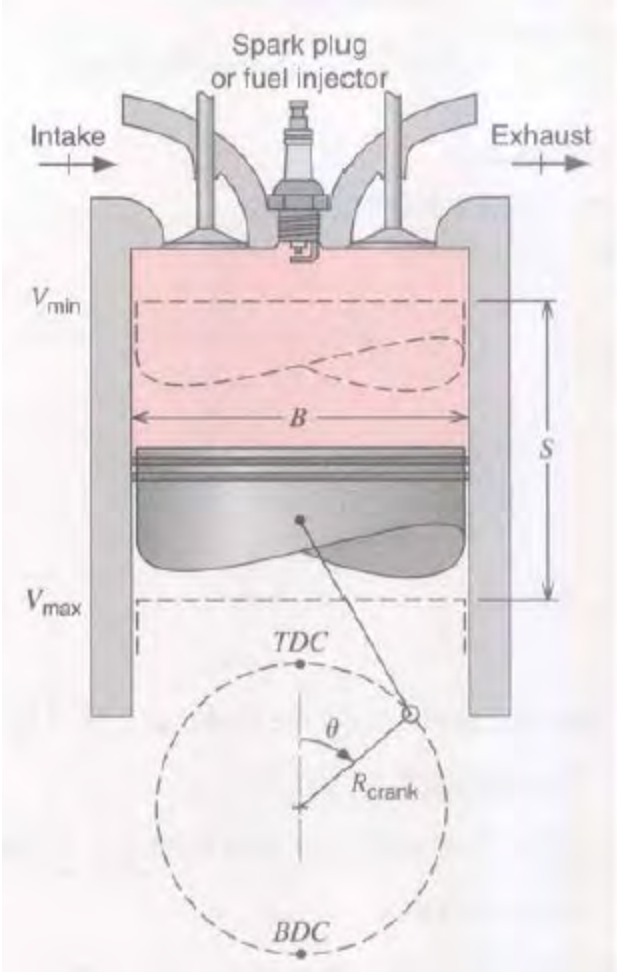



# RECIPROCATING ENGINE POWER CYCLES

- OTTO CYCLE
- DIESEL CYCLE
- STIRLING CYCLE

**SOME DEFINITIONS AND TERMS**

Bore B : cylinder diameter
Crank angle
TDC : Top dead center
BDC : bottom dead center
Clearance volume
Displacement volume
Compression ratio
Air--fuel ratio
Mean effective pressure (mep))



# RECIPROCATING ENGINE POWER CYCLES

- OTTO CYCLE
- DIESEL CYCLE
- STIRLING CYCLE

**SOME DEFINITIONS AND TERMS**

$$S = 2R_{crank}$$

$$V_{displ} = N_{cyl}(V_{max} - V_{min}) = N_{cyl}A_{cyl}S$$

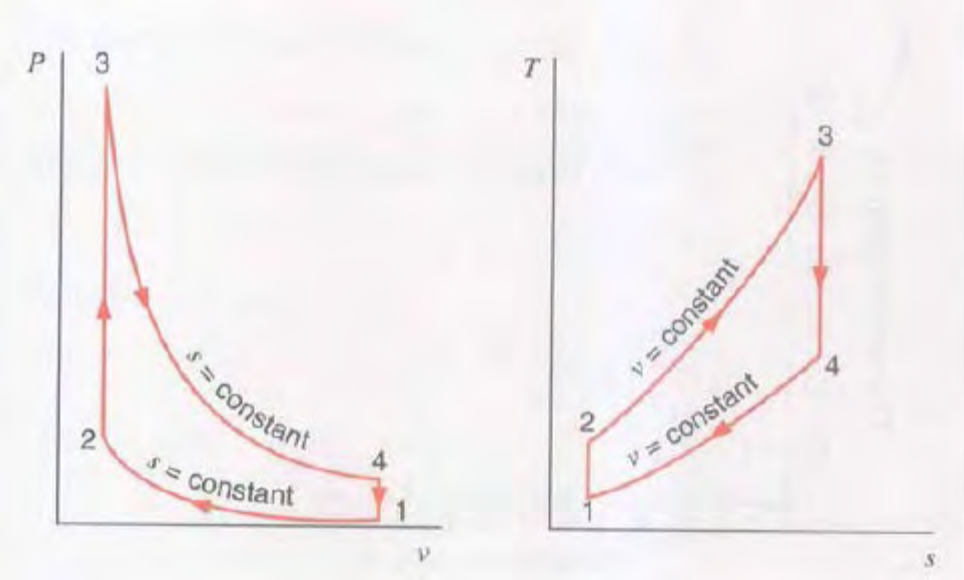
$$r_v = CR = V_{max}/V_{min}$$

$$w_{net} = \oint Pdv = P_{mef} f(v_{max} - v_{min})$$

$$W_{net} = mw_{net} = P_{meff}(V_{max} - V_{min})$$

$$\dot{W} = N_{cyl} mw_{net} \frac{RPM}{60} - P_{meff} V_{displ} \frac{RPM}{60}$$

# THE OTTO CYCLE



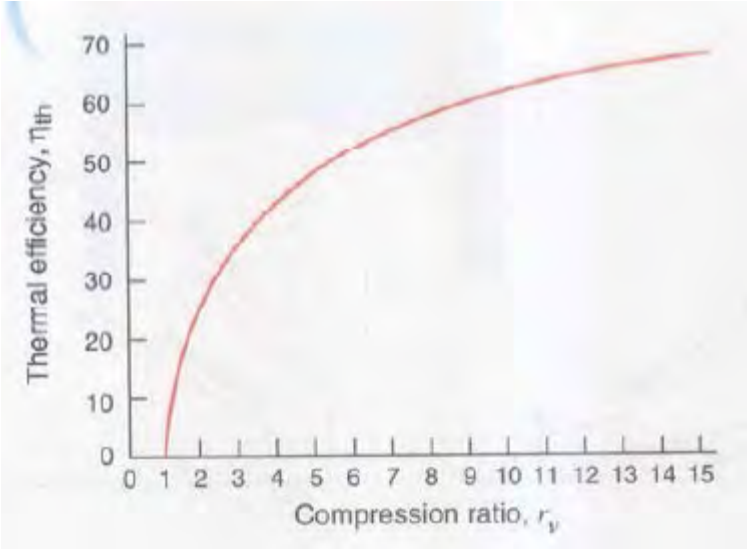

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = \left(\frac{V_1}{V_3}\right)^{k-1} = \frac{T_3}{T_4}$$

$$\therefore \frac{T_3}{T_2} = \frac{T_4}{T_1}$$

$$\eta_{th} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

# THE OTTO CYCLE

**THERMAL EFFICIENCY OF THE OTTO CYCLE AS A FUNCTION OF COMPRESSION RATIO**




$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - (r_v)^{1-k} = 1 - \frac{1}{(r_v)^{k-1}}$$

$$where, \quad r_v = \frac{V_1}{V_2} = \frac{V_4}{V_3}$$

**NOTE 1;**

**1. higher compression ratio, higher thermal efficiency**

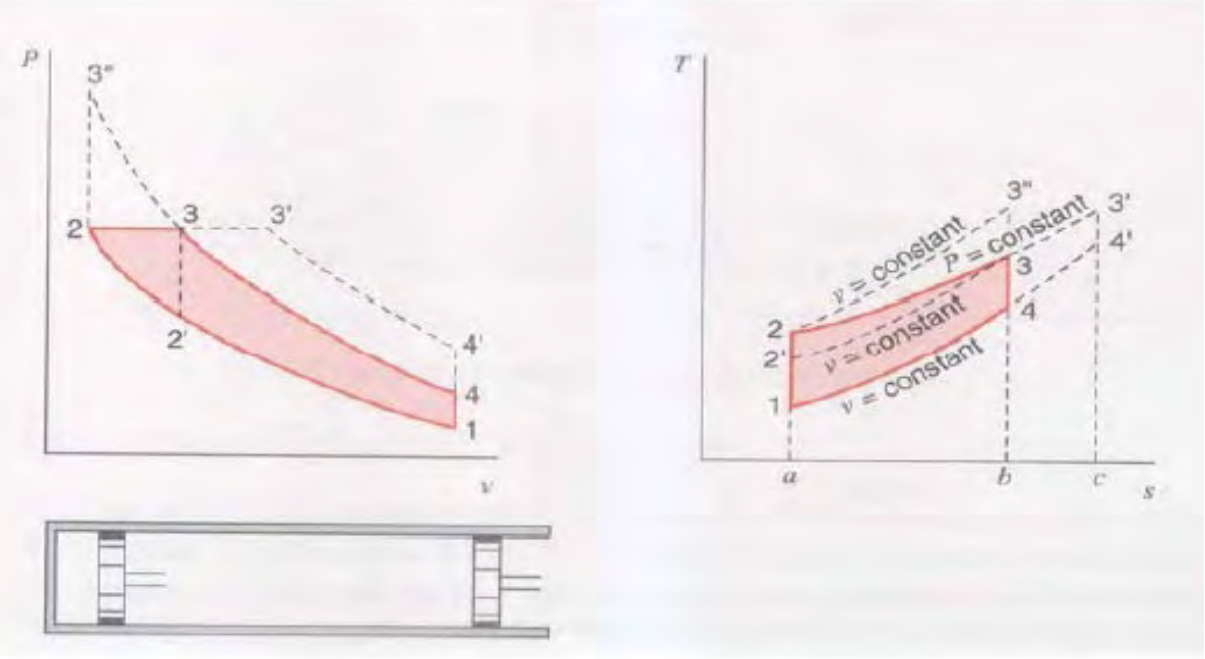
**2. detonation occurs at very high compression ratio, - negative respect in actual engines : strong pressure wave (spark knock)**

# THE OTTO CYCLE

NOTE 2 ; Deviation of actual engine from air-standard cycle

1. specific heat – increases with temperature
2. combustion process is present – incomplete : producing pollutant such as Nox, Soot, and particulate matter (PM)
3. inlet and outlet processes + a certain amount of work is required because of pressure drops
4. considerable heat transfer
5. irreversibilities (pressure and temperature gradients)

# THE DIESEL CYCLE
## (COMPRESSION IGNITION —CI ENGINE)




$$\eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{C_P(T_4 - T_1)}{C_P(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}$$

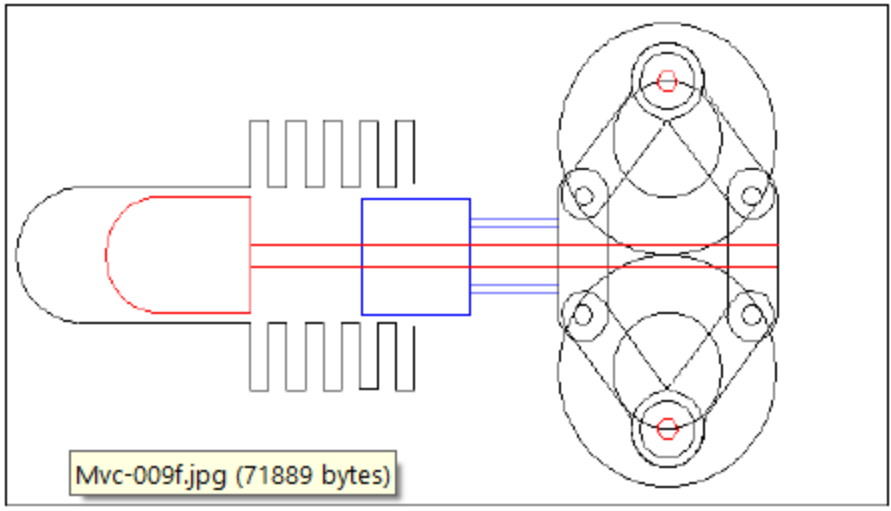
# THE DIESEL CYCLE
## (COMPRESSION IGNITION —CI ENGINE)

**NOTE**

**1. there is no knocking problem because only air is compressed during the compression stroke**
**2. constant pressure – heat transferring (combustion process)**
**Cf) Otto cycle – constant volume process**

**Some losses**
**- pumping loss**
**- some losses during inlet and exhaust processes**
**- heat transfer**
**- not constant pressure process during combustion process**

# STIRLING CYCLE



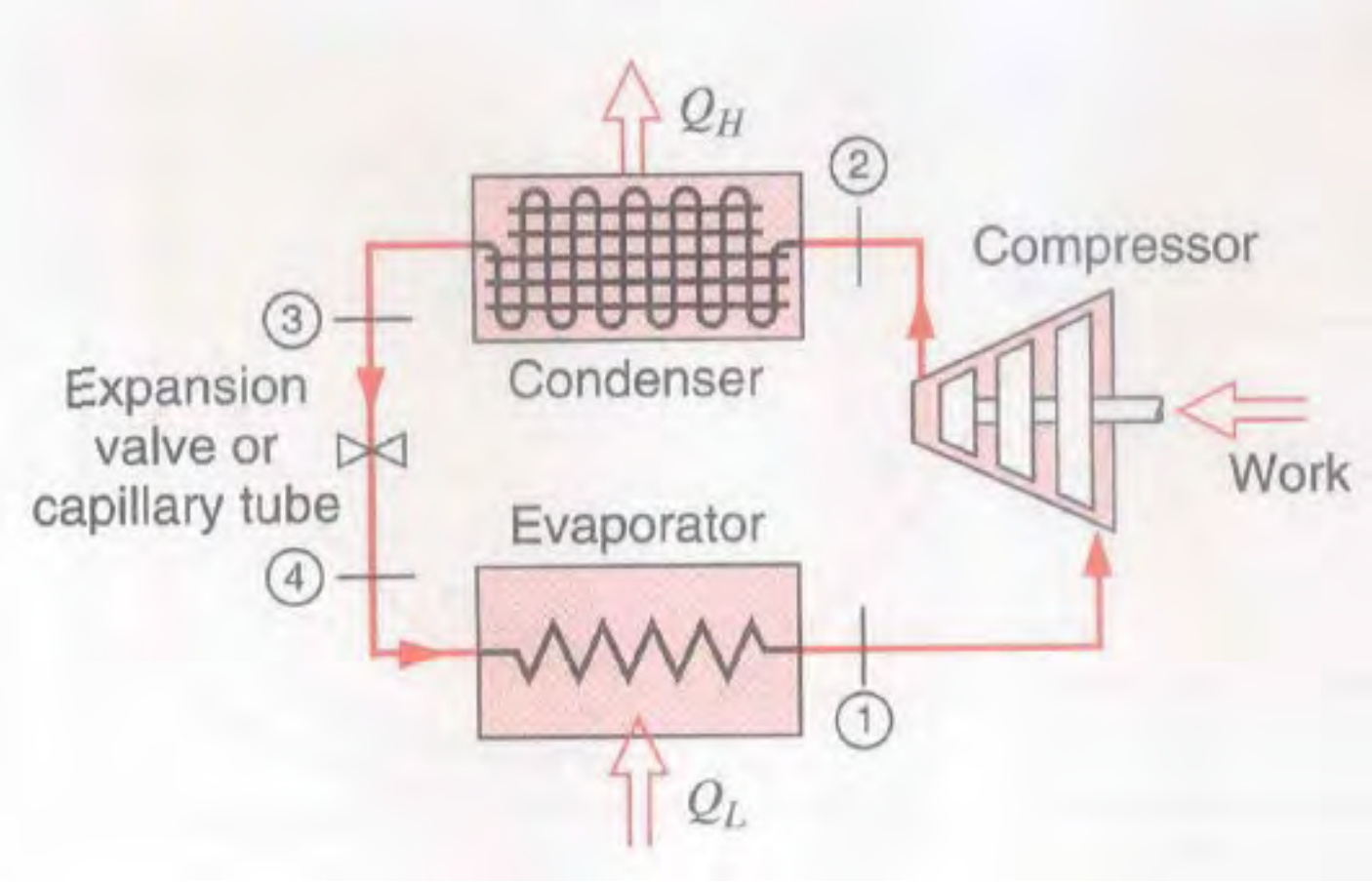
# STIRLING CYCLE

**NOTE**

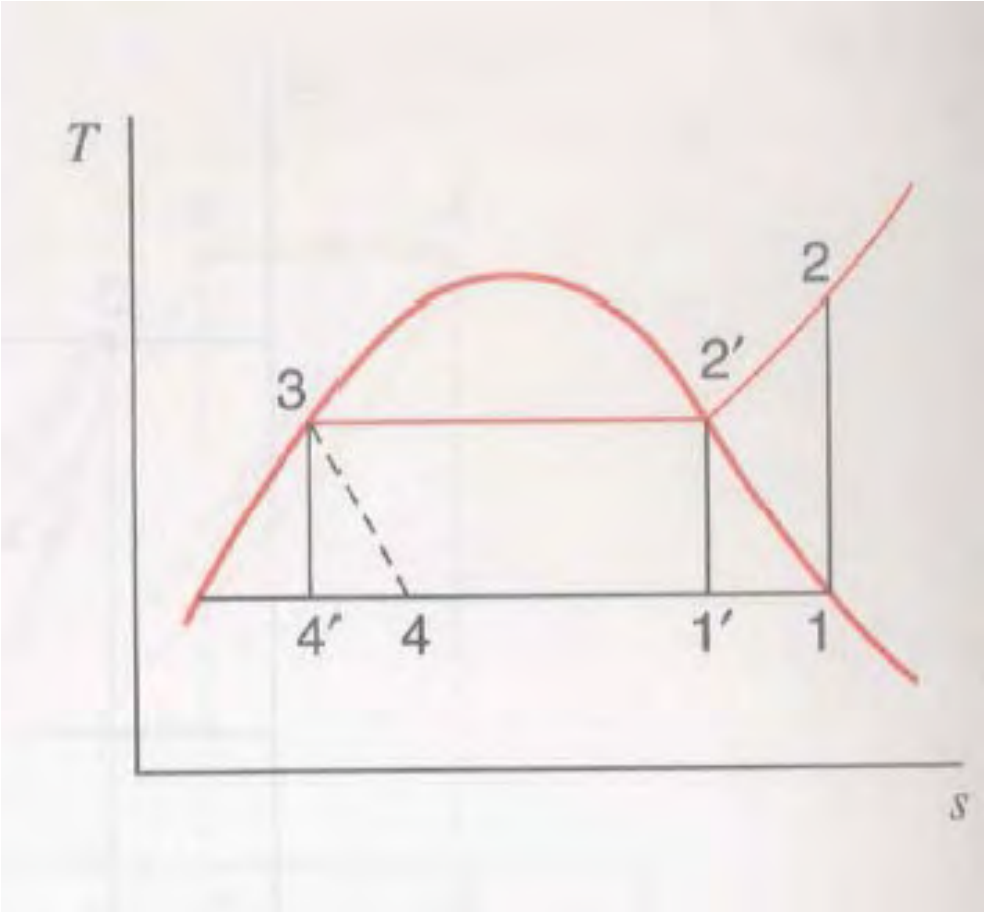
1. Strictly, the Stirling cycle engine is not an internal-combustion engine but external-combustion engine with regeneration

2. Two gas chambers are connected to pistons
3. Constant volume process – heat transferred by external combustors

# VAPOR-COMPRESSION REFRIGERATION CYCLE



# VAPOR-COMPRESSION REFRIGERATION CYCLE



# VAPOR-COMPRESSION REFRIGERATION CYCLE

4 processes

1-2 : isentropic compression (pump)
2-3 : constant pressure heat rejection (condenser)
3-4 : adiabatic throttling process (irreversible)
4-1 : constant pressure evaporation (heat absorption)

Cycle performance : Coefficient of Performance (COP) $\beta = \frac{q_L}{w_c}, \quad \beta' = \frac{q_H}{w_c}$

# VAPOR-COMPRESSION REFRIGERATION CYCLE

## WORKING FLUIDS (REFRIGERANTS)

**Ammonia & Sulfur-Dioxide (early days) – but not used ; highly toxic and dangerous**

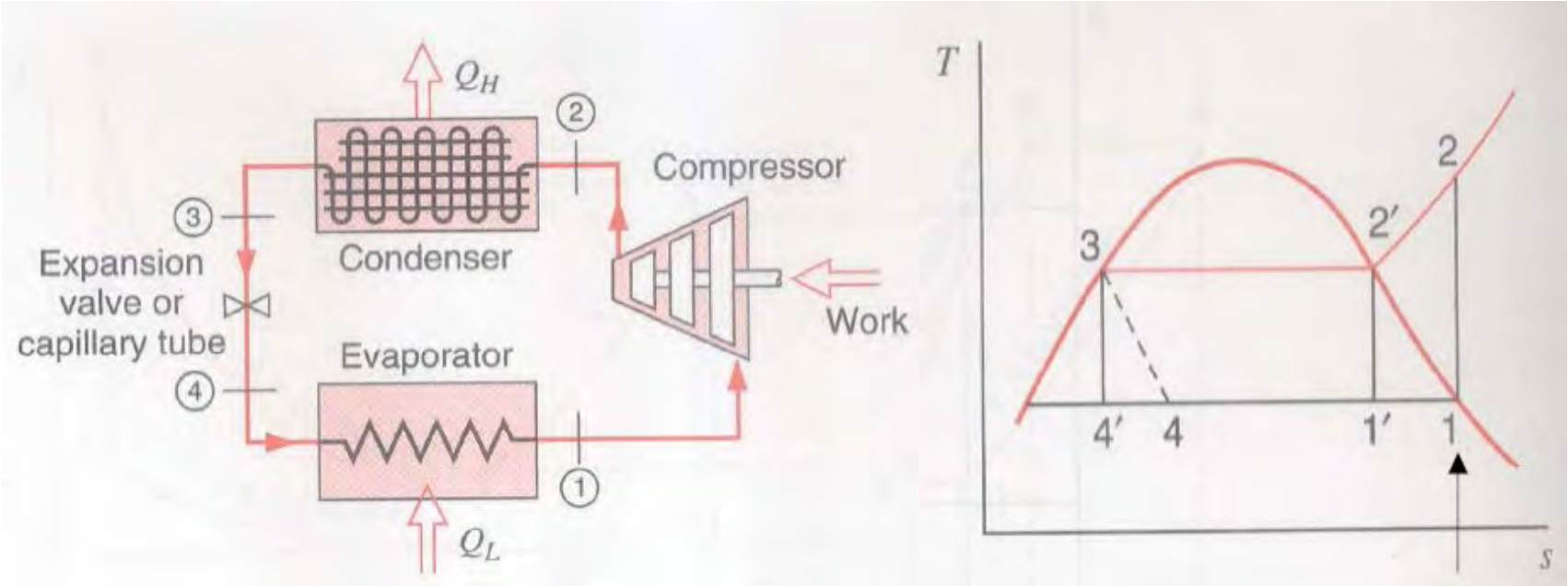
**Chlorofluorocarbons (CFCs) – $CCl_2F_2$ (Freon-12, Genatron-12) ; R-11 and R-12 : but destroying the protective ozone layer of the stratosphere**

**The most desirable fluids – HFCs (CFCs containing hydrogen) R-22**

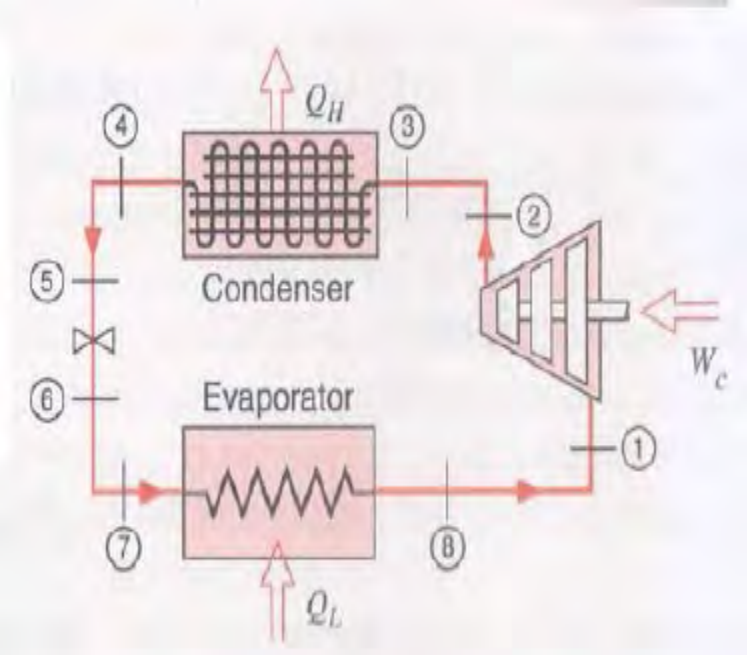
**Two important considerations when selecting refrigerant working fluids**

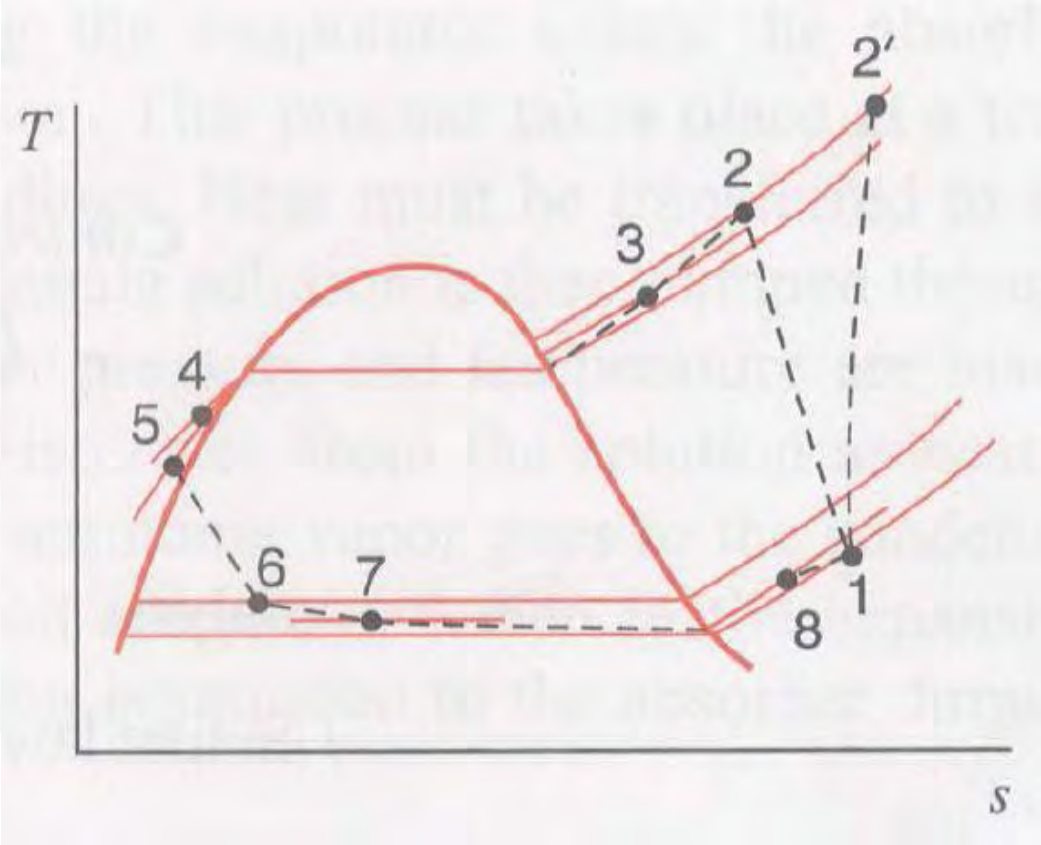
**A. Temperature at which refrigeration is needed**
**B. Type of equipment to be used**

# DEVIATION OF THE ACTUAL VAPOR--COMPRESSOR REFRIGERATION CYCLE FROM IDEAL CYCLE

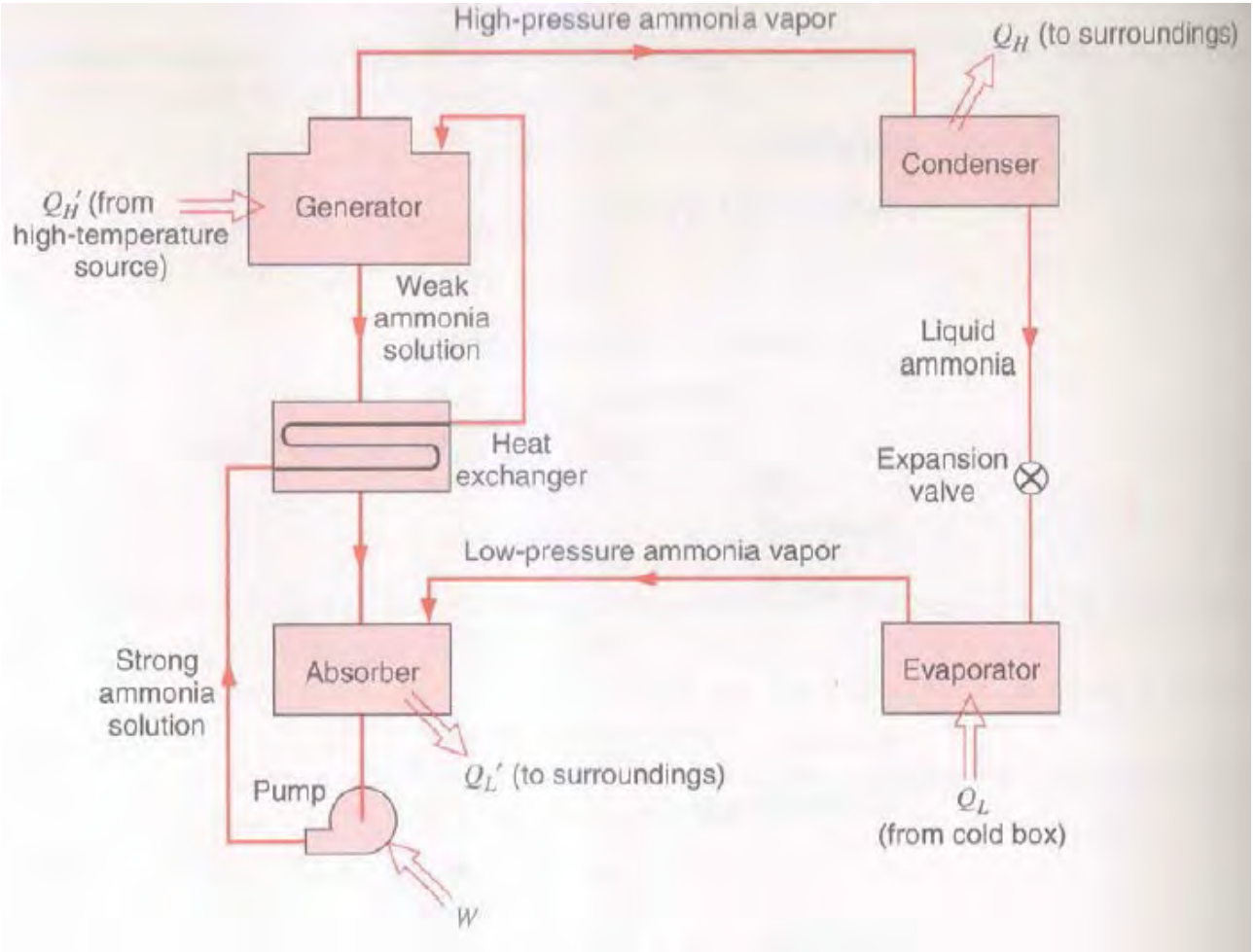


# DEVIATION OF THE ACTUAL VAPOR--COMPRESSOR REFRIGERATION CYCLE FROM IDEAL CYCLE

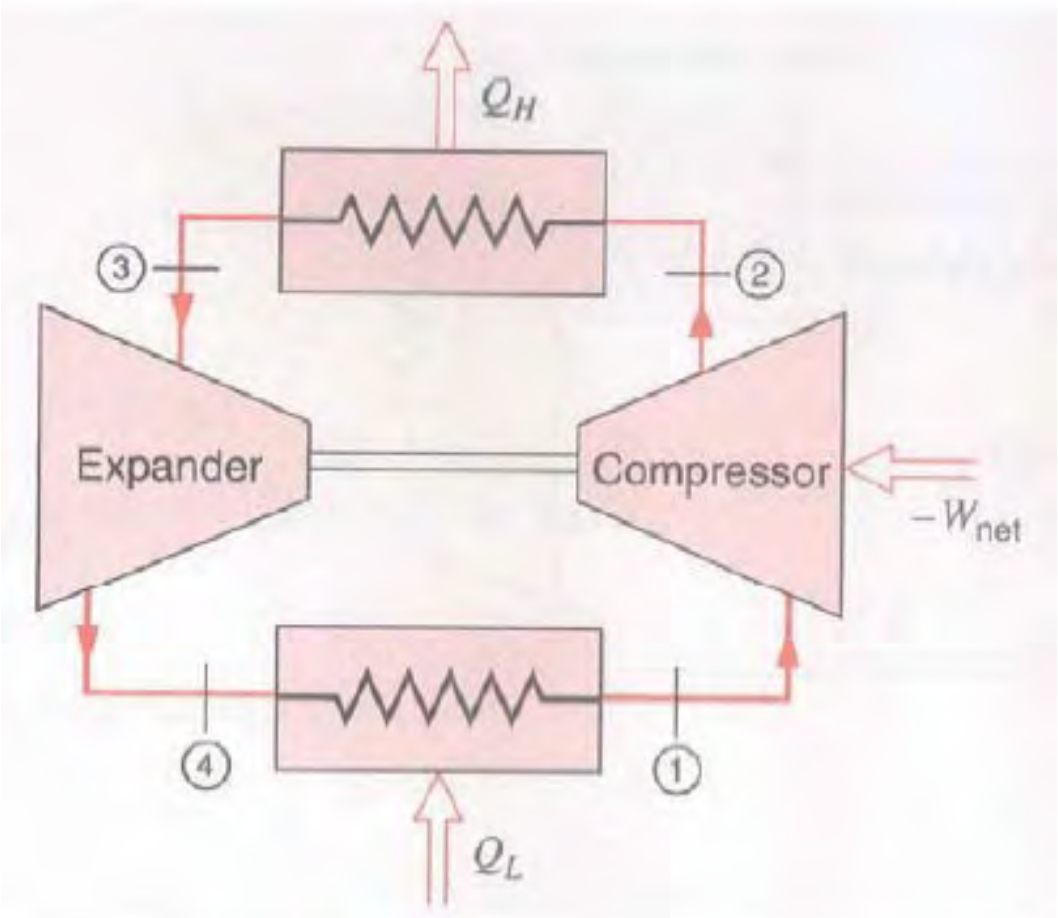


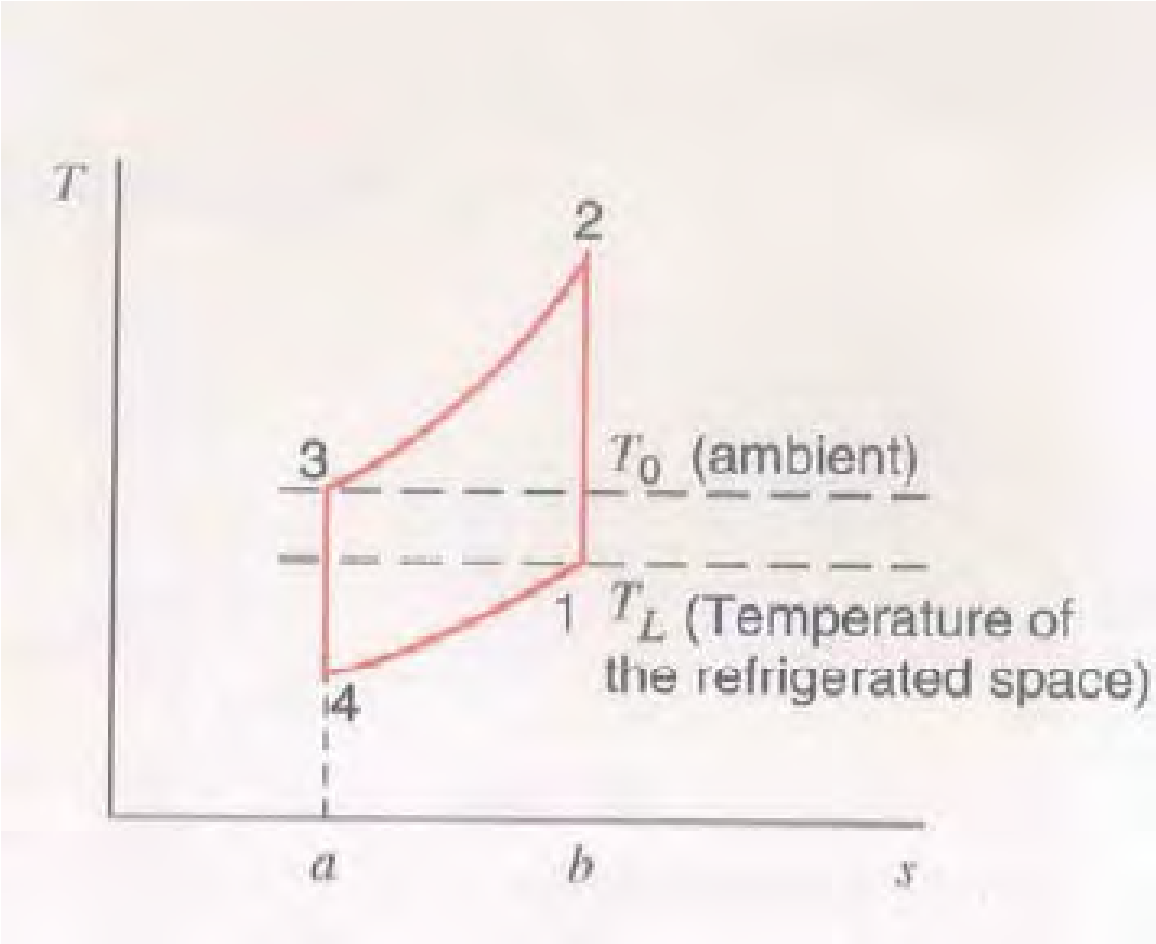
# AMMONIA ABSORPTION REFRIGERATION CYCLE



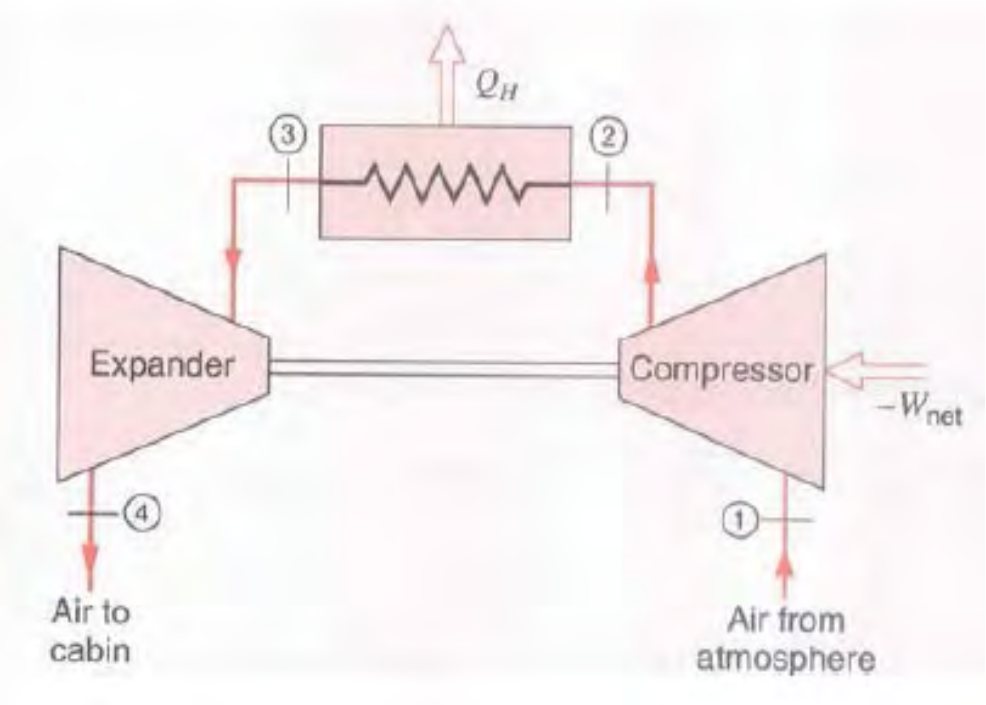
# THE AIR-STANDARD REFRIGERATION CYCLE

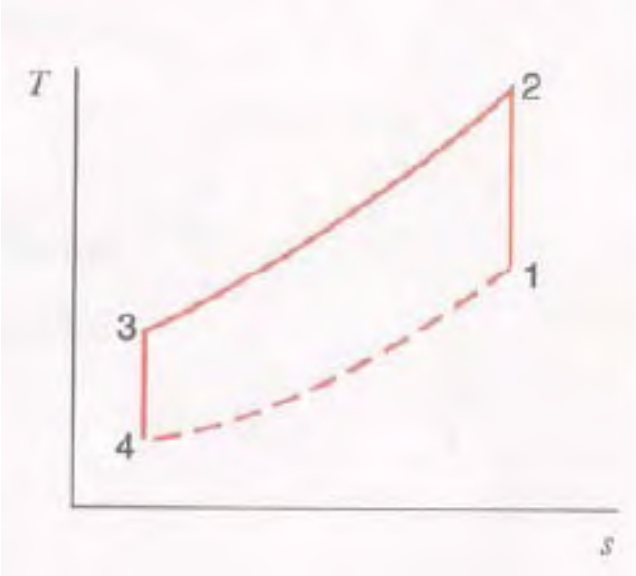


# THE AIR-STANDARD REFRIGERATION CYCLE

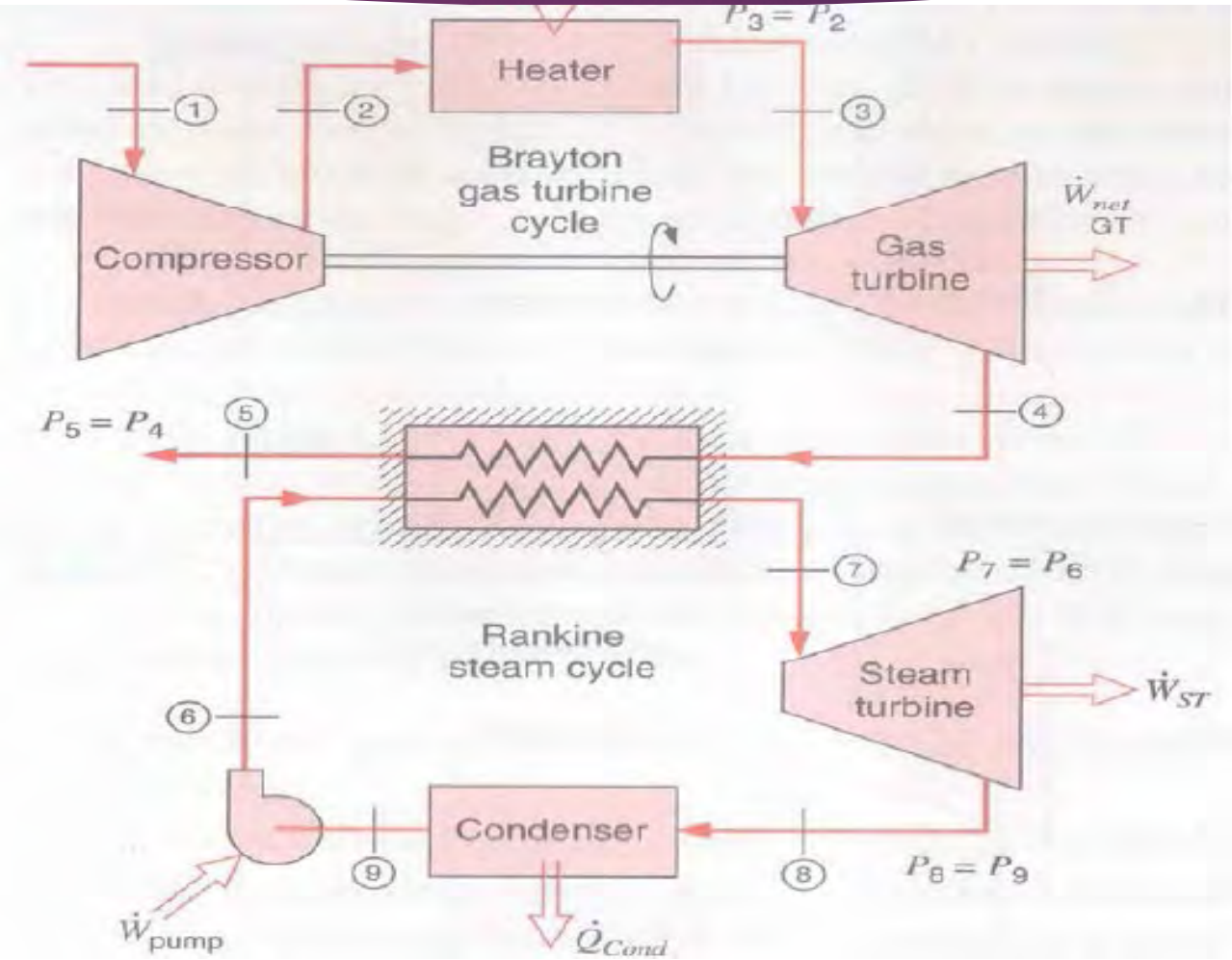


# THE AIR-STANDARD REFRIGERATION CYCLE (FOR AIRCRAFT COOLING)

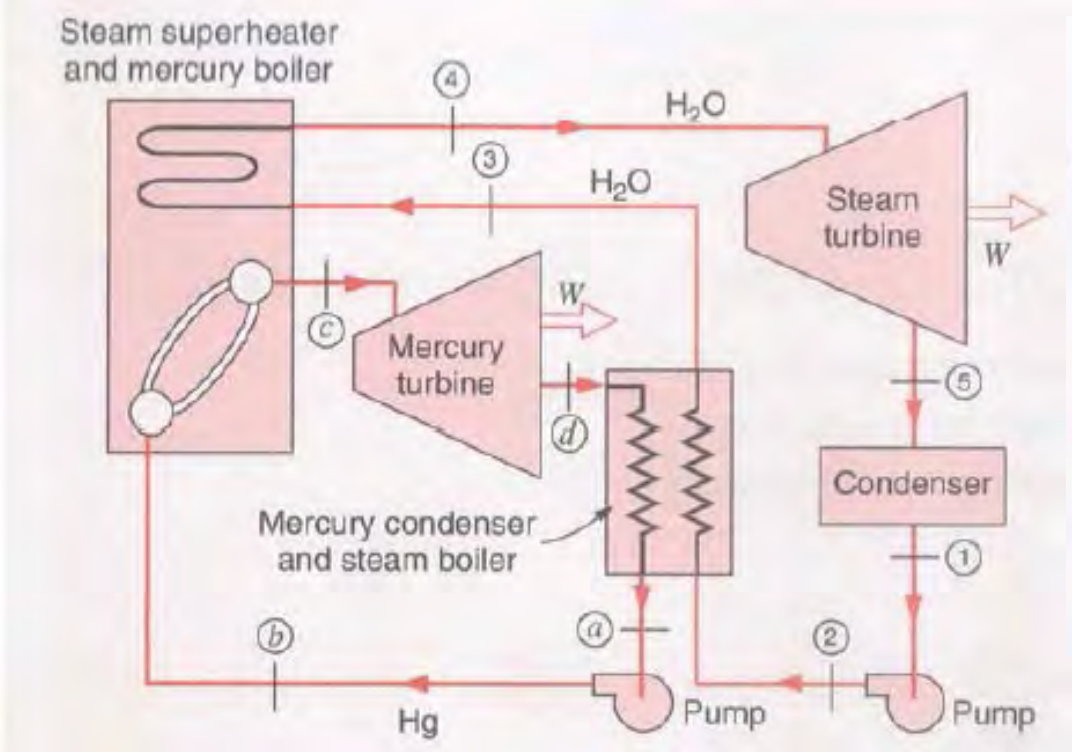


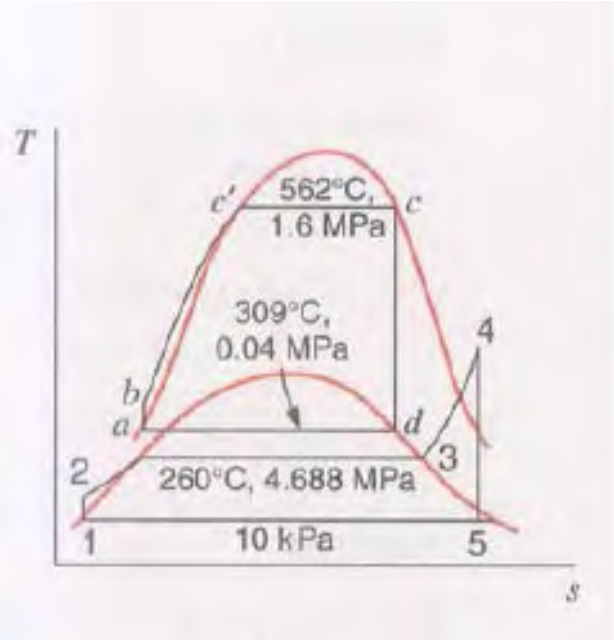



# COMBINED BRAYTON/RANKINE CYCLE POWER SYSTEM



# COMBINED-CYCLE POWER AND REFRIGERATION SYSTEM



# What is Psychometric

**The behavior of mixtures of air and water vapor under varying conditions of heat**

**Enthalpy = Total heat in the air = Sensible plus Latent heat**

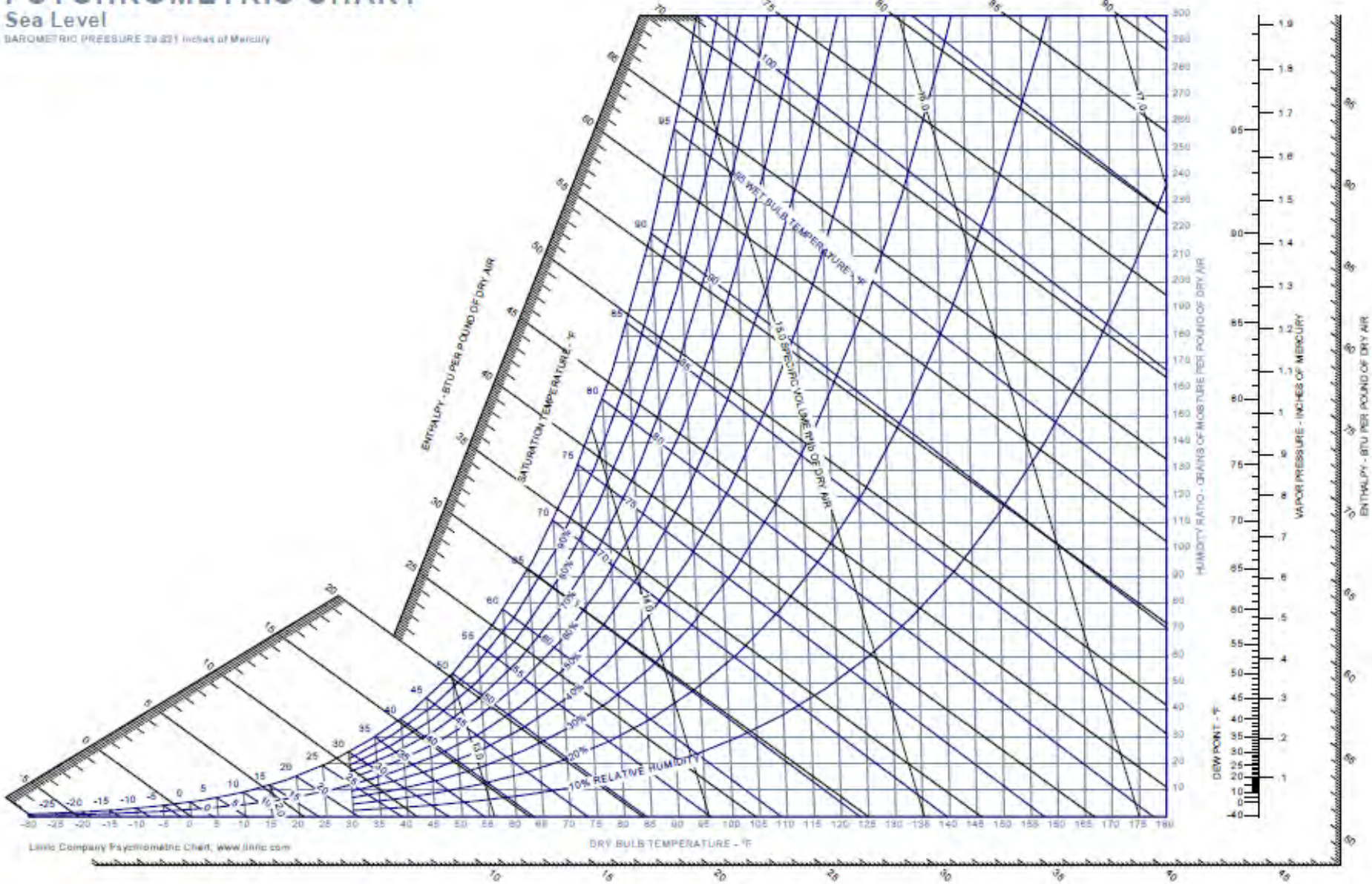
**Sensible Heat – Changes in temperature that do not alter the moisture content of air**

**Latent Heat – Related to level of moisture in the air**

# PSYCHROMETRIC CHART

## Sea Level

BAROMETRIC PRESSURE 29.921 Inches of Mercury



# The Psychrometric Chart – an Overview

Before we proceed further in our study, let's learn or refresh regarding the Psychrometric chart. The following is a summary of the major elements on the chart.

•Constant Dry bulb Temperature: Vertical Lines

•Constant Dew Point and Humidity Ratio: Horizontal Lines

•Constant Wet bulb temperature: Upward left sloping lines

•Relative humidity: Curving lines (100% line is the saturation curve or correlates with Dew Point.

# The Psychrometric Chart – an Overview

•Constant specific volume of dry air: Nearly-vertical sloping lines

•Enthalpy or total heat: Staggered scale left of saturation curve and left sloping Lines

•Humidity Ratio: Right hand scale, grains of moisture/pound of dry air

•Saturation Curve: 100% RH Curve (or the point at which an air mixture can hold no additional moisture at a given temperature); temperature on the curve is the Dew Point

# Latent versus Sensible Changes

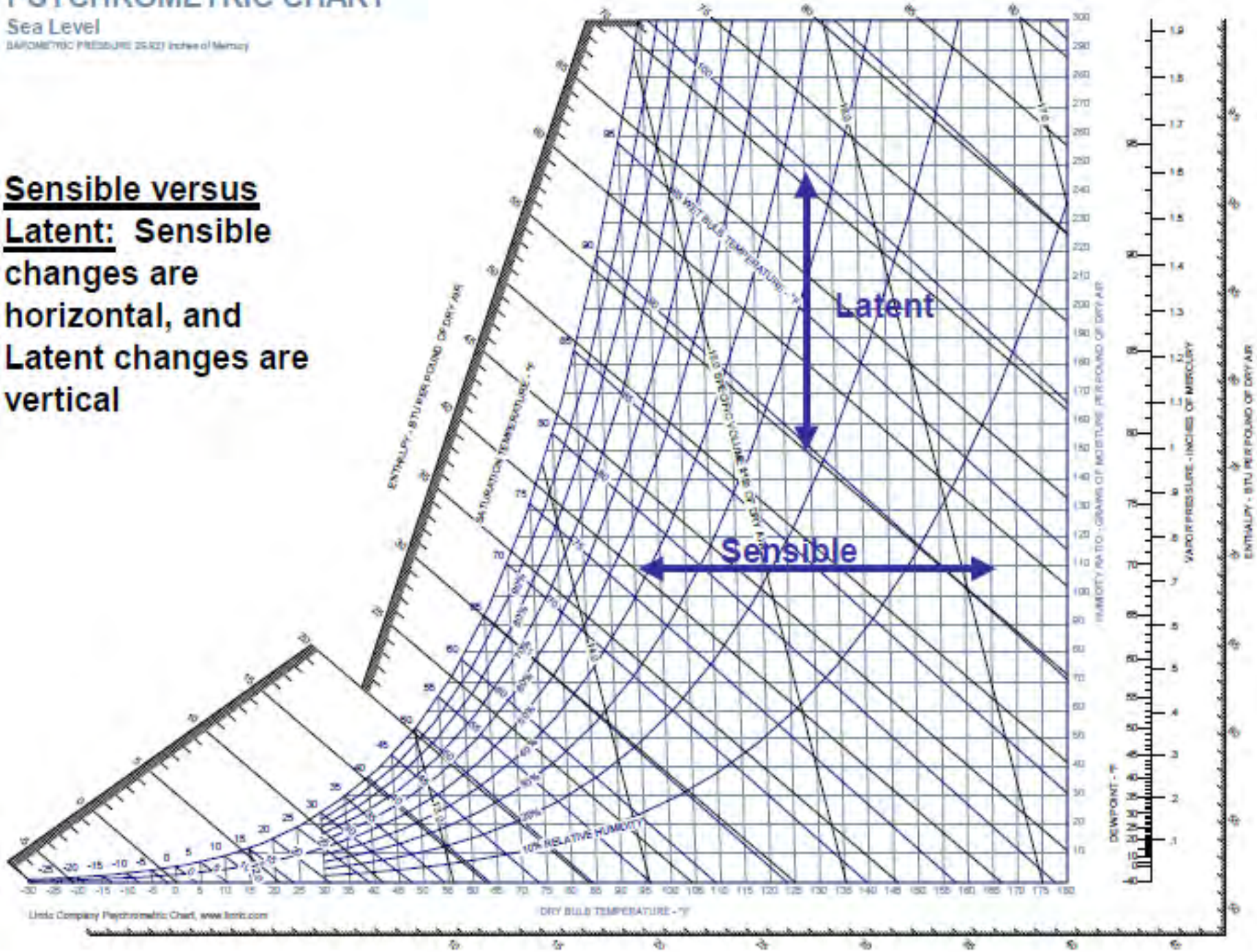
In the next graph, let’s look at changes in Latent versus Sensible changes. Latent changes move in the “Y” axis (associated with moisture content changes), and Sensible changes move in the “X” axis (associated with temperature but not moisture content changes.)

# PSYCHROMETRIC CHART

Sea Level

BAROMETRIC PRESSURE 29.921 Inches of Mercury

**Sensible versus Latent:** Sensible changes are horizontal, and Latent changes are vertical



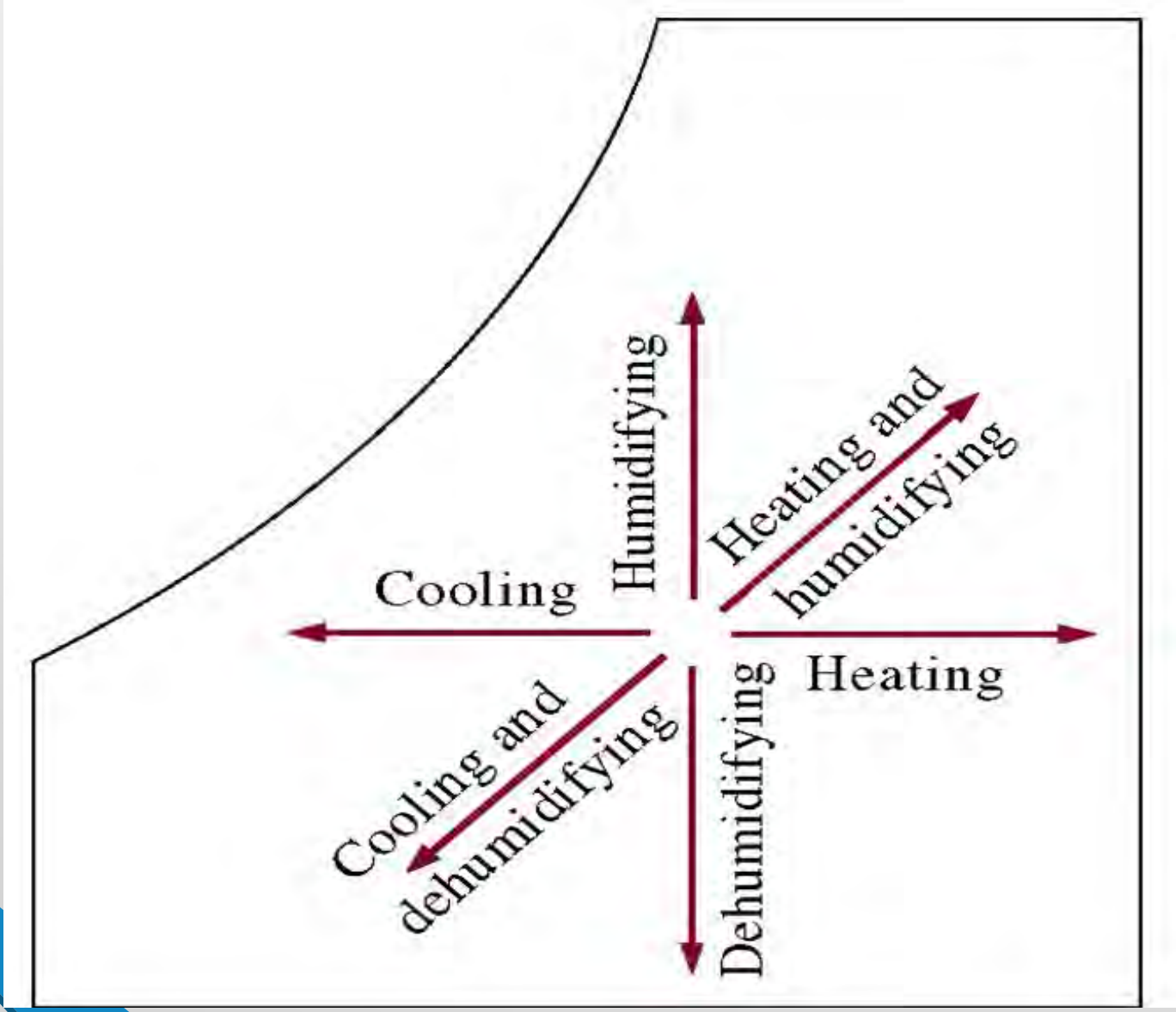
# AIR-CONDITIONING PROCESSES

Maintaining a living space or an industrial facility at the desired temperature and humidity requires some processes called air- conditioning processes.

These processes include ***simple heating*** (raising the temperature),***simple cooling*** (lowering the temperature), ***humidifying*** (adding moisture), and ***dehumidifying*** (removing moisture).

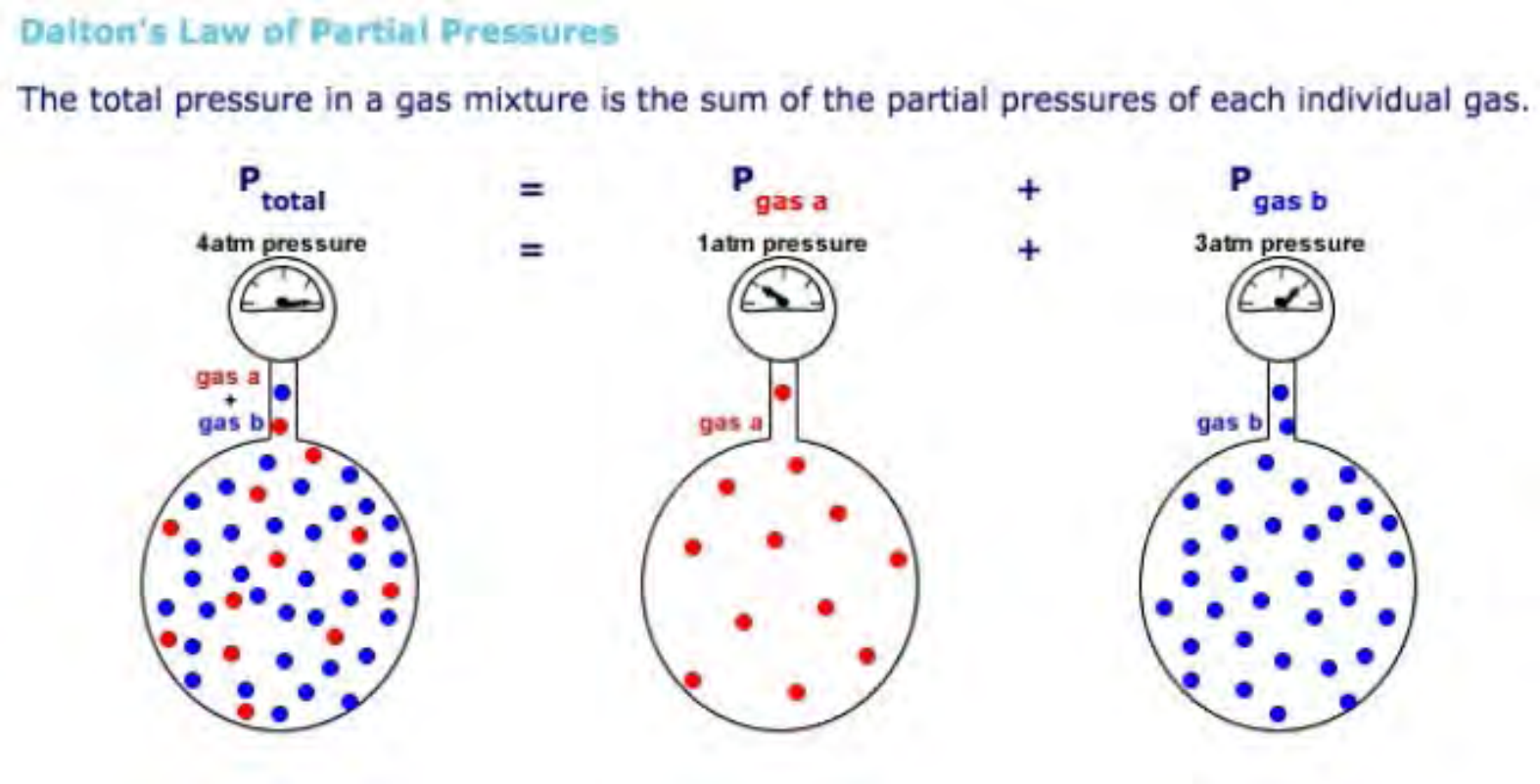
Sometimes two or more of these processes are needed to bring the air to a desired temperature and humidity level. Air is commonly heated & humidified in winter and cooled & dehumidified in the summer time

# VARIOUS PROCESSES.



# GAS MIXTURE - DALTON'S MODEL PARTIAL PRESSURES

Each gas existed alone at the mixture temperature and volume.



# DALTON'S LAW

Def:

Mole Fraction = $\frac{\text{mols of one substance}}{\text{total moles}}$ = X

$(X_A + X_B + X_C = 1)$

Partial Pressure = pressure exerted by only one gas in a mixture.

$P_A = n_A R_u T/V$ or $P_A = X_A P_{TOT}$

# DRY AND ATMOSPHERIC AIR

***Atmospheric air***: Air in the atmosphere containing some water vapor (or moisture).

***Dry air***: Air that contains no water vapor. Water vapor in the air plays a major role in human comfort. Therefore, it is an important consideration in air-conditioning applications.

$$h_{\text{dry air}} = c_p T = (1.005\ \text{kJ/kg}\cdot{}^\circ\text{C})T \qquad (\text{kJ/kg})$$

$$\Delta h_{\text{dry air}} = c_p \Delta T = (1.005\ \text{kJ/kg}\cdot{}^\circ\text{C})\ \Delta T \qquad (\text{kJ/kg})$$

Water vapor in air behaves as if it existed alone and obeys the ideal-gas relation Pv = RT. Then the atmospheric air can be treated as an ideal-gas mixture:

$$P = P_a + P_v \qquad \text{(kPa)}$$

$P_a$ = Partial pressure of dry air

$P_v$ = Partial pressure of vapor (vapor pressure)

Dry air

| $T$, °C | $c_p$, kJ/kg·°C |
|---|---|
| −10 | 1.0038 |
| 0 | 1.0041 |
| 10 | 1.0045 |
| 20 | 1.0049 |
| 30 | 1.0054 |
| 40 | 1.0059 |
| 50 | 1.0065 |

The $c_p$ of air can be assumed to be constant at 1.005 kJ/kg·°C in the temperature range −10 to 50°C with an error under 0.2%.

*For water*

$h_g$ = 2500.9 kJ/kg at 0°C

$c_{p,avg}$ = 1.82 kJ/kg · °C at −10 to 50°C range

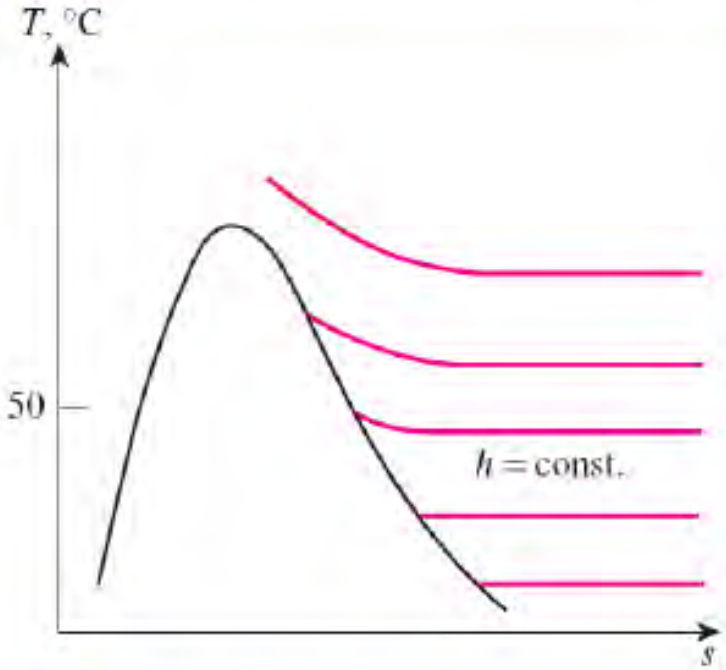
$$h_g(T) \cong 2500.9 + 1.82T \quad \text{(kJ/kg)} \quad T \text{ in °C}$$

$$h_g(T) \cong 1060.9 + 0.435T \quad \text{(Btu/lbm)} \quad T \text{ in °F}$$

$h = h(T)$ since water vapor is an ideal gas

$$h_v(T, \text{low } P) \cong h_g(T)$$

In the temperature range −10 to 50°C, the $h_g$ of water can be determined from Eq. 14-4 with negligible error.




Below 50°C, the $h$ = const. lines coincide with the $T$ = const. lines in the superheated vapor region of water.

Water vapor

| $T$, °C | $h_g$, kJ/kg Table A-4 | $h_g$, kJ/kg Eq. 14-4 | Difference, kJ/kg |
|---|---|---|---|
| −10 | 2482.1 | 2482.7 | −0.6 |
| 0 | 2500.9 | 2500.9 | 0.0 |
| 10 | 2519.2 | 2519.1 | 0.1 |
| 20 | 2537.4 | 2537.3 | 0.1 |
| 30 | 2555.6 | 2555.5 | 0.1 |
| 40 | 2573.5 | 2573.7 | −0.2 |
| 50 | 2591.3 | 2591.9 | −0.6 |

# SPECIFIC AND RELATIVE HUMIDITY OF AIR

**Absolute** or **specific humidity** (*humidity ratio*): The mass of water vapor present in a unit mass of dry air.

$$\omega = \frac{m_v}{m_a} \qquad \text{(kg water vapor/kg dry air)}$$

$$\omega = \frac{m_v}{m_a} = \frac{P_v V/R_v T}{P_a V/R_a T} = \frac{P_v/R_v}{P_a/R_a} = 0.622\frac{P_v}{P_a}$$

$$\omega = \frac{0.622 P_v}{P - P_v} \qquad \text{(kg water vapor/kg dry air)}$$

**Saturated air:** The air saturated with moisture.

**Relative humidity:** The ratio of the amount of moisture the air holds ($m_v$) to the maximum amount of moisture the air can hold at the same temperature ($m_g$).

$$\phi = \frac{m_v}{m_g} = \frac{P_v V/R_v T}{P_g V/R_v T} = \frac{P_v}{P_g} \qquad P_g = P_{sat\,@\,T}$$

AIR
25°C, 100 kPa

($P_{sat,H_2O\,@\,25°C} = 3.1698$ kPa)

$P_v = 0 \rightarrow$ dry air
$P_v < 3.1698$ kPa $\rightarrow$ unsaturated air
$P_v = 3.1698$ kPa $\rightarrow$ saturated air

For saturated air, the vapor pressure is equal to the saturation pressure of water.

AIR
25°C, 1 atm

$m_a = 1$ kg
$m_v = 0.01$ kg
$m_{v,\,max} = 0.02$ kg

Specific humidity: $\omega = 0.01 \dfrac{\text{kg } H_2O}{\text{kg dry air}}$
Relative humidity: $\phi = 50\%$

The difference between specific and relative humidities.

$$\phi = \frac{\omega P}{(0.622 + \omega)P_g} \quad \text{and} \quad \omega = \frac{0.622\phi P_g}{P - \phi P_g}$$

What is the relative humidity of dry air and saturated air?

In most practical applications, the amount of dry air in the air–water-vapor mixture remains constant, but the amount of water vapor changes.

Therefore, the enthalpy of atmospheric air is expressed *per unit mass of dry air.*

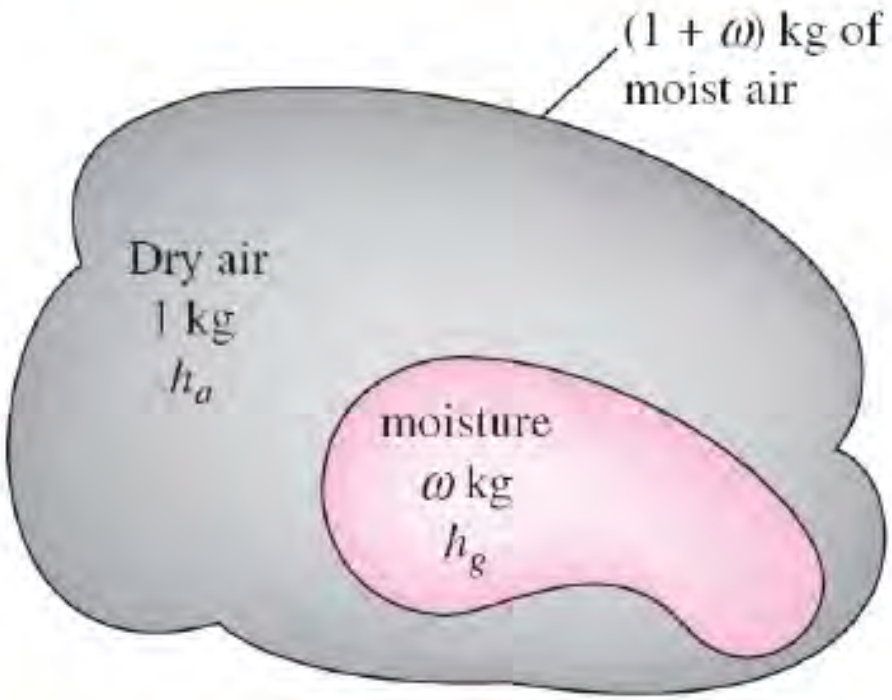
$$H = H_a + H_v = m_a h_a + m_v h_v$$

$$h = \frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$$

$$h_v \cong h_g$$

$$h = h_a + \omega h_g \qquad \text{(kJ/kg dry air)}$$

**Dry-bulb temperature:**
The ordinary temperature of atmospheric air.




$h = h_a + \omega h_g$, kJ/kg dry air

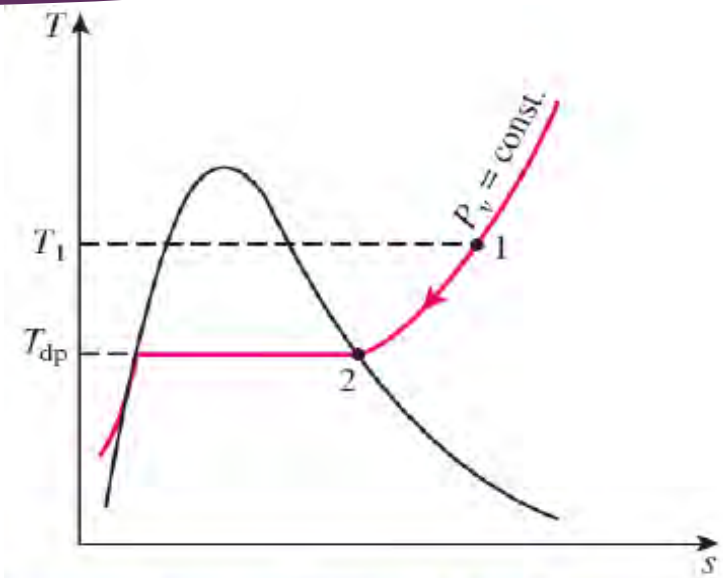
The enthalpy of moist (atmospheric) air is expressed per unit mass of dry air, not per unit mass of moist air.

# DEW-POINT TEMPERATURE

## Dew-point temperature *T*dp:

The temperature at which condensation begins when the air is cooled at constant pressure (i.e., the saturation temperature of water corresponding to the vapor pressure.)

$$T_{dp} = T_{sat\ @\ P_v}$$




Constant-presssure cooling of moist air and the dew-point temperature on the *T-s* diagram of water.


When the temperature of a cold drink is below the dew-point temperature of the surrounding air, it "sweats."

# ADIABATIC SATURATION TEMPERATURE AND WET-BULB TEMPERATURE

$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$ (The mass flow rate of dry air remains constant)

$\dot{m}_{w_1} + \dot{m}_f = \dot{m}_{w_2}$ (The mass flow rate of vapor in the air increases by an amount equal to the rate of evaporation $\dot{m}_f$)

$$\dot{m}_a\omega_1 + \dot{m}_f = \dot{m}_a\omega_2 \longrightarrow \dot{m}_f = \dot{m}_a(\omega_2 - \omega_1)$$

$$\dot{E}_{in} = \dot{E}_{out}$$

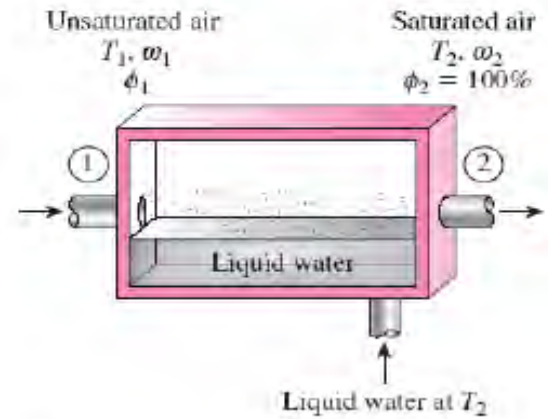
$$\dot{m}_a h_1 + \dot{m}_f h_{f_1} = \dot{m}_a h_2 \longrightarrow \dot{m}_a h_1 + \dot{m}_a(\omega_2 - \omega_1)h_{f_2} = \dot{m}_a h_2$$

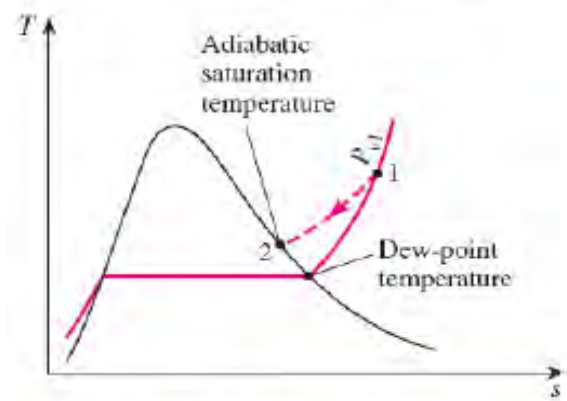
$$h_1 + (\omega_2 - \omega_1)h_{f_2} = h_2$$

$$(c_pT_1 + \omega_1 h_{g_1}) + (\omega_2 - \omega_1)h_{f_2} = (c_pT_2 + \omega_2 h_{g_2})$$

$$\omega_1 = \frac{c_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_1}} \qquad \omega_2 = \frac{0.622P_{g_2}}{P_2 - P_{g_2}}$$

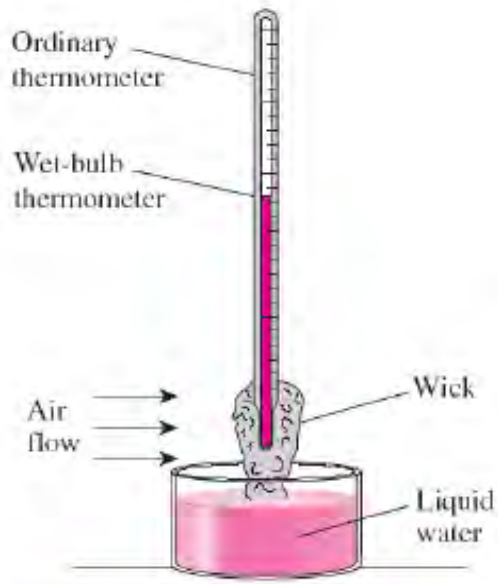
The specific humidity (and relative humidity) of air can be determined from these equations by measuring the pressure and temperature of air at the inlet and the exit of an adiabatic saturator.



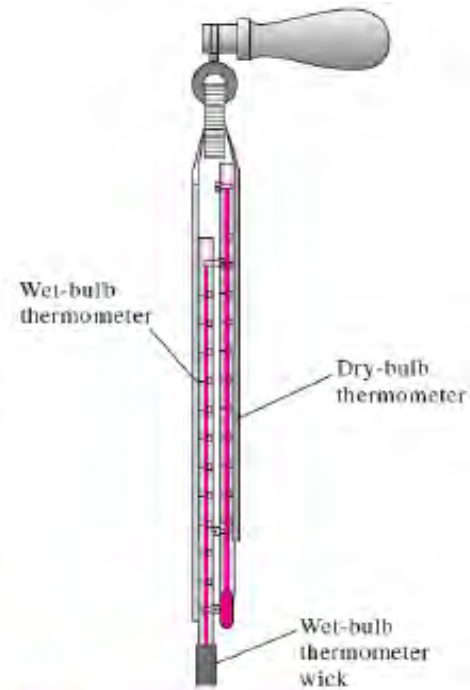




The adiabatic saturation process and its representation on a *T-s* diagram of water.

The adiabatic saturation process is not practical. To determine the absolute and relative humidity of air, a more practical approach is to use a thermometer whose bulb is covered with a cotton wick saturated with water and to blow air over the wick. The temperature measured is the **wet-bulb temperature *Twb*** and it is commonly used in A-C applications.




A simple arrangement to measure the wet-bulb temperature.

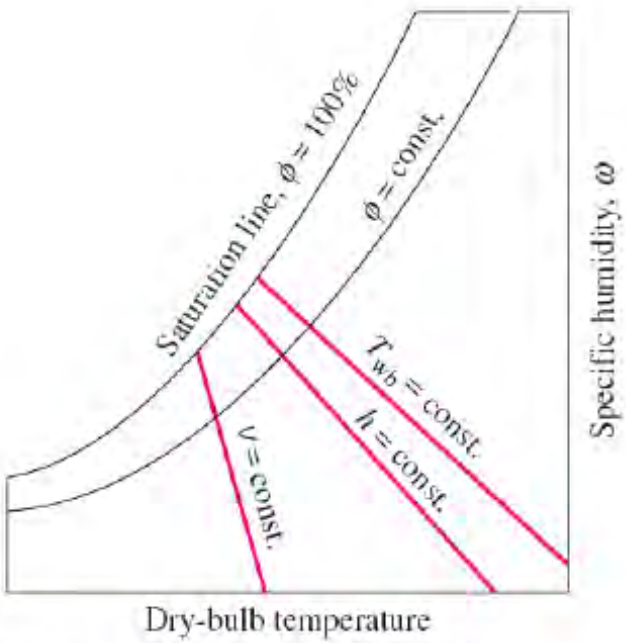



Sling psychrometer

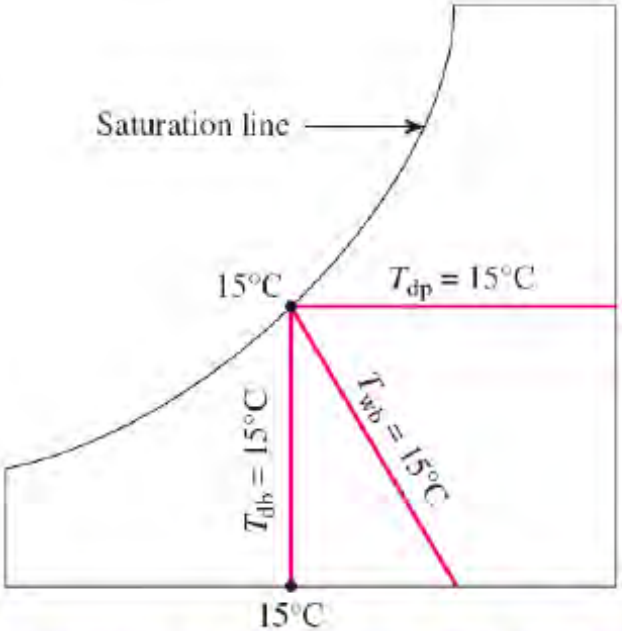
For air–water vapor mixtures at atmospheric pressure, $T_{wb}$ is approximately equal to the adiabatic saturation temperature.

# THE PSYCHROMETRIC CHART

**Psychrometric charts:** Present moist air properties in a convenient form. They are used extensively in A-C applications. The psychrometric chart serves as a valuable aid in visualizing the A-C processes such as heating, cooling, and humidification.

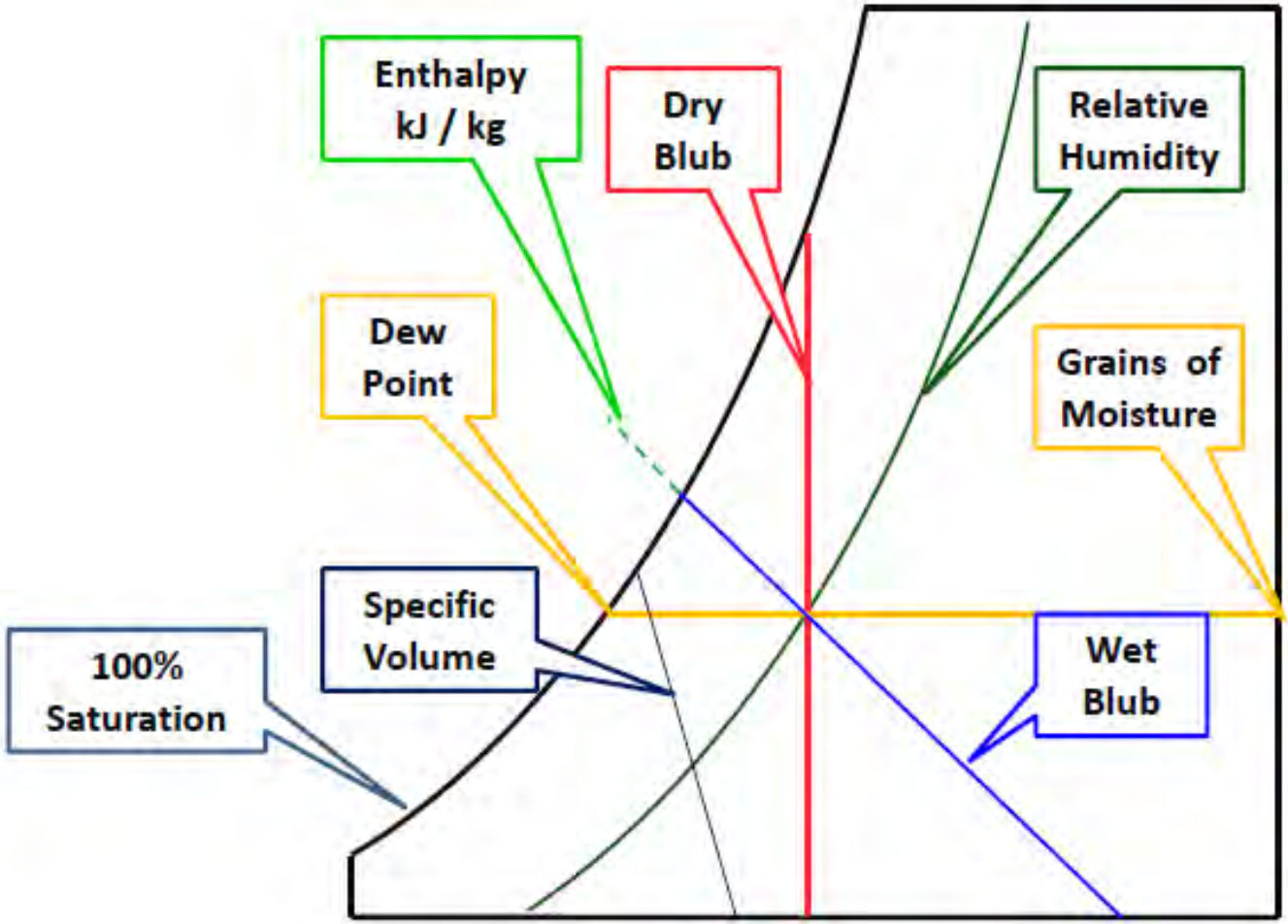



Schematic for a psychrometric chart.




For saturated air, the dry-bulb, wet-bulb, and dew-point temperatures are identical.

# BREAKDOWN OF THE LINES



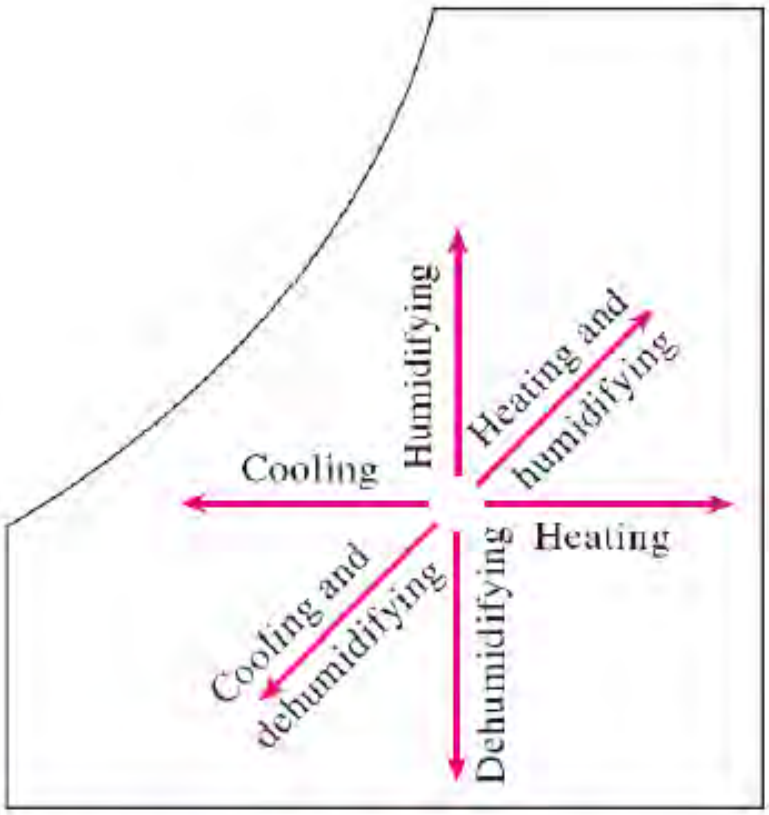
# AIR-CONDITIONING PROCESSES

Maintaining a living space or an industrial facility at the desired temperature and humidity requires some processes called air-conditioning processes.

These processes include simple heating (raising the temperature), simple cooling (lowering the temperature), humidifying (adding moisture), and dehumidifying (removing moisture).

Sometimes two or more of these processes are needed to bring the air to a desired temperature and humidity level.

Air is commonly heated and humidified in winter and cooled and dehumidified in summer.




Various air-conditioning processes.

Most air-conditioning processes can be modeled as steady-flow processes with the following general mass and energy balances:

Mass balance $\dot{m}_{in} = \dot{m}_{out}$

*Mass balance for dry air:* $\sum_{in} \dot{m}_a = \sum_{out} \dot{m}_a$ (kg/s)

*Mass balance for water:* $\sum_{in} \dot{m}_w = \sum_{out} \dot{m}_w$ or $\sum_{in} \dot{m}_a \omega = \sum_{out} \dot{m}_a \omega$

Energy balance $\dot{E}_{in} = \dot{E}_{out}$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m} h = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m} h$$

The work term usually consists of the *fan work input*, which is small relative to the other terms in the energy balance relation.

# SIMPLE HEATING AND COOLING

Many residential heating systems consist of a stove, a heat pump, or an electric resistance heater. The air in these systems is heated by circulating it through a duct that contains the tubing for the hot gases or the electric resistance wires. Cooling can be accomplished by passing the air over some coils through which a refrigerant or chilled water flows.

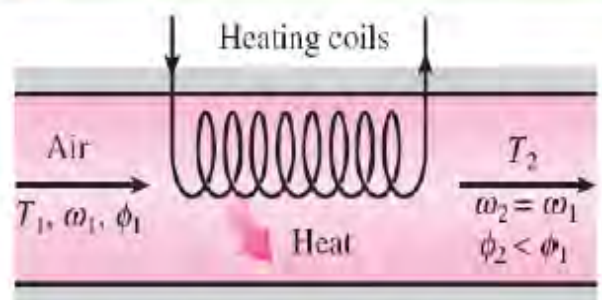
Heating and cooling appear as a horizontal line since no moisture is added to or removed from the air.

Dry air mass balance $\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}a$

Water mass balance $\omega_1 = \omega_2$

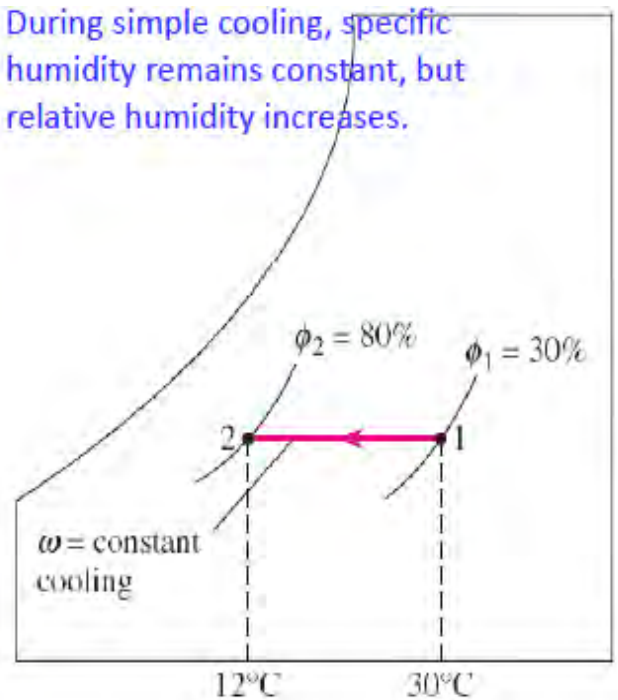
Energy balance

$$\dot{Q} = \dot{m}_a(h_2 - h_1) \quad \text{or} \quad q = h_2 - h_1$$




During simple heating, specific humidity remains constant, but relative humidity decreases.

During simple cooling, specific humidity remains constant, but relative humidity increases.



# HEATING WITH HUMIDIFICATION

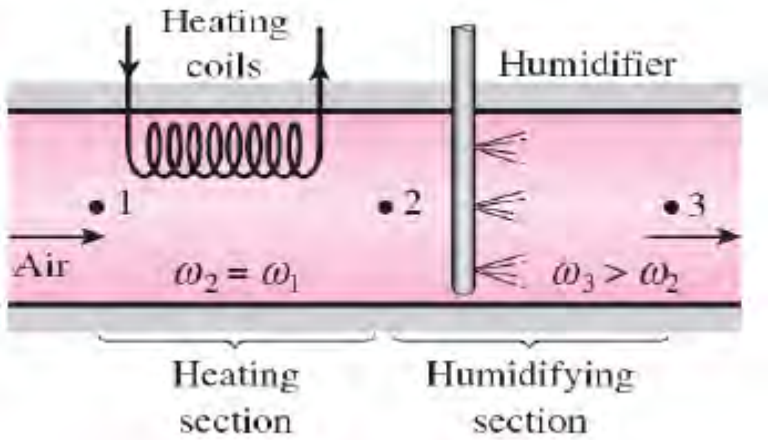
Problems with the low relative humidity resulting from simple heating can be eliminated by humidifying the heated air. This is accomplished by passing the air first through a heating section and then through a humidifying section.

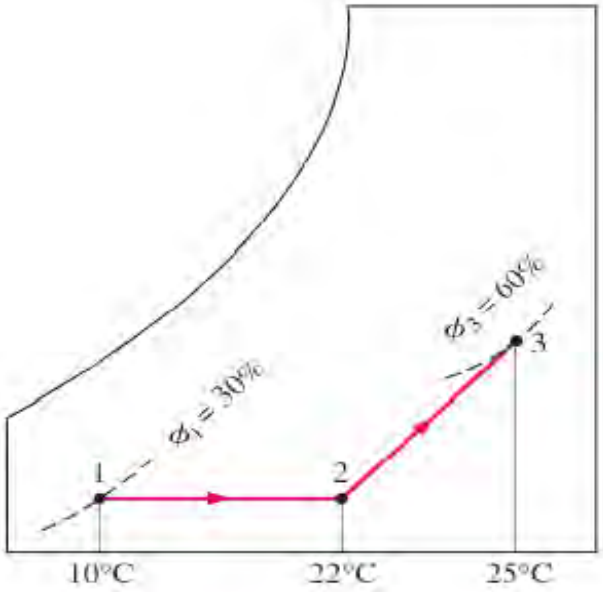
*Dry air mass balance:* $\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$

*Water mass balance:* $\dot{m}_{a_1}\omega_1 = \dot{m}_{a_2}\omega_2 \rightarrow \omega_1 = \omega_2$

*Energy balance:* $\dot{Q}_{in} + \dot{m}_a h_1 = \dot{m}_a h_2 \rightarrow \dot{Q}_{in} = \dot{m}_a(h_2 - h_1)$

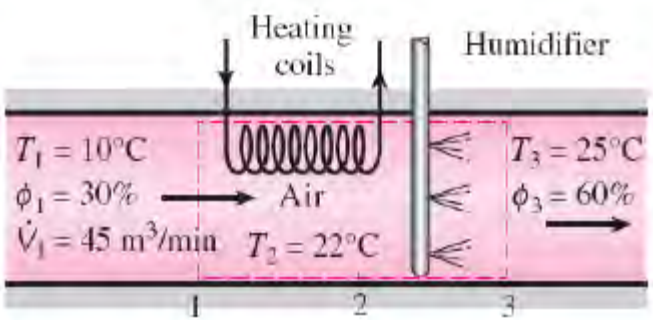
$\dot{m}_{a_2}\omega_2 + \dot{m}_w = \dot{m}_{a_3}\omega_3 \longrightarrow \dot{m}_w = \dot{m}_a(\omega_3 - \omega_2)$

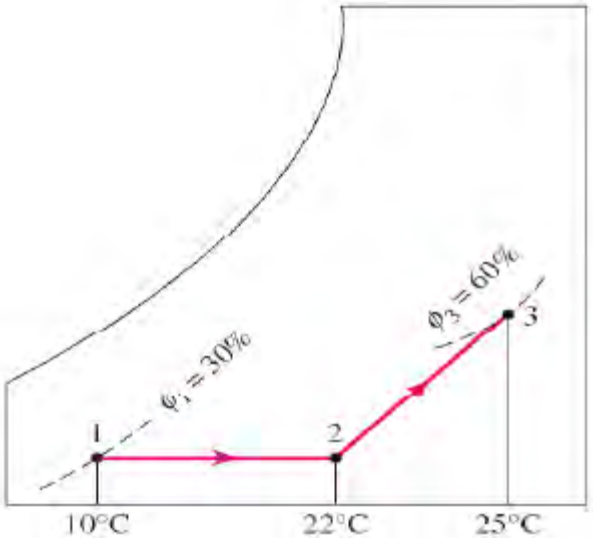


# EXAMPLE

An air-conditioning system is to take in outdoor air at 10°C and 30 percent relative humidity at a steady rate of 45 $m^3$/min and to condition it to 25°C and 60 percent relative humidity. The outdoor air is first heated to 22°C in the heating section and then humidified by the injection of hot steam in the humidifying section. Assuming the entire process takes place at a pressure of 100 kPa, determine (*a*) the rate of heat supply in the heating section and (*b*) the mass flow rate of the steam required in the humidifying section.



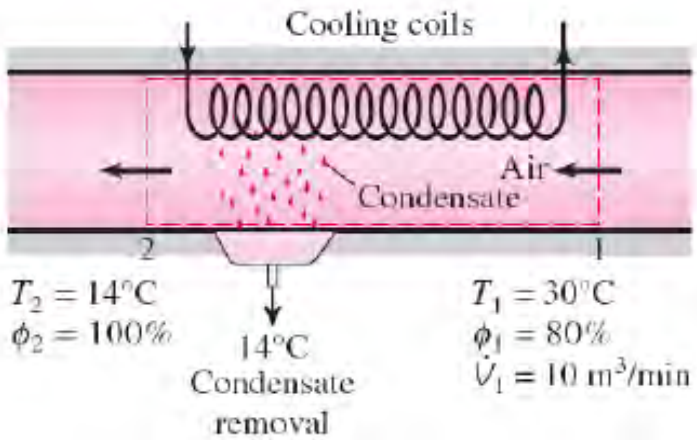
# COOLING WITH DEHUMIDIFICATION

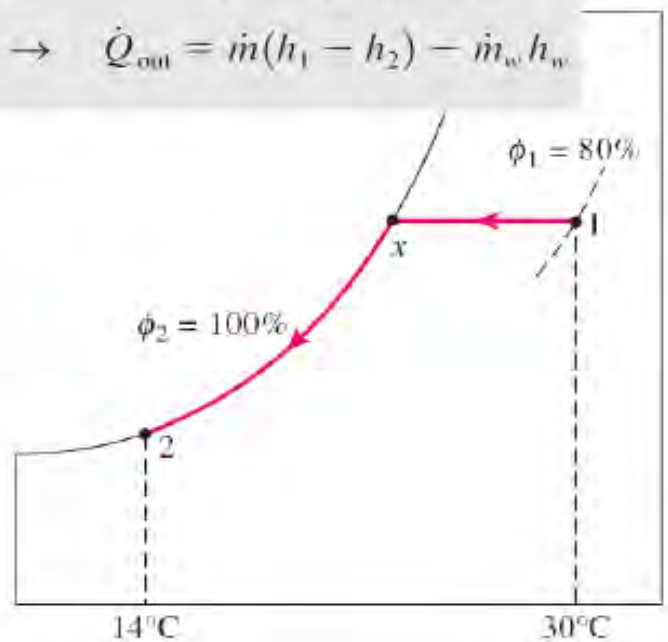
The specific humidity of air remains constant during a simple cooling process, but its relative humidity increases. If the relative humidity reaches undesirably high levels, it may be necessary to remove some moisture from the air, that is, to dehumidify it. This requires cooling the air below its dew-point temperature.

*Dry air mass balance:* $\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$

*Water mass balance:* $\dot{m}_{a_1}\omega_1 = \dot{m}_{a_2}\omega_2 + \dot{m}_w \rightarrow \dot{m}_w = \dot{m}_a(\omega_1 - \omega_2)$

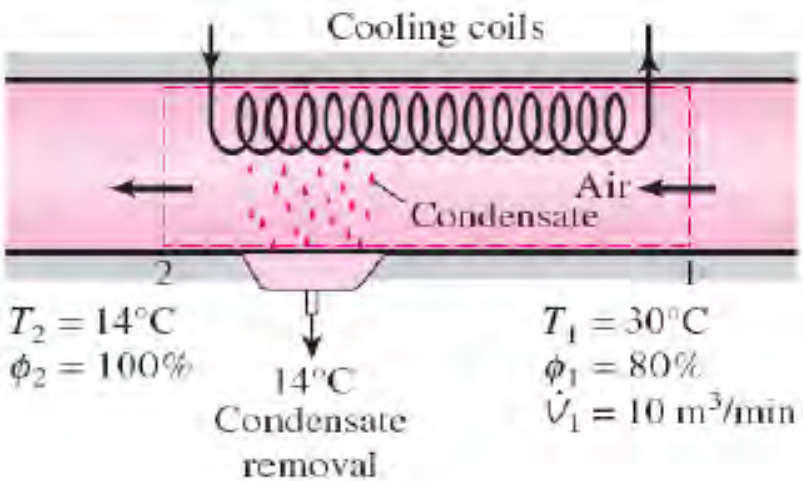
*Energy balance:* $\sum_{in} \dot{m}h = \dot{Q}_{out} + \sum_{out} \dot{m}h \rightarrow \dot{Q}_{out} = \dot{m}(h_1 - h_2) - \dot{m}_w h_w$

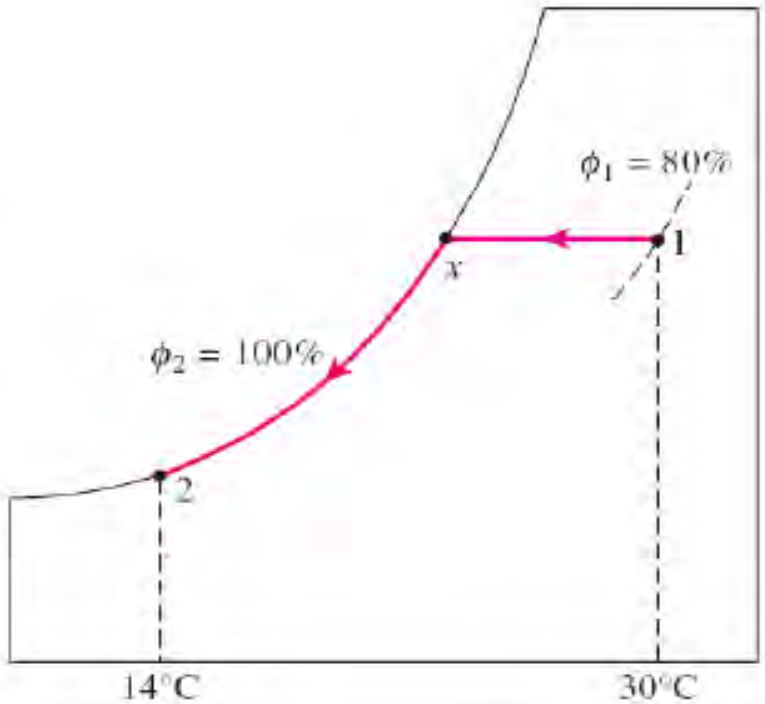


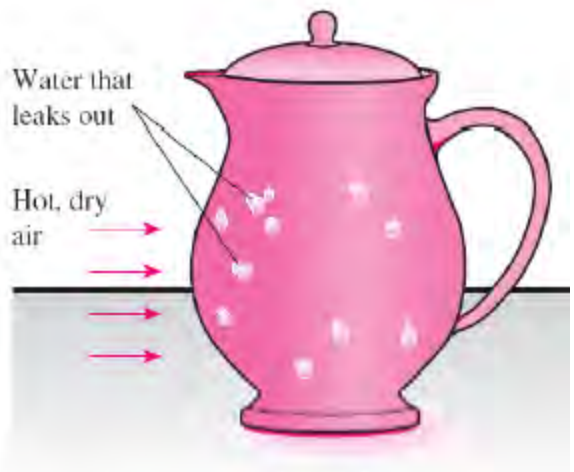
# EXAMPLE

Air enters a window air conditioner at 1 atm, 30°C, and 80 percent relative humidity at a rate of 10 $m^3$/min, and it leaves as saturated air at 14°C. Part of the moisture in the air that condenses during the process is also removed at 14°C. Determine the rates of heat and moisture removal from the air.



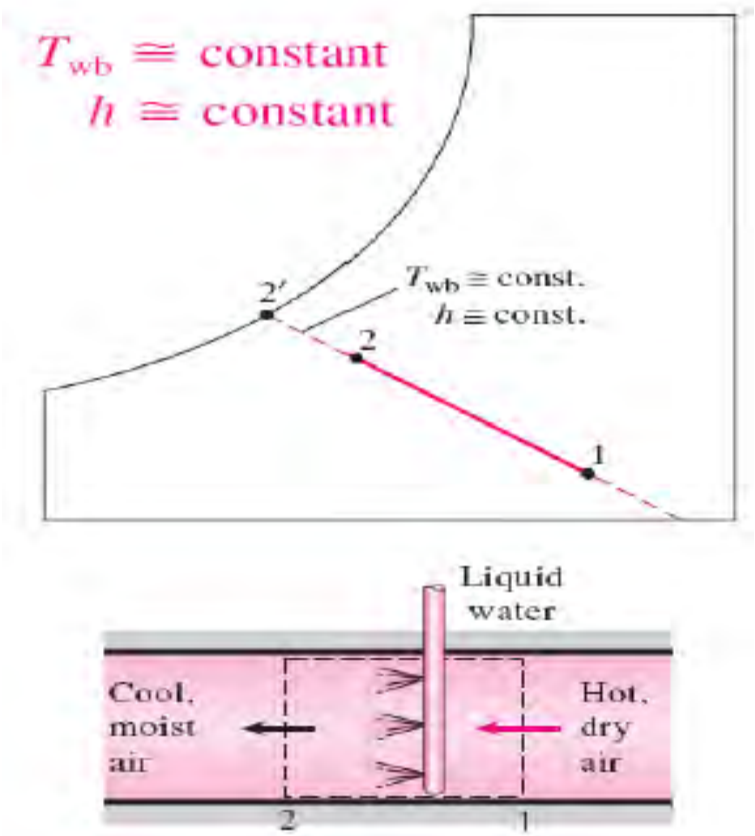
In desert (*hot and dry*) climates, we can avoid the high cost of conventional cooling by using *evaporative coolers*, also known as *swamp coolers*. As water evaporates, the latent heat of vaporization is absorbed from the water body and the surrounding air. As a result, both the water and the air are cooled during the process.




Water in a porous jug left in an open, breezy area cools as a result of evaporative cooling.

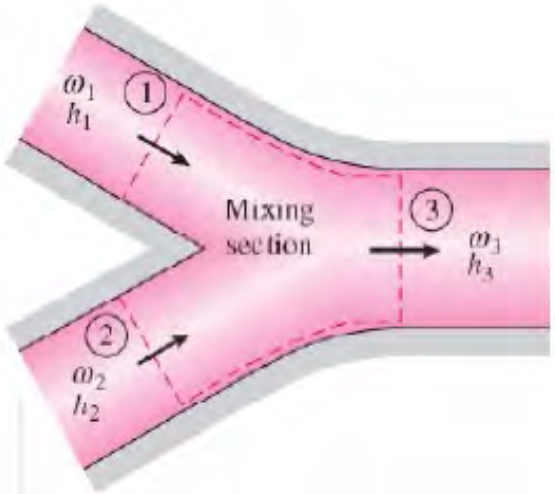
# EVAPORATIVE COOLING

This process is essentially identical to adiabatic saturation process.



# ADIABATIC MIXING OF AIRSTREAMS

Many A-C applications require the mixing of two airstreams. This is particularly true for large buildings, most production and process plants, and hospitals, which require that the conditioned air be mixed with a certain fraction of fresh outside air before it is routed into the living space.

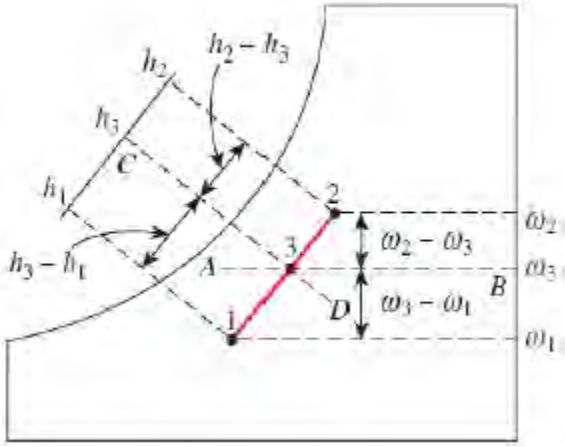



*Mass of dry air:* $\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$

*Mass of water vapor:* $\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \dot{m}_{a_3}$

*Energy:* $\dot{m}_{a_1} h_1 + \dot{m}_{a_2} h_2 = \dot{m}_{a_3} h_3$

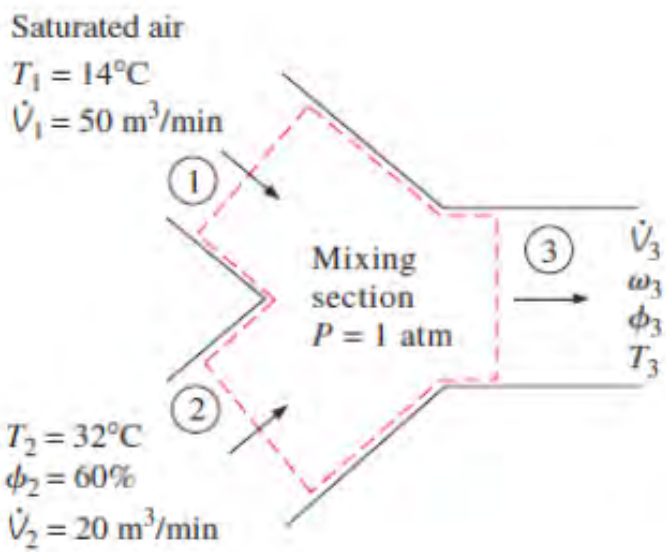
$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1}$$

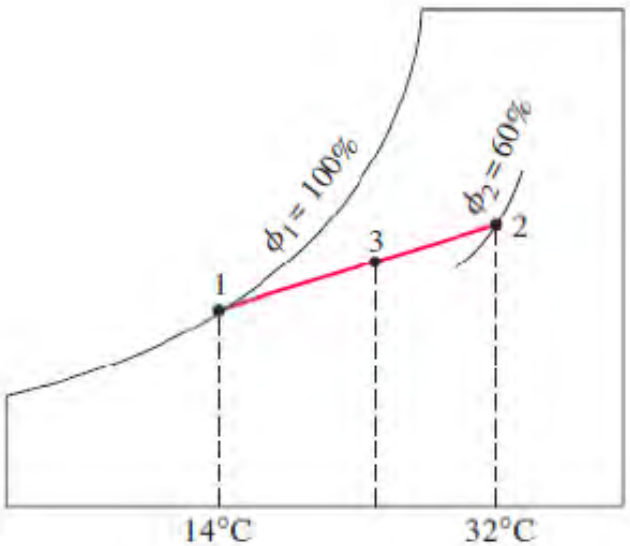



When two airstreams at states 1 and 2 are mixed adiabatically, the state of the mixture lies on the straight line connecting the two states.

# EXAMPLE

Saturated air leaving the cooling section of an air-conditioning system at 14°C at a rate of 50 $m^3$/min is mixed adiabatically with the outside air at 32°C and 60 percent relative humidity at a rate of 20 $m^3$/min. Assuming that the mixing process occurs at a pressure of 1 atm, determine the specific humidity, the relative humidity, the dry-bulb temperature, and the volume flow rate of the mixture.



# WET COOLING TOWERS

Power plants, large air-conditioning systems, and some industries generate large quantities of waste heat that is often rejected to cooling water from nearby lakes or rivers. In some cases, however, the cooling water supply is limited or thermal pollution is a serious concern. In such cases, the waste heat must be rejected to the atmosphere, with cooling water recirculating and serving as a transport medium for heat transfer between the source and the sink (the atmosphere). One way of achieving this is through the use of wet cooling towers. A **wet cooling tower** is essentially a semi-enclosed evaporative cooler.


An induced-draft counterflow cooling tower.

# COMPRESSIBLE FLOWS

Being the property of a fluid, the gases have high values of compressibility $\left(\kappa_T = 10^{-5}\ \mathrm{m^2/N}\ \text{for air at 1atm}\right)$ while liquids have low values of compressibility much less than that of gases $\left(\kappa_T = 5\times10^{-10}\ \mathrm{m^2/N}\ \text{for water at 1atm}\right)$. From the basic definition (Eq. 1.12), it is seen that whenever a fluid experiences a change in pressure $dp$, there will be a corresponding change in $d\rho$. Normally, high speed flows involve large pressure gradient. For a given change in $dp$, the resulting change in density will be small for liquids (low values of $\kappa$) and more for gases (high values of $\kappa$). Therefore, for the flow of liquids, the relative large pressure gradients can create much high velocities without much change in densities. Thus, the liquids are treated to be incompressible. On the other hand, for the flow of gases, the moderate to strong pressure gradient leads to substantial changes in the density (Eq. 1.12) and at the same time, it can create large velocity changes. Such flows are defined as compressible flows where the density is a variable property and the fractional change in density $(d\rho/\rho)$ is too large to be ignored.

# FUNDAMENTAL EQUATIONS FOR COMPRESSIBLE FLOW

Consider a compressible flow passing through a rectangular control volume as shown in Fig. 1.1. The flow is one-dimensional and the properties change as a function of $x$, from the region '1' to '2' and they are velocity $(u)$, pressure $(p)$, temperature $(T)$, density $(\rho)$ and internal energy $(e)$. The following assumptions are made to derive the fundamental equations;

- Flow is uniform over left and right side of control volume.
- Both sides have equal area $(A)$, perpendicular to the flow.
- Flow is inviscid, steady and nobody forces are present.
- No heat and work interaction takes place to/from the control volume.

Let us apply mass, momentum and energy equations for the one dimensional flow as shown in Fig. .1.1.

<u>Conservation of Mass</u>:

$$-\rho_1 u_1 A + \rho_2 u_2 A = 0 \quad \Rightarrow \rho_1 u_1 = \rho_2 u_2 \qquad (.1.14)$$

# FUNDAMENTAL EQUATIONS FOR COMPRESSIBLE FLOW

Conservation of Momentum:

$$\rho_1(-u_1A)u_1 + \rho_2(u_2A)u_2 = -(-p_1A + p_2A) \quad \Rightarrow p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \qquad (.1.15)$$

Steady Flow Energy Conservation:

$$\frac{p_1}{\rho_1} + e_1 + \frac{u_1^2}{2} = \frac{p_2}{\rho_2} + e_2 + \frac{u_2^2}{2} \quad \Rightarrow h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \qquad (.1.16)$$

# FUNDAMENTAL EQUATIONS FOR COMPRESSIBLE FLOW

Here, the enthalpy $h\left(=e+\frac{p}{\rho}\right)$ is defined as another thermodynamic property of the gas.

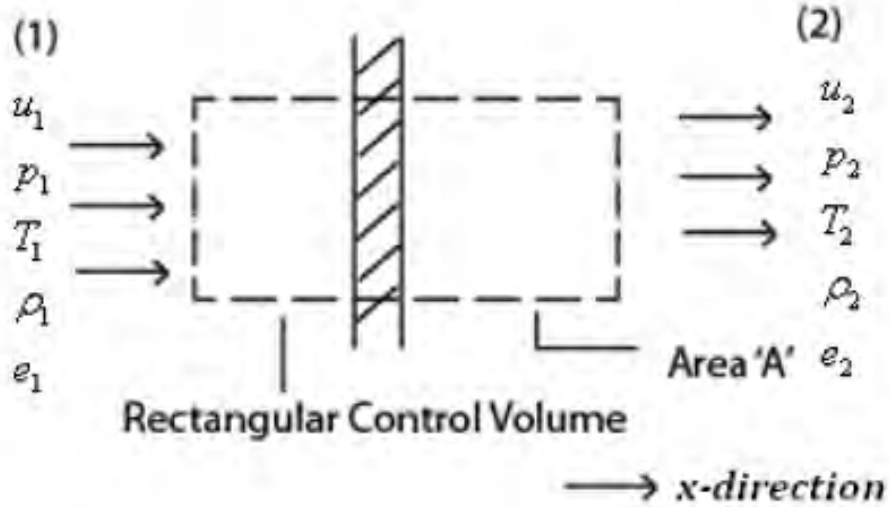



**Fig. 1.1: Schematic representation of one-dimensional flow.**

# CONTINUITY EQUATION

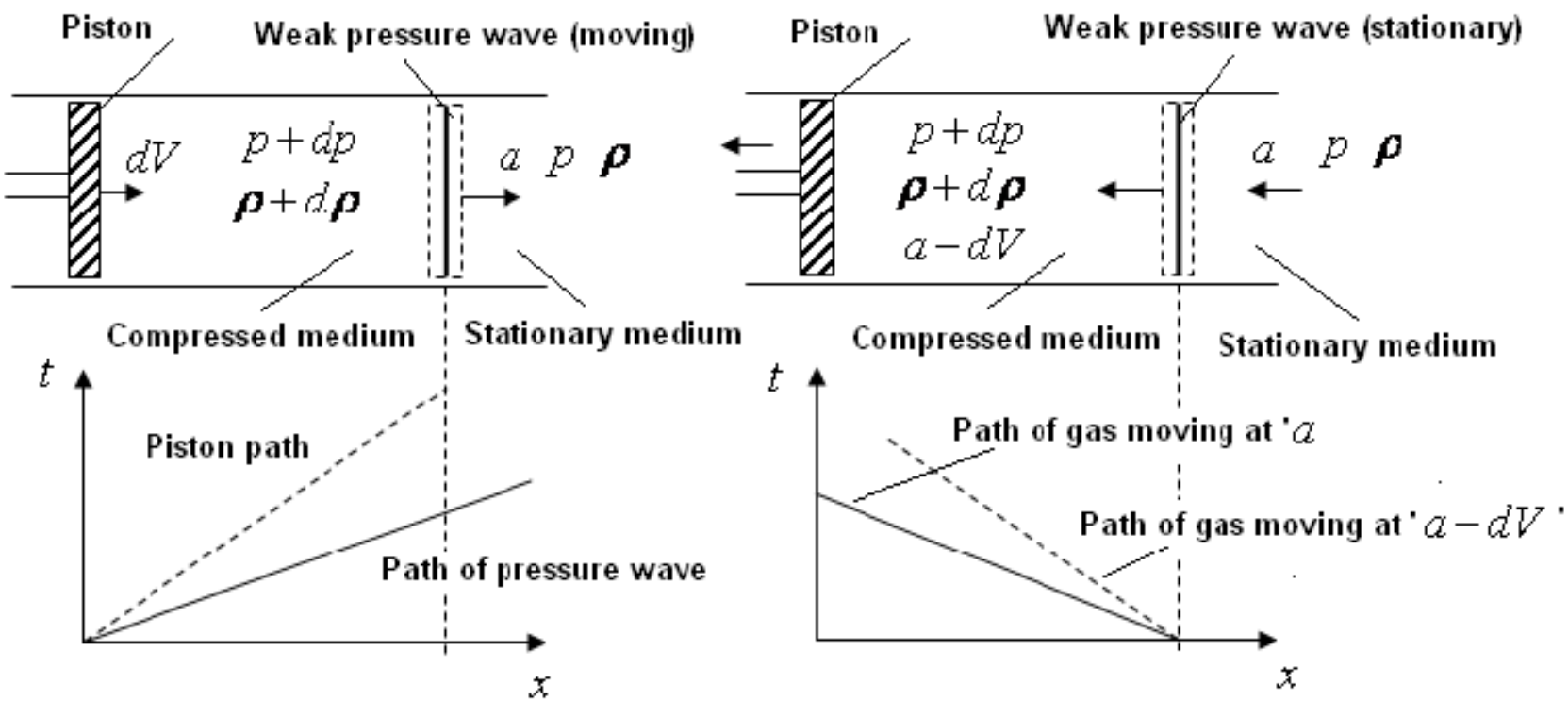
Mass flow rate $(\dot{m})$ is conserved across the stationary wave.

$$\dot{m} = \rho a A = (\rho + d\rho)(a - dV)A \;\Rightarrow dV = \left(\frac{a}{\rho}\right) d\rho \qquad (\ .2.1)$$

<u>Momentum equation</u>: As long as the compression wave is thin, the shear forces on the control volume are negligibly small compared to the pressure force. The momentum balance across the control volume leads to the following equation;

$$(p + dp)A - pA = \dot{m}a - \dot{m}(a - dV) \quad \Rightarrow dV = \left(\frac{1}{\rho a}\right) dp \qquad (\ .2.2)$$

# CONTINUITY EQUATION



# CONTINUITY EQUATION

Energy equation: Since the compression wave is thin, and the motion is very rapid, the heat transfer between the control volume and the surroundings may be neglected and the thermodynamic process can be treated as *adiabatic*. Steady flow energy equation can be used for energy balance across the wave.

$$h+\frac{a^2}{2}=(h+dh)+\frac{(a-dV)^2}{2} \quad \Rightarrow dV=\left(\frac{1}{a}\right)dh \qquad (\cdot.2.3)$$

Entropy equation: In order to decide the direction of thermodynamic process, one can apply $T-ds$ relation along with Eqs ( .2.2 & 2.3) across the compression wave.

$$T\,ds=dh-\frac{dp}{\rho}=0 \quad \Rightarrow ds=0 \qquad (\cdot.2.4)$$

Thus, the flow is isentropic across the compression wave and this compression wave can now be called as sound wave. The speed of the sound wave can be computed by equating Eqs.( .2.1 & .2.2).

$$\left(\frac{a}{\rho}\right)=\left(\frac{1}{\rho a}\right) \quad \Rightarrow a^2=\frac{d\,p}{d\,\rho}=\left(\frac{\partial p}{\partial \rho}\right)_s \qquad (\cdot.2.5)$$

# CONTINUITY EQUATION

Further simplification of Eq. ( .2.5) is possible by evaluating the differential with the use of isenropic equation.

$$\frac{p}{\rho^{\gamma}} = \text{constant} \Rightarrow \ln p - \gamma \ln \rho = \text{constant} \qquad (\ .2.6)$$

Differentiate Eq. ( .2.6) and apply perfect gas equation ( $p = \rho RT$ ) to obtain the expression for speed of sound. is obtained as below:

$$\left(\frac{\partial p}{\partial \rho}\right)_s = \frac{\gamma p}{\rho} \quad \Rightarrow a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT} \qquad (\ .2.7)$$

# MACH NUMBER

It may be seen that the speed of sound is the thermodynamic property that varies from point to point. When there is a large relative speed between a body and the compressible fluid surrounds it, then the compressibility of the fluid greatly influences the flow properties. Ratio of the local speed $(V)$ of the gas to the speed of sound $(a)$ is called as local Mach number $(M)$.

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma RT}} \qquad (\cdot\ .2.8)$$

There are few physical meanings for Mach number:

(a) It shows the compressibility effect for a fluid i.e. $M < 0.3$ implies that fluid is incompressible.

(b) It can be shown that Mach number is proportional to the ratio of kinetic to internal energy.

$$\frac{(V^2/2)}{e} = \frac{V^2/2}{c_v T} = \frac{V^2/2}{RT/(\gamma-1)} = \frac{(\gamma/2)V^2}{a^2/(\gamma-1)} = \frac{\gamma(\gamma-1)}{2}M^2 \qquad (\ \ 2.9)$$

# MACH NUMBER

(c) It is a measure of directed motion of a gas compared to the random thermal motion of the molecules.

$$M^2 = \frac{V^2}{a^2} = \frac{\text{directed kinetic energy}}{\text{random kinetic energy}} \qquad (\ .2.10)$$

# COMPRESSIBLE FLOW REGIMES

In order to illustrate the flow regimes in a compressible medium, let us consider the flow over an aerodynamic body.

The flow is uniform far away from the body with free stream velocity ($V_\infty$) while the speed of sound in the uniform stream is ($a_\infty$)

Then, the free stream Mach number becomes $M_\infty = (V_\infty)/(a_\infty)$ . The streamlines can be drawn as the flow passes over the body and the local Mach number can also vary along the streamlines. Let us consider the following distinct flow regimes commonly dealt with in compressible medium.

# SUBSONIC FLOW

It is a case in which an airfoil is placed in a free stream flow and the local Mach number is less than unity everywhere in the flow field

The flow is characterized by smooth streamlines with continuous varying properties. Initially, the streamlines are straight in the free stream, but begin to deflect as they approach the body. The flow expands as it passed over the airfoil and the local Mach number on the top surface of the body is more than the free stream value. Moreover, the local Mach number $M$ in the surface of the airfoil remains always less than 1.

when the free stream Mach number $M\infty$ is sufficiently less than 1. This regime is defined as subsonic flow which falls in the range of free stream Mach number less than 0.8 i.e. $M\infty \leq 0.8$

# TRANSONIC FLOW

If the free stream Mach number increases but remains in the subsonic range close to 1, then the flow expansion over the air foil leads to supersonic region locally on its surface. Thus, the entire regions on the surface are considered as mixed flow in which the local Mach number is either less or more than 1 and thus called as *sonic pockets*

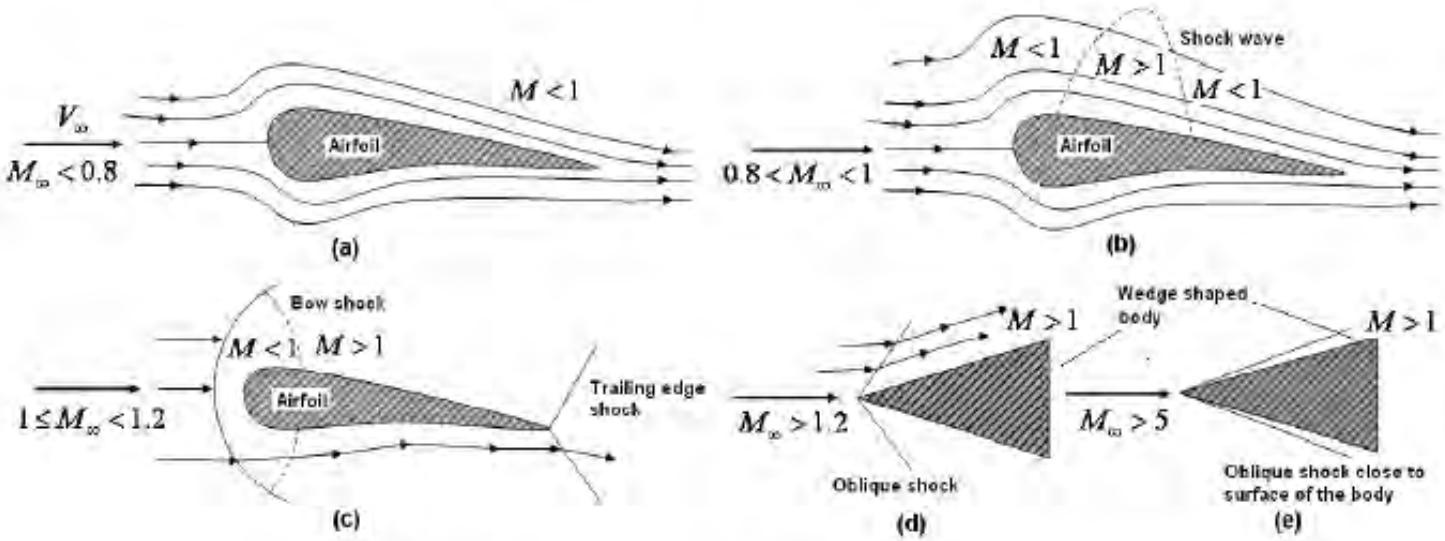
The phenomena of sonic pocket is initiated as soon as the local Mach number reaches 1 and subsequently terminates in the downstream with a shock wave across which there is discontinuous and sudden change in flow properties. If the free stream Mach number is slightly above unity

the shock pattern will move towards the trailing edge and a second shock wave appears in the leading edge which is called as *bow shock*. In front of this bow shock, the streamlines are straight and parallel with a uniform supersonic free stream Mach number. After passing through the bow shock, the flow becomes subsonic close to the free stream value. Eventually, it further expands over the airfoil

# TRANSONIC FLOW

surface to supersonic values and finally terminates with trailing edge shock in the downstream. The mixed flow patterns sketched in Figs.

(b & c), is defined as the *transonic regime*.




**compressible flow regime: (a) subsonic flow; (b & c) transonic flow; (d) supersonic flow; (d) hypersonic flow.**

# SUPERSONIC FLOW

In a flow field, if the Mach number is more than 1 everywhere in the domain, then it defined as supersonic flow. In order to minimize the drag, all aerodynamic bodies in a supersonic flow, are generally considered to be sharp edged tip. Here, the flow field is characterized by straight, oblique shock as shown in Fig. (d)

The stream lines ahead of the shock the streamlines are straight, parallel and horizontal. Behind the oblique shock, the streamlines remain straight and parallel but take the direction of wedge surface. The flow is supersonic both upstream and downstream of the oblique shock. However, in some exceptional strong oblique shocks, the flow in the downstream may be subsonic.

# HYPERSONIC FLOW

When the free stream Mach number is increased to higher supersonic speeds, the oblique shock moves closer to the body surface (Fig. (e) ). At the same time, the pressure, temperature and density across the shock increase explosively. So, the flow field between the shock and body becomes hot enough to ionize the gas. These effects of thin shock layer, hot and chemically reacting gases and many other complicated flow features are the characteristics of *hypersonic flow*. In reality, these special characteristics associated with hypersonic flows appear gradually as the free stream Mach numbers is increased beyond 5.

As a rule of thumb, the compressible flow regimes are classified as below;

$$M < 0.3 \text{ (incompressible flow)}$$

$$M < 1 \text{ (subsonic flow)}$$

$$0.8 < M < 1.2 \text{ (transonic flow)}$$

$$M > 1 \text{ (supersonic flow)}$$

$$M > 5 \text{ and above (hypersonic flow)}$$

# RAREFIED AND FREE MOLECULAR FLOW

In general, a gas is composed of large number of discrete atoms and molecules and all move in a random fashion with frequent collisions. However, all the fundamental equations are based on overall macroscopic behavior where the continuum assumption is valid. If the mean distance between atoms/molecules between the collisions is large enough to be comparable in same order of magnitude as that of characteristics dimension of the flow, then it is said to be low density/rarefied flow. Under extreme situations, the mean free path is much larger than the characteristic dimension of the flow. Such flows are defined as free molecular flows. These are the special cases occurring in flight at very high altitudes (beyond 100 km) and some laboratory devices such as electron beams.

# MACH WAVES

Consider an aerodynamic body moving with certain velocity (V) in a still air. When the pressure at the surface of the body is greater than that of the surrounding air, it results an infinitesimal compression wave that moves at speed of sound (a). These disturbances in the medium spread out from the body and become progressively weaker away from the body. If the air has to pass smoothly over the surface of the body, the disturbances must 'warn' the still air, about the approach of the body.

Now, let us analyze two situations: (a) the body is moving at subsonic speed($V<a$; $M<1$); (b) the body is moving at supersonic speed. ($V>a$; $M>1$);

# MACH WAVES

Case 1.

During the motion of the body, the sound waves are generated at different time intervals $(t)$ as shown in Fig. 4.4.1. The distance covered by the sound waves can be represented by the circle of radius $(at, 2at, 3at.....\text{so on})$. During same time intervals $(t)$, the body will cover distances represented by, $Vt$, $2Vt$, $3Vt.....\text{so on}$. At subsonic speeds $(V < a; M < 1)$, the body will always remains inside the family of circular sound waves. In other words, the information is propagated through the sound wave in all directions. Thus, the surrounding still air becomes aware of the presence of the body due to the disturbances induced in the medium. Hence, the flow adjusts itself very much before it approaches the body.

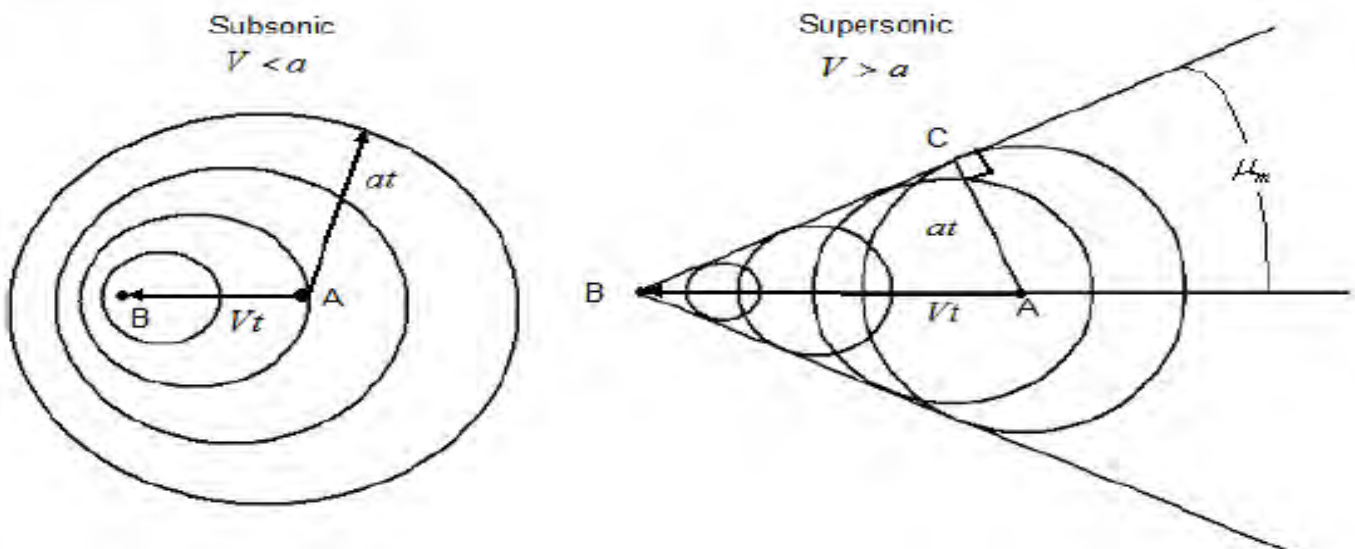
# MACH WAVES

Case 2.

Consider the case, when the body is moving at supersonic speed ($V$>a;M>1). With a similar manner, the sound waves are represented by circle of radius(at,2at,3at……so on) after different time(t) intervals. By this time, the body would have moved to a different location much faster from its initial position. At any point of time, the location of the body is always outside the family of circles of sound waves. The pressure disturbances created by the body always lags behind the body that created the disturbances. In other words, the information reaches the surrounding

# MACH WAVES

air much later because the disturbances cannot overtake the body. Hence, the flow cannot adjust itself when it approaches the body. The nature induces a wave across which the flow properties have to change and this line of disturbance is known as "Mach wave". These mach waves are initiated when the speed of the body approaches the speed of sound $(V = a; M = 1)$. They become progressively stronger with increase in the Mach number.



# MACH WAVES

Some silent features of a *Mach wave* are listed below;

- The series of wave fronts form a disturbance envelope given by a straight line which is tangent to the family of circles. It will be seen that all the disturbance waves lie within a cone having a *vertex/apex* at the body at time considered. The locus of all the leading surfaces of the waves of this cone is known as *Mach cone*.

- All disturbances confine inside the Mach cone extending downstream of the moving body is called as *zone of action*. The region outside the Mach cone and extending upstream is known as *zone of silence*. The pressure disturbances are largely concentrated in the neighborhood of the Mach cone that forms the outer limit of the zone of action

# MACH WAVES

The half angle of the Mach cone is called as the Mach angle $(\mu_m)$ that can be easily calculated from the geometry

$$\sin\mu_m = \frac{at}{Vt} = \frac{a(2t)}{V(2t)} = \frac{a(3t)}{V(3t)} \ldots\ldots = \frac{a}{V} = \frac{1}{M} \Rightarrow \mu_m = \sin^{-1}\left(\frac{1}{M}\right)$$

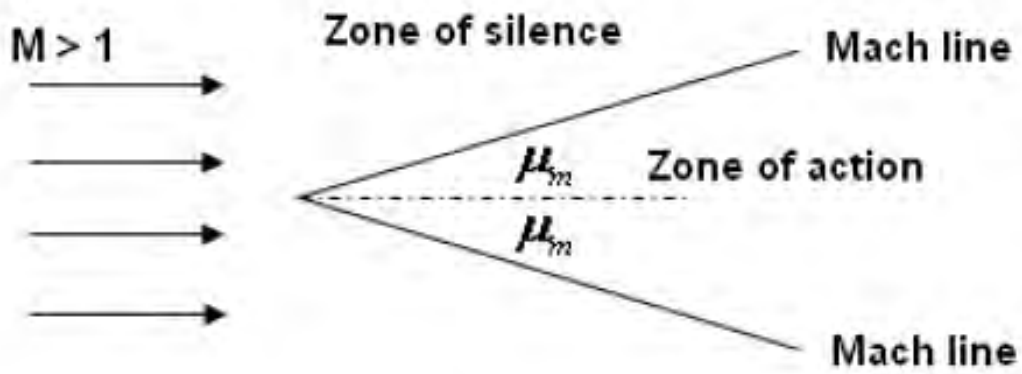



**Illustration of a Mach wave.**

# SHOCK WAVES

the flow medium gets compressed at a very short distance ahead of the body in a very thin region that may be comparable to the mean free path of the molecules in the medium. Since, these compression waves propagate upstream, so they tend to merge as *shock wave*. Ahead of the shock wave, the flow has no idea of presence of the body and immediately behind the shock; the flow is subsonic

The thermodynamic definition of a shock wave may be written as “the instantaneous compression of the gas”. The energy for compressing the medium, through a shock wave is obtained from the kinetic energy of the flow upstream the shock wave. The reduction in kinetic energy is accounted as heating of the gas to a static temperature above that corresponding to the isentropic compression value. Consequently, in flowing through the shock wave, the gas experiences a decrease in its available energy and accordingly, an increase in entropy. So, the compression through a shock wave is considered as an irreversible process.

# SHOCK WAVES

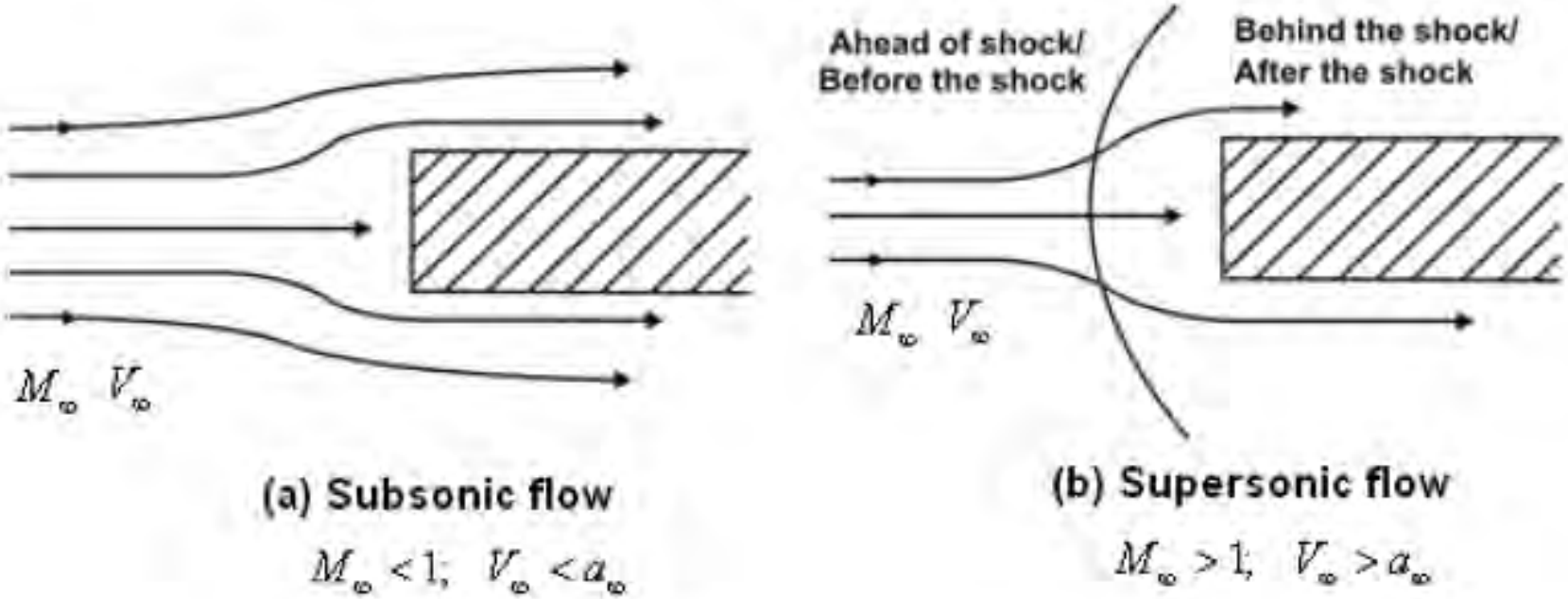
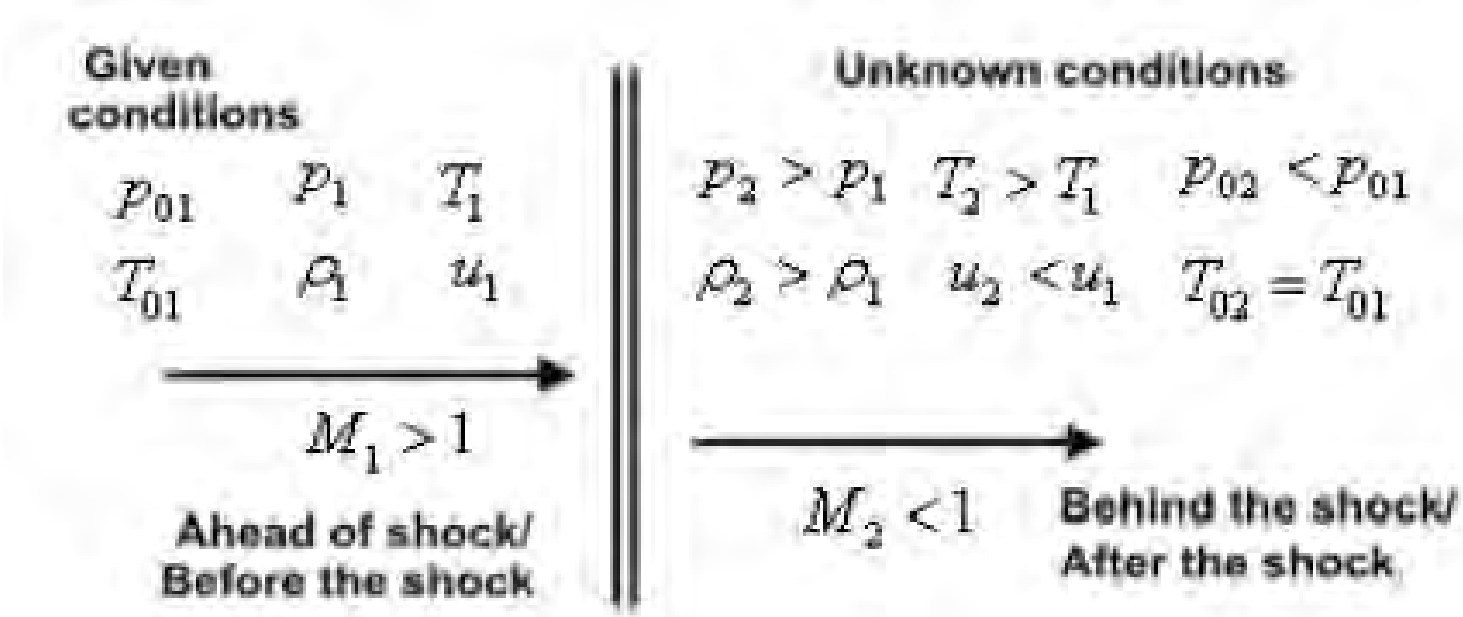



**Illustration of shock wave phenomena.**

# NORMAL SHOCK WAVES

A normal shock wave is one of the situations where the flow properties change drastically in one direction. The shock wave stands perpendicular to the flow as shown in Fig. The quantitative analysis of the changes across a normal shock wave involves the determination of flow properties. All conditions of are known ahead of the shock and the unknown flow properties are to be determined after the shock. There is no heat added or taken away as the flow traverses across the normal shock. Hence, the flow across the shock wave is adiabatic(q=0).

# NORMAL SHOCK WAVES




Schematic diagram of a standing normal shock wave.

# ENTROPY ACROSS A NORMAL SHOCK

The compression through a shock wave is considered as irreversible process leading to an increase in entropy. The change in entropy can be written as a function of static pressure and static temperature ratios across the normal shock.

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

Mathematically, it can be seen that the entropy change across a normal shock is also a function of the upstream Mach number. The *second law of thermodynamics* puts the limit that 'entropy' must increase $(s_2 - s_1 \geq 0)$ for a process to occur in a certain direction.

# ENTROPY ACROSS A NORMAL SHOCK

The entropy change across a normal shock can also be calculated from another simple way by expressing the thermodynamic relation in terms of total pressure. Referring to Fig it is seen that the discontinuity occurs only in the thin region across the normal shock. If the fluid elements is brought to rest isentropically from its real state (for both upstream and downstream conditions), then they will reach an imaginary state '1a and 2a'. The expression for entropy change between the imaginary states can be written as,

$$s_{2a} - s_{1a} = c_p \ln\left(\frac{T_{2a}}{T_{1a}}\right) - R\ln\left(\frac{p_{2a}}{p_{1a}}\right)$$

Since, $s_{2a} = s_2; s_{1a} = s_1; T_{2a} = T_{1a} = T_0;\ p_{2a} = p_{02}$ and $p_{1a} = p_{01}$, the Eq. reduces to,

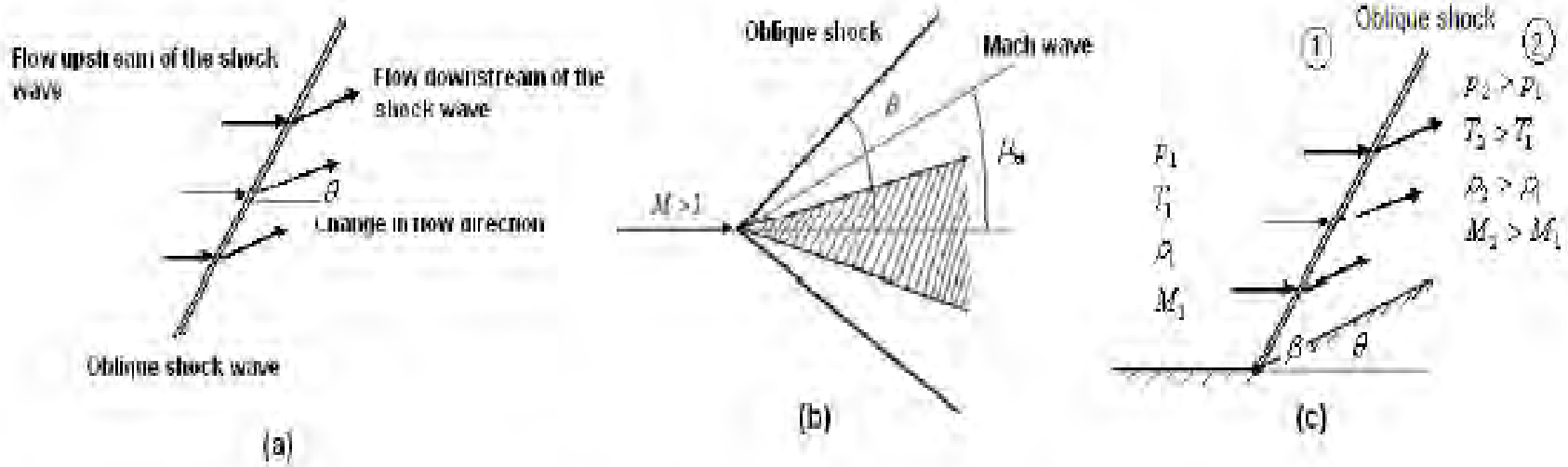
$$s_2 - s_1 = -R\ln\left(\frac{p_{02}}{p_{01}}\right) \quad \Rightarrow \frac{p_{02}}{p_{01}} = e^{-(s_2 - s_1)/R}$$

Because of the fact $s_2 > s_1$, Eq. implies that $p_{02} < p_{01}$. Hence, the stagnation pressure always decreases across a normal shock.

# OBLIQUE SHOCK WAVE

The normal shock waves are straight in which the flow before and after the wave is normal to the shock. It is considered as a special case in the general family of oblique shock waves that occur in supersonic flow. In general, oblique shock waves are straight but inclined at an angle to the upstream flow and produce a change in flow direction an oblique shock generally occurs, when a supersonic flow is ‘turned into itself” Here, a supersonic flow is allowed to pass over a surface, which is inclined at an angle(θ) to the horizontal. The flow streamlines are deflected upwards and aligned along the surface. Since, the upstream flow is supersonic; the streamlines are adjusted in the downstream an oblique shock wave angle (β) with the horizontal such that they are parallel to the surface in the downstream. All the streamlines experience same deflection angle across the oblique shock.

# OBLIQUE SHOCK WAVE




Schematic representation of an oblique shock.

# OBLIQUE EXPANSION WAVES

Another class of two dimensional waves occurring in supersonic flow shows the opposite effects of oblique shock. Such types of waves are known as *expansion waves*. When the supersonic flow is “turned away from itself”, Here, the flow is allowed to pass over a surface which is inclined at an angle (θ) to the horizontal and all the flow streamlines are deflected downwards. The change in flow direction takes place across an expansion fan centered at point ‘A’. The flow streamlines are smoothly curved till the downstream flow becomes parallel to the wall surface behind the point ‘A’. Here, the flow properties change smoothly through the expansion fan except at point ‘A’. An infinitely strong oblique expansion wave may be called as a *Mach wave*. An expansion wave emanating from a sharp convex corner is known as a *centered expansion* which is commonly known as *Prandtl-Meyer expansion wave*. Few features of PM expansion waves are as follows;

# OBLIQUE EXPANSION WAVES

- Streamlines through the expansion wave are smooth curved lines.

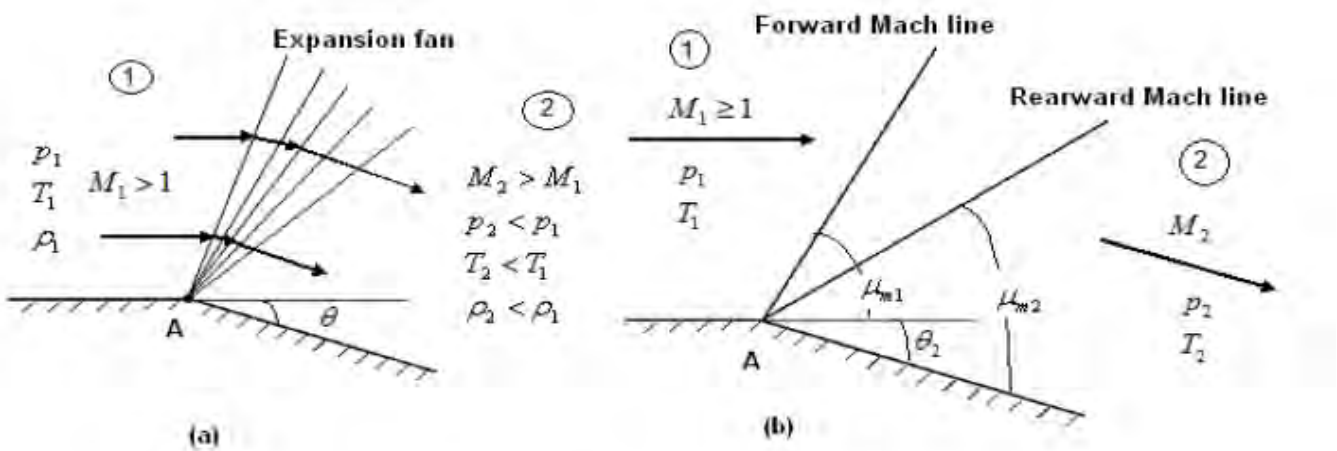
- The expansion of the flow takes place though an infinite number of Mach waves emitting from the center 'A'. It is bounded by forward and rearward Mach lines. These Mach lines are defined by Mach angles i.e.

Forward Mach angle: $\mu_{m1} = \sin^{-1}(1/M_1)$

Rearward Mach angle: $\mu_{m2} = \sin^{-1}(1/M_2)$

# OBLIQUE EXPANSION WAVES

- The expansion takes place through a continuous succession of Mach waves such that there is no change in entropy for each Mach wave. Thus, the expansion process is treated as isentropic.
- The Mach number increases while the static properties such as pressure, temperature and density decrease during the expansion process.




Schematic representation of an expansion fan.

# INTRODUCTION TO HYPERSONIC FLOW

The hypersonic flows are different from the conventional regimes of supersonic flows. As a rule of thumb, when the Mach number is greater than 5, the flow is classified as hypersonic. However, the flow does not change its feature all of a sudden during this transition process. So, the more appropriate definition of hypersonic flow would be regime of the flow where certain physical flow phenomena become more important with increase in the Mach number. One of the physical meanings may be given to the Mach number as the measure of the ordered motion of the gas to the random thermal motion of the molecules. In other words, it is the ratio of ordered energy to the random energy as given in Eq.

$$M^2 = \frac{(1/2)V^2}{(1/2)a^2} = \frac{\text{Ordered kinetic energy}}{\text{Random kinetic energy}}$$

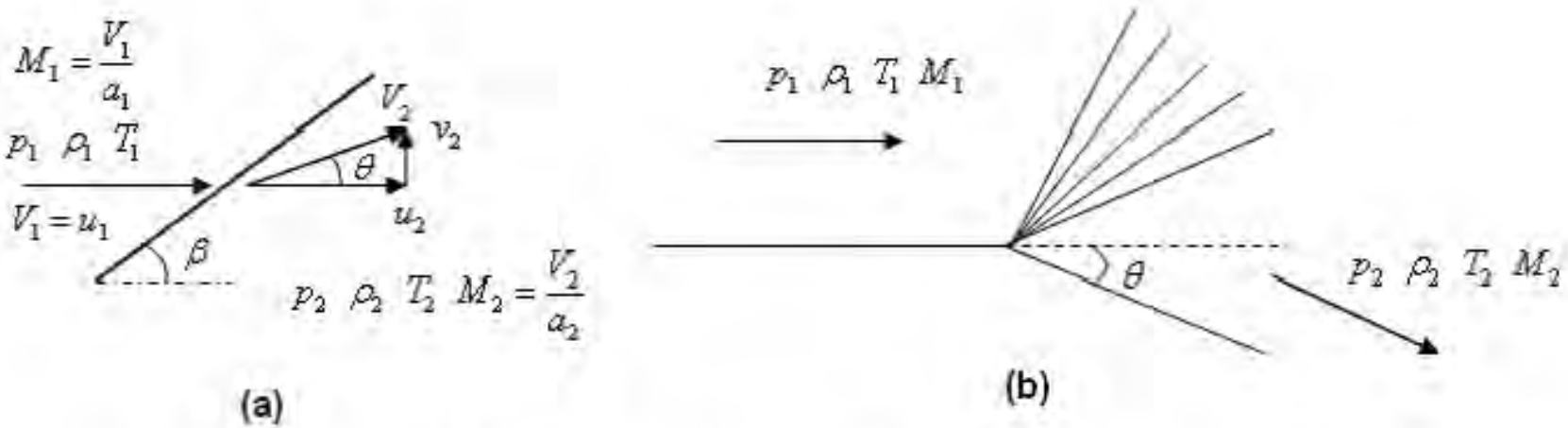
# INTRODUCTION TO HYPERSONIC FLOW

The study/research on hypersonic flows revels many exciting and unknown flow features of aerospace vehicles in the twenty-first century. The presence of special features in a hypersonic flow is highly dependent on type of trajectory, configuration of the vehicle design, mission requirement that are decided by the nature of hypersonic atmosphere encountered by the flight vehicle. Therefore, the hypersonic flight vehicles are classified in four different types, based on the design constraints imposed from mission specifications.

- Reentry vehicles (uses the rocket propulsion system)
- Cruise and acceleration vehicle (air-breathing propulsion such as ramjet/scramjet)
- Reentry vehicles (uses both air-breathing and rocket propulsion)
- Aero-assisted orbit transfer vehicle (presence of ions and plasma in the vicinity of spacecraft)

# HYPERSONIC SHOCK RELATIONS

Consider the flow through a straight oblique shock as shown in Fig. The notations have their usual meaning and upstream and downstream conditions are denoted by subscripts ‘1’ and ‘2’, respectively. Let us revisit the exact oblique shock relations and simplify them in the limit of high Mach numbers.




**Geometry of shock and expansion wave: (a) oblique shock; (b) centered expansion wave.**

# HYPERSONIC SHOCK RELATIONS

The exact oblique shock relations for pressure, temperature and density ratio across the wave are given by,

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 \sin^2\beta - 1\right); \quad \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2 \sin^2\beta}{2+(\gamma-1)M_1^2 \sin^2\beta}; \quad \frac{T_2}{T_1} = \frac{(p_2/p_1)}{(\rho_2/\rho_1)}$$

As, $M_1 \to \infty \quad \Rightarrow M_1^2 \sin^2\beta \square\ 1$, so that Eq. becomes,

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 \sin^2\beta; \quad \frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1}; \quad \frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_1^2 \sin^2\beta$$

It may be noted that for air (γ=1.4) flow in the hypersonic speed limit, the density ratio approaches to a fixed value of 6. The velocity components behind the shock wave, parallel and perpendicular to the upstream flow, may be computed from the following relations;

# HYPERSONIC SHOCK RELATIONS

$$\frac{u_2}{V_1} = 1 - \frac{2\left(M_1^2 \sin^2\beta - 1\right)}{(\gamma+1)M_1^2}; \quad \frac{v_2}{V_1} = \frac{2\left(M_1^2 \sin^2\beta - 1\right)\cot\beta}{(\gamma+1)M_1^2}$$

For large values of $M_1$, the Eq. can be approximated by the following relations;

$$\frac{u_2}{V_1} = 1 - \frac{2\sin^2\beta}{\gamma+1}; \quad \frac{v_2}{V_1} = \frac{2\sin\beta\cos\beta}{(\gamma+1)} = \frac{\sin 2\beta}{\gamma+1}$$

The non-dimensional parameter $c_p$ is defined as the pressure coefficient which is the ratio of static pressure difference across the shock to the dynamic pressure $(q_1)$.

$$c_p = \frac{p_2 - p_1}{q_1}$$

# HYPERSONIC SHOCK RELATIONS

The dynamic pressure can also be expressed in the form of Mach number as given below;

$$q_1 = \frac{1}{2}\rho_1 V_1^2 = \frac{1}{2}V_1^2 \frac{\gamma p_1}{(\gamma p_1/\rho_1)} = \frac{\gamma p_1}{2}\left(\frac{V_1}{a_1}\right)^2 = \frac{\gamma}{2}p_1 M_1^2$$

Now, Eq. can be simplified as,

$$c_p = \frac{2}{\gamma M_1^2}\left(\frac{p_2}{p_1} - 1\right) = \frac{4}{\gamma + 1}\left(\sin^2\beta - \frac{1}{M_1^2}\right)$$

# HYPERSONIC SHOCK RELATIONS

In the hypersonic limit of $M_1 \to \infty$,, Eq. is approximated as below;

$$c_p = \left(\frac{4}{\gamma+1}\right)\sin^2\beta$$

The relationship between Mach number $(M)$, shock angle $(\beta)$ and deflection angle $(\theta)$ is expressed by $\theta-\beta-M$ equation.

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}\right]$$

In the hypersonic limit, when, $\theta$ is small, $\beta$ is also small. Thus, the small angle approximation can be used

$$\sin\beta \approx \beta; \quad \cos 2\beta \approx 1; \quad \tan\theta \approx \sin\theta \approx \theta$$

It leads

# HYPERSONIC SHOCK RELATIONS

It leads to simplification of Eq. as below;

$$\theta = \frac{2}{\beta}\left[\frac{M_1^2\beta^2 - 1}{M_1^2(\gamma+1)+2}\right]$$

In the high Mach number limit, Eq may be approximated for $\gamma = 1.4$.

$$\theta = \frac{2}{\beta}\left[\frac{M_1^2\beta^2}{M_1^2(\gamma+1)}\right] = \frac{2\beta}{\gamma+1}; \quad \frac{\beta}{\theta} = \frac{\gamma+1}{2} \text{ and } \beta = 1.2\theta$$

It is interesting to observe that in the hypersonic limit of a slender wedge, the shock wave angle is only 20% larger than the wedge angle which is the typical physical features of thin shock layer in the hypersonic flow.

# HYPERSONIC EXPANSION WAVE RELATIONS

Consider the flow through an expansion corner . The expansion fan consists of infinite number of Mach waves originating at the corner and spreading downstream. The notations have their usual meaning and upstream and downstream conditions are denoted by subscripts '1' and '2', respectively. Let us revisit the exact relations for a *Prandtl-Meyer* expansion. The relation for deflection angleθ. $M_1$ and $M_2$ is expressed through *Prandtl-Meyer* function. [v(M)]

For large Mach numbers, $\sqrt{M_1^2 - 1} \approx M$ and series expansion can be approximated for the trigonometric functions.

$$\nu(M) = \left(\sqrt{\frac{\gamma+1}{\gamma-1}}\right)\left(\frac{\pi}{2}\right) - \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{1}{M}\right) - \frac{\pi}{2} + \frac{1}{M}$$

$$\text{and} \quad \theta = \frac{1}{M_2} - \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{1}{M_2}\right) - \frac{1}{M_1} + \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{1}{M_1}\right)$$

Further, simplification of Eq can be done and the final expression for $\theta$ may be written as below;

$$\theta = \frac{2}{\gamma-1}\left(\frac{1}{M_1} - \frac{1}{M_2}\right)$$

# HYPERSONIC SIMILARITY PARAMETER

In the study of hypersonic flow over slender bodies, the product of $M_1\,\theta$ is a controlling parameter which is known as the similarity parameter denoted by $K$. All the hypersonic shock and expansion relations can be expressed in terms of this parameter. Introducing this parameter, Eq. is rewritten in the limit of high values of Mach number;

$$M_1^2\beta^2 - 1 = \left[\frac{M_1^2(\gamma+1)}{2} + 1\right]\beta\theta \Rightarrow M_1^2\beta^2 - 1 = \left(\frac{\gamma+1}{2}\right)M_1^2\beta\theta$$

Rearranging Eq., one may obtain a quadratic equation in terms of (β/θ) , which may be easily solved.

$$\left(\frac{\beta}{\theta}\right)^2 - \frac{\gamma+1}{2}\left(\frac{\beta}{\theta}\right) - \frac{1}{M_1^2\theta^2} = 0 \Rightarrow \frac{\beta}{\theta} = \frac{\gamma+1}{4} + \sqrt{\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{M_1^2\theta^2}}$$

# HYPERSONIC SIMILARITY PARAMETER

Within the framework of hypersonic assumption, the hypersonic shock relation for pressure ratio , may be reduced in terms of $K$

$$\frac{p_2}{p_1} = 1 + \frac{\gamma(\gamma+1)}{4}K^2 + \gamma K^2\sqrt{\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{K^2}}$$

Similarly, the pressure coefficient may also be expressed as a function of similarity parameter.

$$c_p = 2\theta^2\left[\frac{\gamma+1}{4} + \sqrt{\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{K^2}}\right] \Rightarrow \frac{c_p}{\theta^2} = f(K,\gamma)$$

# HYPERSONIC SIMILARITY PARAMETER

The similarity relations for *Prandtl-Meyer expansion wave* may also be written in terms of the similarity parameter. The flow through an expansion fan is isentropic. Hence, the isentropic relations for pressure can be used for the conditions on both sides of expansion fan. When approximated to hypersonic flows, the static pressure relation across the expansion fan can be written as below;

$$\frac{p_2}{p_1} = \left[ \frac{1 + \left( \frac{\gamma - 1}{2} \right) M_1^2}{1 + \left( \frac{\gamma - 1}{2} \right) M_2^2} \right]^{\frac{\gamma}{\gamma - 1}} \quad \Rightarrow \frac{p_2}{p_1} = \left( \frac{M_1}{M_2} \right)^{\frac{2\gamma}{\gamma - 1}}$$

Rearranging , the ratio of Mach numbers across the expansion wave can be obtained.

$$\frac{M_1}{M_2} = 1 - \left( \frac{\gamma - 1}{2} \right) M_1 \theta$$

Combine Eqs. to obtain pressure ratio across the expansion fan in terms of similarity parameter.

$$\frac{p_2}{p_1} = \left( 1 - \frac{\gamma - 1}{2} K \right)^{\frac{2\gamma}{\gamma - 1}}$$

# HYPERSONIC SIMILARITY PARAMETER

Further, the pressure coefficient across the expansion fan, may be expressed as a function of similarity parameter.

$$c_p = \frac{2}{\gamma M_1^2}\left(\frac{p_2}{p_1} - 1\right) = \frac{2}{\gamma M_1^2}\left[\left(1 - \frac{\gamma - 1}{2}K\right)^{\frac{2\gamma}{\gamma-1}} - 1\right]$$

Multiply and divide the right-hand side by $\theta^2$ and simplify to obtain the following relation.

$$c_p = \frac{2\theta^2}{\gamma K^2}\left[\left(1 - \frac{\gamma - 1}{2}K\right)^{\frac{2\gamma}{\gamma-1}} - 1\right] \Rightarrow \frac{c_p}{\theta^2} = g(K, \gamma)$$

It may be seen that pressure coefficient for hypersonic shock and expansion wave, are related through the similarity parameter in the limit of hypersonic Mach numbers.

# NEWTONIAN THEORY FOR HYPERSONIC FLOWS

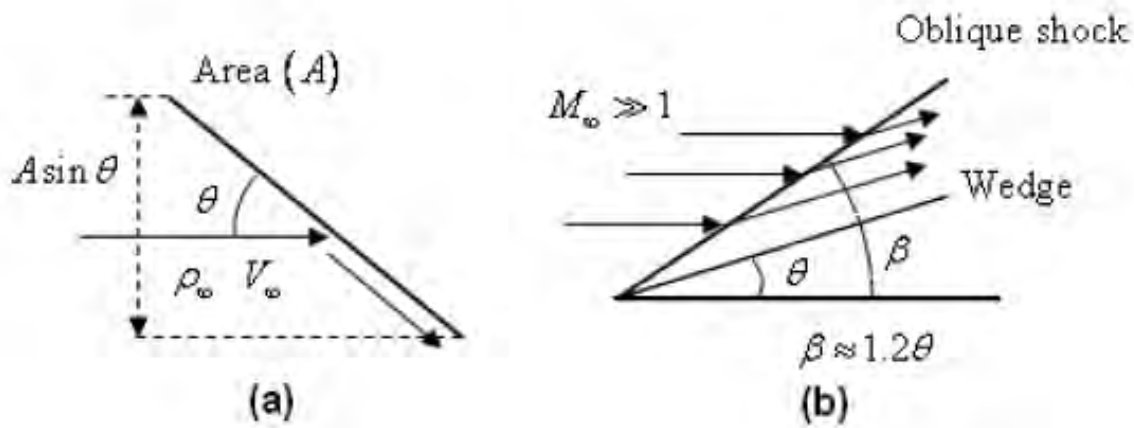
The hypersonic flows are highly nonlinear due to many physical phenomena leading to complexity in the mathematical formulation and its solution. One can get rid of the complex nature of aerodynamic theories with the simple approximation of inviscid flow to obtain the linear relationship. It is interesting to note that the invicid compressible flow theory for high Mach number flows, resemble the fundamental Newtonian law of classical mechanics.

When a fluid as a stream of particles in rectilinear motion, strikes a plate, it loses all its momentum normal to the surface and moves tangentially to the surface without the loss of tangential momentum.

# NEWTONIAN THEORY FOR HYPERSONIC FLOWS

This is known as the Newtonian impact theory as shown in Fig Let a fluid stream of density $\rho_\infty$ strikes a surface of area A , with a velocity $V_\infty$ . This surface is inclined at an angle θ with the free stream. By Newton's law, the time rate of change of momentum of this mass flux is equal to the force (F) exerted on the surface.

$$F = (\rho_\infty A)(V_\infty \sin\theta)(V_\infty \sin\theta) = \rho_\infty V_\infty^2 A \sin^2\theta \quad \Rightarrow \frac{F}{A} = \rho_\infty V_\infty^2 \sin^2\theta$$




**Newtonian impact theory and hypersonic flow over a wedge: (a) schematic representation of a jet striking a plate; (b) streamlines in a thin shock layer.**

# NEWTONIAN THEORY FOR HYPERSONIC FLOWS

Since the motion is rectilinear and the individual particles do not interact with each other, the force per unit area, associated with the random motion may be interpreted as the difference in surface pressure( P) and the free stream pressure ($P_\infty$ ). So, the Eq. may be simplified in terms of pressure coefficient($C_P$ ) .

$$c_p = \frac{p - p_\infty}{(1/2)\rho_\infty V_\infty^2} = 2\sin^2\theta$$

Both the upstream and downstream side of the shock wave, the streamlines are straight and parallel. But, the stream lines are deflected by an angle θ in the downstream. Since, the difference in the shock wave angle (β) and the flow deflection is very small at hypersonic speeds, it may be visualized as the upstream incoming flow impinging on the wedge surface and then running parallel to the wedge surface in the downstream. This phenomenon is analogous to Newtonian theory and Eq. may be used for hypersonic flow as well to predict the surface pressures. It is known as the *Newtonian Sine-Squared Law* for hypersonic flow.

# INVISCID HYPERSONIC FLOW OVER A FLAT PLATE

Consider a two-dimensional flat plate of certain length(l) , inclined at angle(θ) with respect to free stream hypersonic flow in Fig. Now, the Newtonian theory can be applied at the lower and upper surface of the plate to obtain the pressure coefficient ( $C_p$ ) .

$$c_{pl} = 2\sin^2\theta;\ c_{pu} = 0$$

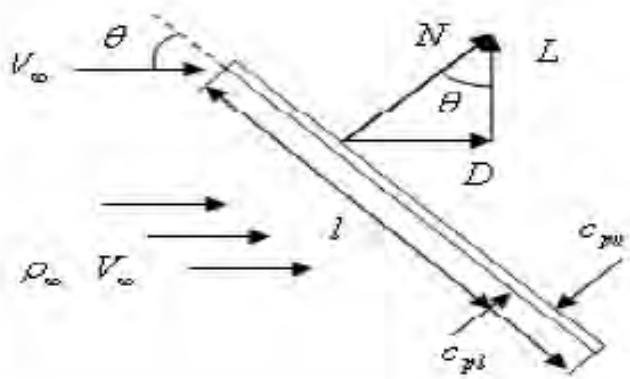



**Illustration of aerodynamic forces for a flat plate in hypersonic flow.**

# INVISCID HYPERSONIC FLOW OVER A FLAT PLATE

The difference in pressures in the upper and lower surface of the plate, gives rise to a normal force (N) . The normal force coefficient ($C_n$ ) can also be readily defined through the following formula.

$$c_n = \frac{1}{l}\int_0^l \left(c_{pl} - c_{pu}\right) dx = \frac{N}{q_\infty S}$$

Here, $q_\infty \left(= \frac{1}{2}\rho_\infty V_\infty^2\right)$ is the free stream dynamic pressure S (=l), is the frontal area per unit width and x is the distance along the length of the plate from the leading edge. the simplified relations is;

$$c_n = \frac{1}{l}\left(2\sin^2\theta\right)l = 2\sin^2\theta$$

# INVISCID HYPERSONIC FLOW OVER A FLAT PLATE

If L and D are defined as the lift and drag as shown in Fig. then the other aerodynamic parameters such as lift coefficient ($C_l$ ) and drag coefficient ($C_d$ ) can be expressed in the following fashion.

$$c_l = \frac{L}{q_\infty S} = c_n \cos\theta = 2\sin^2\theta\cos\theta; \; c_d = \frac{D}{q_\infty S} = c_d \cos\theta = 2\sin^3\theta$$

Referring to geometry of Fig., the other important parameter *lift-to-drag* is obtained through the following relation;

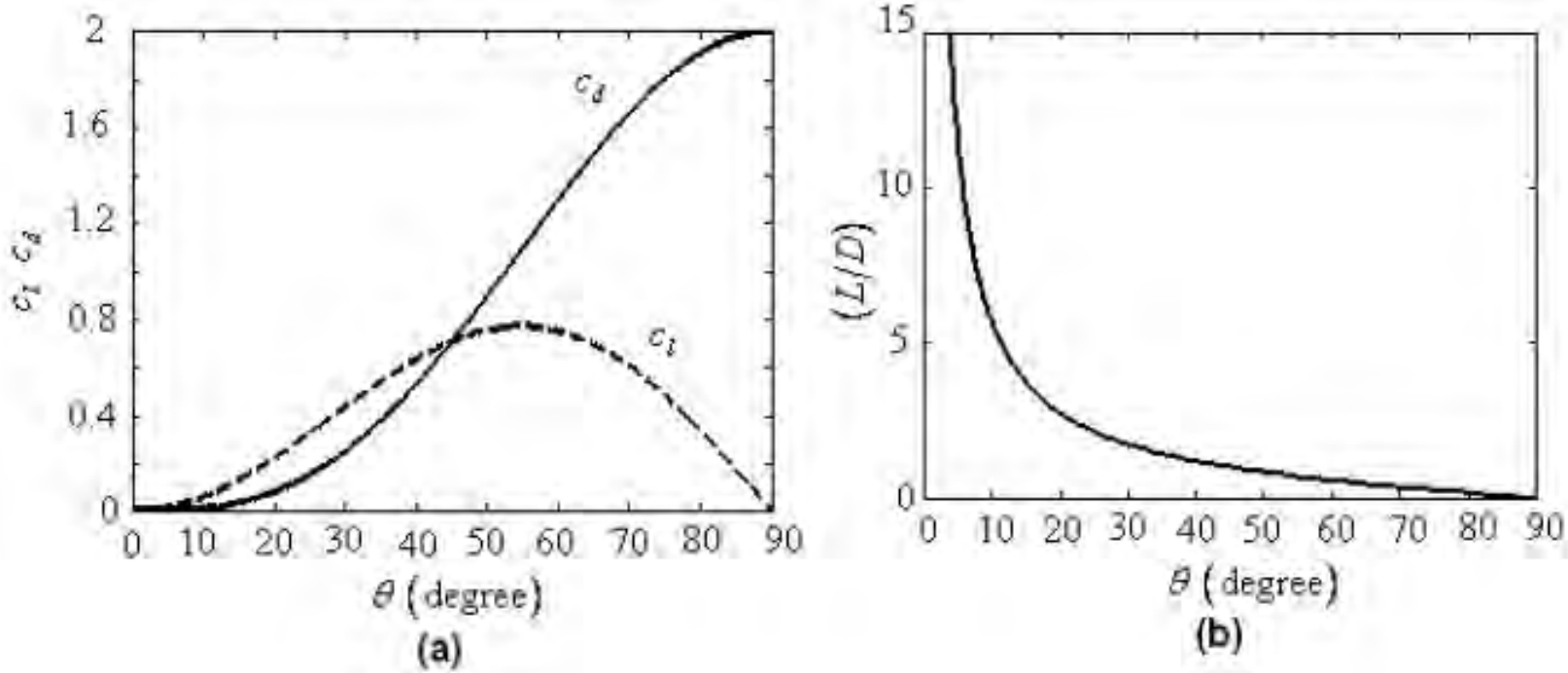
$$\frac{L}{D} = \cot\theta$$

# RESULTS OF NEWTONIAN THEORY

The results of Newtonian theory for the inviscid flow over a flat plate are plotted in Fig.  and the following important observations can be made;

- The value of *lift-to-drag* ratio increases monotonically when the inclination angle decreases. It is mainly due to the fact that the Newtonian theory does not account for skin friction drag in the calculation. When skin friction is added, the drag becomes a finite value at $0^o$ inclination angle and the ratio approaches zero.
- The lift curve reaches its peak value approximately at an angle of $55^0$. It is quite realistic, because most of the practical hypersonic vehicles get their maximum lift in this vicinity of angle of attack.
- The lift curve at lower angle ($0\text{-}15^0$) shows the non-linear behavior. It is clearly the important characteristics feature of the hypersonic flows.

# RESULTS OF NEWTONIAN THEORY



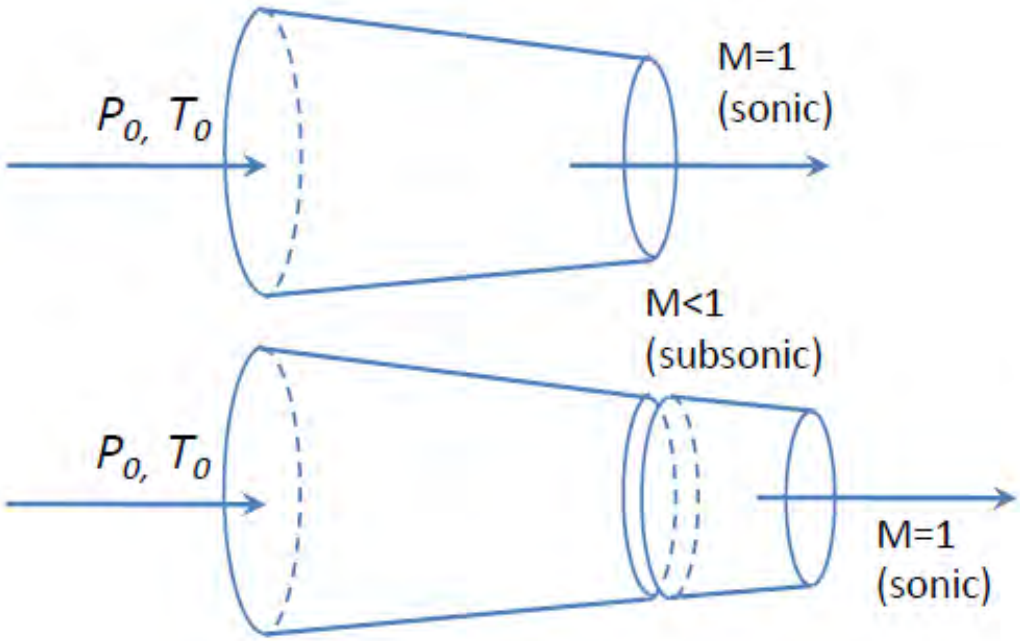

**Aerodynamic parameters for a flat plate inclined at an angle.**

# IN THIS LECTURE...

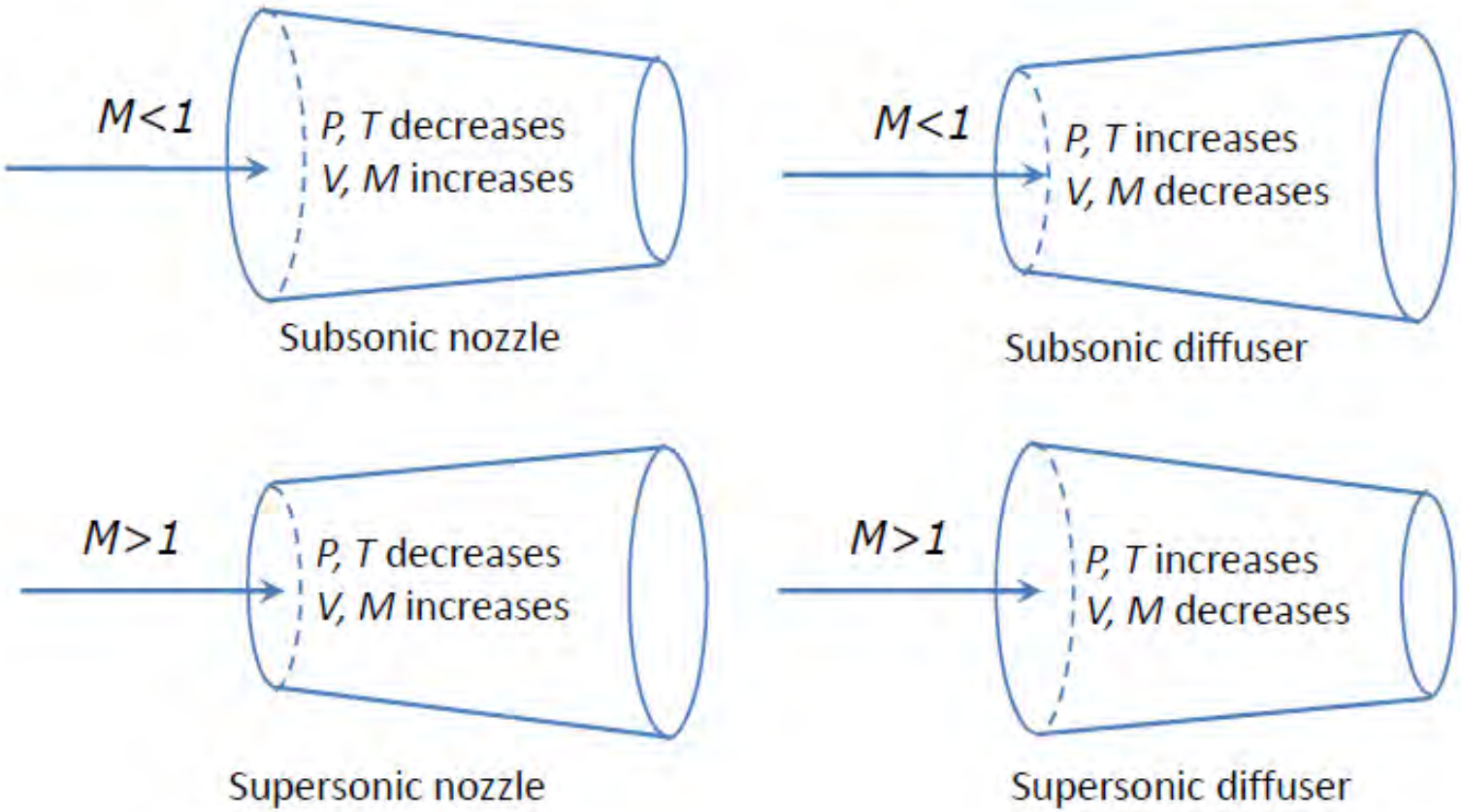
- Subsonic and supersonic nozzles
- Working of these nozzles
- Performance parameters for nozzles

# VARIATION OF FLUID VELOCITY WITH FLOW AREA

Sonic velocity will occur at the exit of the converging extension, instead of the exit of the original nozzle, and the mass flow rate through the nozzle will decrease because of the reduced exit area.



# VARIATION OF FLUID VELOCITY WITH FLOW AREA



# GOVERNING EQUATIONS

Let us consider a calorically perfect gas flow through a nozzle.

The mass flow through the nozzle is

$$\dot{m} = \rho u A = \left(\frac{P}{RT}\right)\left(M\sqrt{\gamma RT}\right)A = (MA)\left(\frac{P}{P_0}\right)P_0\frac{\sqrt{\gamma}}{\sqrt{RT}}\sqrt{\frac{T}{T_0}}$$

$$= \frac{\sqrt{\gamma}P_0}{\sqrt{T_0 R}}MA\frac{\left\{1+((\gamma-1)/2)M^2\right\}^{1/2}}{\left\{1+((\gamma-1)/2)M^2\right\}^{\gamma/(\gamma-1)}}$$

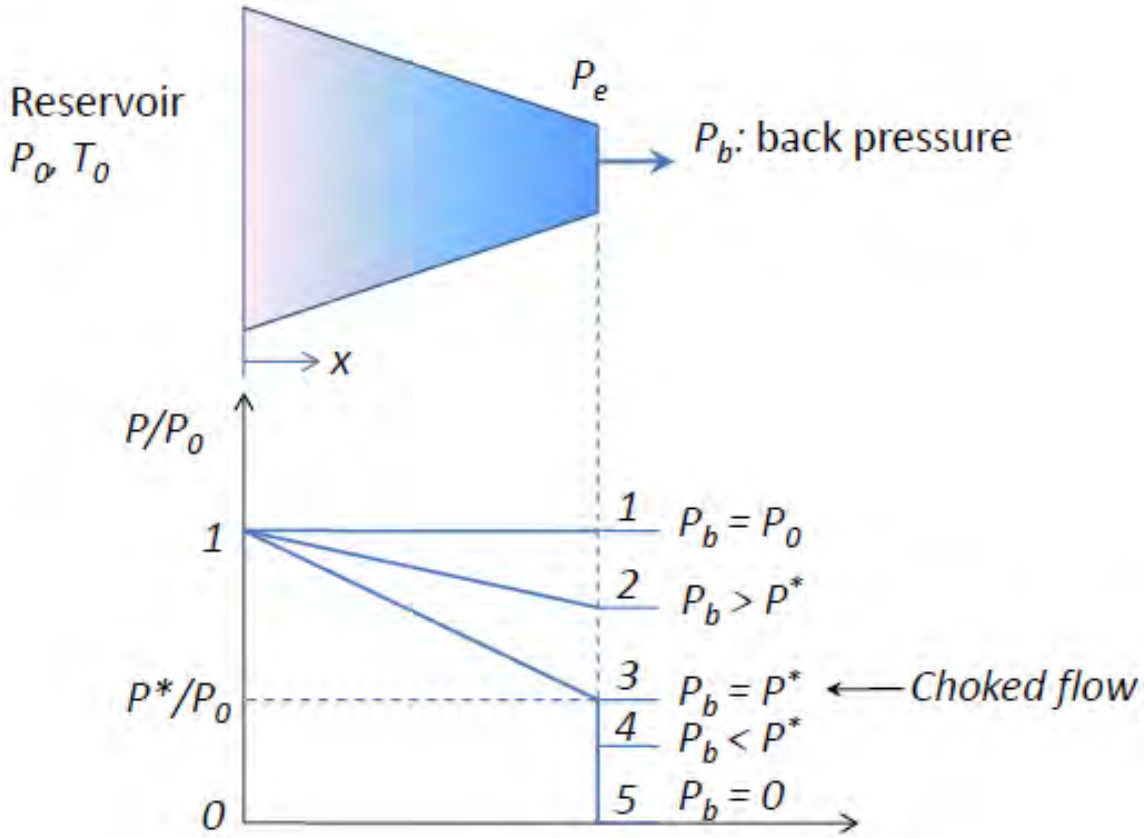
This on simplication reduces to

$$\dot{m} = \frac{AP_0}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}}\frac{M}{\left\{1+((\gamma-1)/2)M^2\right\}^{(\gamma+1)/2(\gamma-1)}}$$

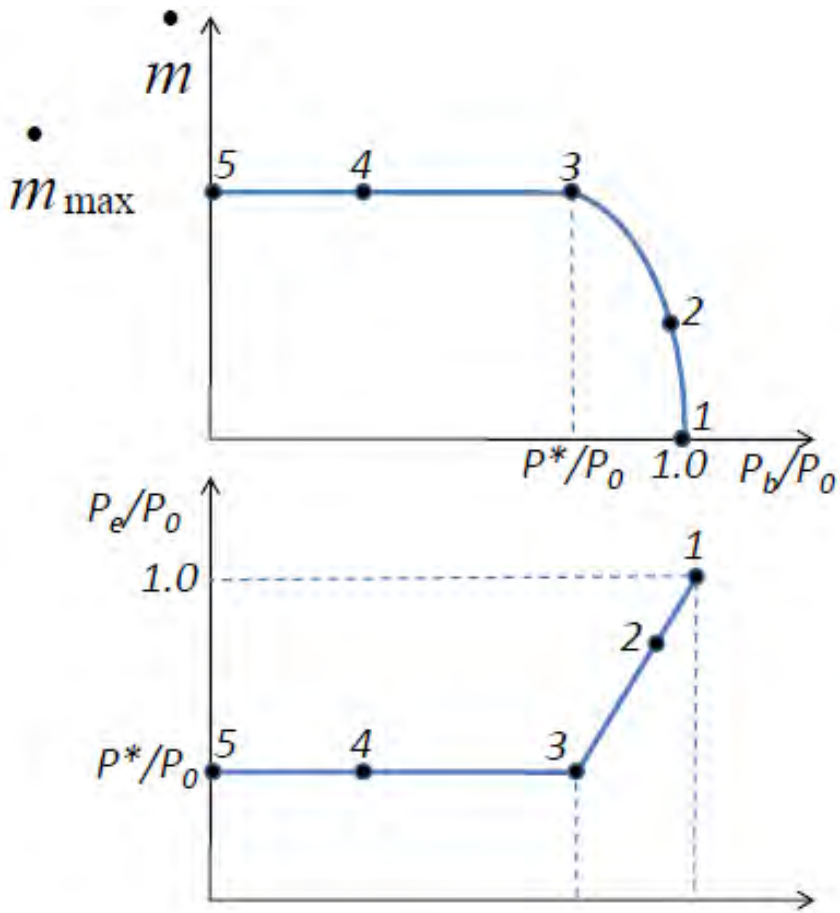
# ISENTROPIC FLOW THROUGH CONVERGING NOZZLES

- Converging nozzle in a subsonic flow will have decreasing area along the flow direction.
- We shall consider the effect of back pressure on the exit velocity, mass flow rate and pressure distribution along the nozzle.
- We assume flow enters the nozzle from a reservoir so that inlet velocity is zero.
- Stagnation temperature and pressure remains unchanged in the nozzle.

# ISENTROPIC FLOW THROUGH CONVERGING NOZZLES



# ISENTROPIC FLOW THROUGH CONVERGING NOZZLES




The effect of back pressure $P_b$ *on the* mass flow rate and the exit pressure $P_e$.

# ISENTROPIC FLOW THROUGH CONVERGING NOZZLES

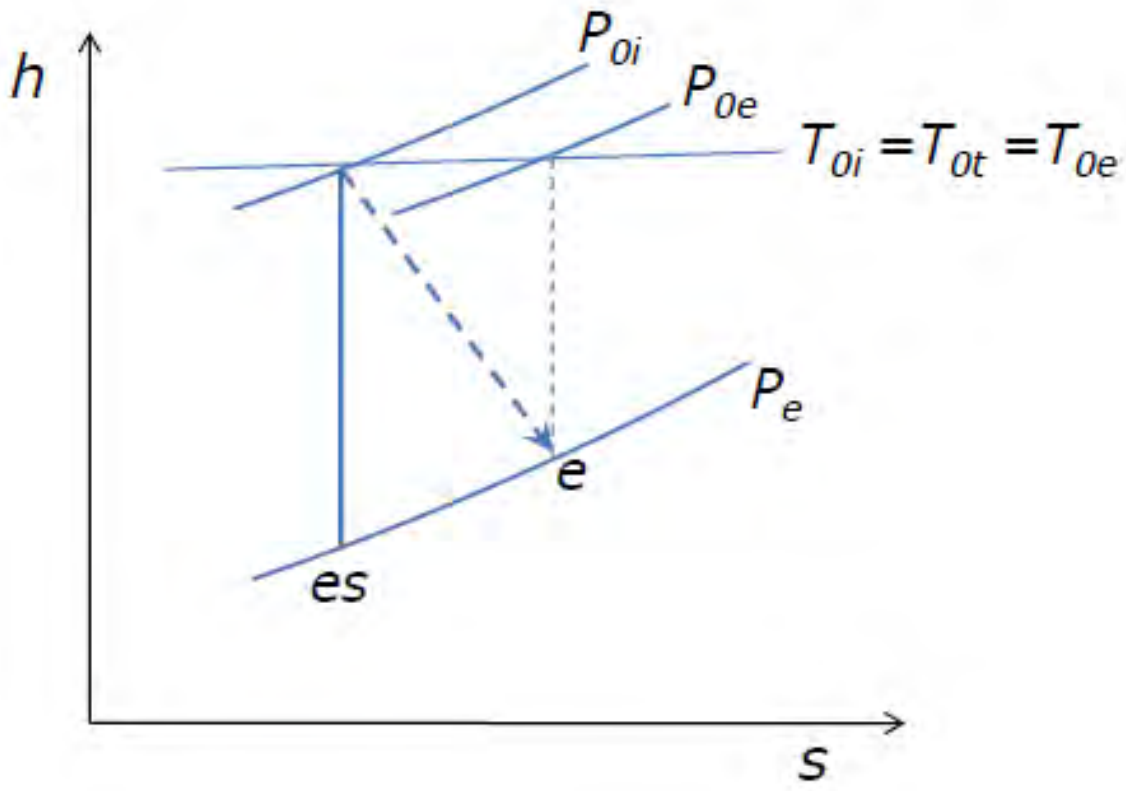
- From the above figure,

$$P_e = \begin{cases} P_b & \text{for } P_b \geq P^* \\ P^* & \text{for } P_b < P^* \end{cases}$$

- For all back pressures lower that the critical pressure, exit pressure = critical pressure, Mach number is unity and the mass flow rate is maximum (choked flow).

- A back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and does not affect the flow rate.

# NOZZLE EFFICIENCY



# CONVERGING NOZZLES

The efficiency of a nozzle is defined as

$\eta_n = \dfrac{h_{0i} - h_e}{h_{0i} - h_{es}}$, where $h_{0i}$ is the stagnation enthalpy at the nozzle inlet, $h_e$ is the actual static enthalpy at the nozzle exit, $h_{es}$ is the isentropic static enthalpy at the nozzle exit.

In terms of the corresponding temperatures,

$$\eta_n = \frac{T_{0i} - T_e}{T_{0i} - T_{es}} = \frac{1 - T_e / T_{0e}}{1 - T_{es} / T_{0i}}$$

# CONVERGING NOZZLES

For choked flow, $M = 1$,

$$\eta_n = \frac{1 - (2/(\gamma + 1))}{1 - (P_C / P_{0i})^{(\gamma - 1)/\gamma}}$$

The pressure ratio is therefore,

$$\frac{P_{0i}}{P_C} = \frac{1}{(1 - (1/\eta_n)((\gamma - 1)/(\gamma + 1)))^{\gamma/(\gamma - 1)}}$$

If $\frac{P_{0i}}{P_C} < \frac{P_{0i}}{P_a}$,

the nozzle is operating under choked condition.

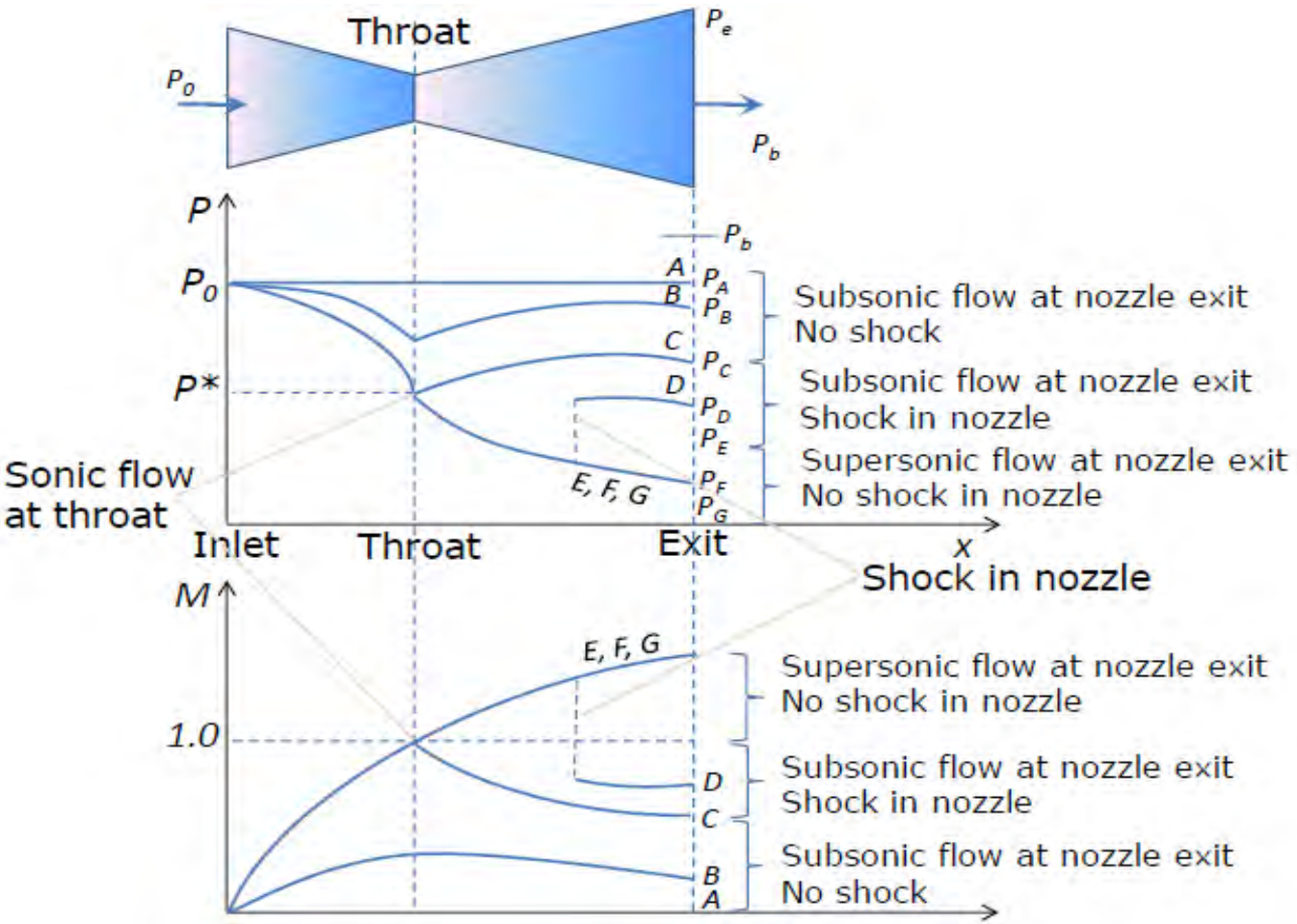
# CONVERGING NOZZLES

- If a convergent nozzle is operating under choked condition, the exit Mach number is unity.
- The exit flow parameters are then defined by the critical parameters.
- To determine whether a nozzle is choked or not, we calculate the actual pressure ratio and then compare this with the critical pressure ratio.
- If the actual pressure ratio > critical pressure ratio, the nozzle is said to be choked.

# ISENTROPIC FLOW THROUGH CONVERGING-DIVERGING NOZZLES

- Maximum Mach number achievable in a converging nozzle is unity.
- For supersonic Mach numbers, a diverging section after the throat is required.
- However, a diverging section alone would not guarantee a supersonic flow.
- The Mach number at the exit of the converging-diverging nozzle depends upon the back pressure.

# ISENTROPIC FLOW THROUGH CONVERGING-DIVERGING NOZZLES



# CONVERGING-DIVERGING NOZZLES

- The flow through nozzles is normally assumed to be adiabatic as the heat transfer per unit mass is much smaller than the difference in enthalpy between the inlet and outlet.

- The flow from the inlet to the throat can be assumed to be isentropic, but the flow from the throat to exit may not be due the possible presence of shocks.

# CONVERGING-DIVERGING NOZZLES

The efficiency of a nozzle is defined as

$$\eta_n = \frac{h_{0i} - h_e}{h_{0i} - h_{es}} = \frac{T_{0i} - T_e}{T_{0i} - T_{es}} = \frac{1 - T_e / T_{0e}}{1 - T_{es} / T_{0i}}$$

$$= \frac{1 - (P_e / P_{0e})^{(\gamma-1)/\gamma}}{1 - (P_e / P_{0i})^{(\gamma-1)/\gamma}}$$

Therefore, $$\left(\frac{P_e}{P_{0e}}\right) = \left[1 - \eta_n \left\{1 - \left(\frac{P_e}{P_{0i}}\right)^{(\gamma-1)/\gamma}\right\}\right]^{\gamma/(\gamma-1)}$$

Since, $$\frac{P_{0i}}{P_{0e}} = \frac{P_e}{P_{0e}} \frac{P_{0i}}{P_e} \Rightarrow \frac{P_{0i}}{P_{0e}} = \frac{P_{0i}}{P_e}\left[1 - \eta_n \left\{1 - \left(\frac{P_e}{P_{0i}}\right)^{(\gamma-1)/\gamma}\right\}\right]^{\gamma/(\gamma-1)}$$

# CONVERGING-DIVERGING NOZZLES

The exit velocity can be calculated from

$$u_e = \sqrt{2(h_{0i} - h_e)} = \sqrt{2\eta_n(h_{0i} - h_{es})}$$

$$= \sqrt{2c_p\eta_n(T_{0i} - T_{es})} = \sqrt{2c_p\eta_n T_{0i}\left\{1 - \left(\frac{P_e}{P_{0i}}\right)^{(\gamma-1)/\gamma}\right\}}$$

$$= \sqrt{\frac{2\gamma R}{(\gamma - 1)}\eta_n T_{0i}\left\{1 - \left(\frac{P_e}{P_{0i}}\right)^{(\gamma-1)/\gamma}\right\}}$$

# CONVERGING-DIVERGING NOZZLES

The exit Mach number is $M_e^2 = \frac{u_e^2}{a_e^2} = \frac{u_e^2}{\gamma R T_e}$

Since, $\frac{T_{0e}}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 = \frac{T_{0i}}{T_e}$

$$M_e^2 = \frac{2\eta_n}{\gamma - 1}\left\{1 + \frac{\gamma - 1}{2} M_i^2\right\}\left\{1 - \left(\frac{P_i}{P_{0i}}\right)^{(\gamma-1)/\gamma}\right\}$$

$$= \frac{2}{\gamma - 1}\left[\frac{\eta_n\left\{1 - (P_e / P_{0i})^{(\gamma-1)/\gamma}\right\}}{1 - \eta_n\left\{1 - (P_e / P_{0i})^{(\gamma-1)/\gamma}\right\}}\right]$$

# CONVERGING-DIVERGING NOZZLES

From the governing equation discussed earlier, and also assuming isentropic flow upto throat, the ratio between the throat area and the exit area is,

$$\frac{A_t}{A_e} = \frac{P_{0e}}{P_{ot}} \frac{M_e}{M_t} \left[ \frac{1 + ((\gamma - 1)/2)M_t^2}{1 + ((\gamma - 1)/2)M_e^2} \right]^{(\gamma+1/2(\gamma-1))}$$

$$= \frac{P_{0e}}{P_{0i}} \frac{M_e}{M_t} \left[ \frac{1 + ((\gamma - 1)/2)M_t^2}{1 + ((\gamma - 1)/2)M_e^2} \right]^{(\gamma+1/2(\gamma-1))}$$

If the throat is choked, $M_t = 1$,

$$\frac{A^*}{A_e} = \frac{P_{0e}M_e}{P_{0i}^*} \left[ \frac{(\gamma + 1)/2)}{1 + ((\gamma - 1)/2)M_e^2} \right]^{(\gamma+1/2(\gamma-1))}$$

# CONVERGING-DIVERGING NOZZLES

The mass flow rate will therefore be,

$$\dot{m} = \frac{A^* P_{0i}^*}{\sqrt{T_{0i}^*}} \sqrt{\frac{\gamma}{R}} \frac{1}{((\gamma+1)/2)^{(\gamma+1)/2(\gamma-1)}}$$

The mass flow rate is a function of the inlet stagnation pressure, temperature and throat area.

By design one would like to keep the area ratio $A_i/A_e$ as close as possible to unity. This is to keep the external drag under control. However this may result in the nozzle exit pressure to be different from the ambient pressure : incomplete expansion.

# CONVERGING-DIVERGING NOZZLES

- Underexpanded nozzle:
  - $P_e > P_a$
  - The flow is capable of additional expansion.
  - Expansion waves originating from the lip of the nozzle.
- Overexpanded nozzle:
  - $P_e < P_a$
  - Shock waves originate from the nozzle lip.

# CONVERGING-DIVERGING NOZZLES

- Fully expanded nozzle:
  - $P_e = P_a$
  - No shock waves/expansion waves.
- If $P_e << P_a$
  - Shock waves will occur within the divergent section of the nozzle.

# COMPRESSORS

A compressor is the most important and often the costliest component (typically 30 to 40 percent of total cost) of any vapour compression refrigeration system (VCRS). The function of a compressor in a VCRS is to continuously draw the refrigerant vapour from the evaporator, so that a low pressure and low temperature can be maintained in the evaporator at which the refrigerant can boil extracting heat from the refrigerated space. The compressor then has to raise the pressure of the refrigerant to a level at which it can condense by rejecting heat to the cooling medium in the condenser.

# CLASSIFICATION OF COMPRESSORS

Compressors used in refrigeration systems can be classified in several ways:

a) Based on the working principle:

i. Positive displacement type

ii. Roto-dynamic type

In positive displacement type compressors, compression is achieved by trapping a refrigerant vapour into an enclosed space and then reducing its volume. Since a fixed amount of refrigerant is trapped each time, its pressure rises as its volume is reduced. When the pressure rises to a level that is slightly higher than the condensing pressure, then it is expelled from the enclosed space and a fresh charge of low-pressure refrigerant is drawn in and the cycle continues. Since the flow of refrigerant to the compressor is not steady, the positive displacement type compressor is a *pulsating flow device*. However, since the operating speeds are normally very high the flow appears to be almost steady on macroscopic time scale. Since the flow is pulsating on a microscopic time scale, positive displacement type compressors are prone to high wear, vibration and noise level. Depending upon the construction, positive displacement type compressors used in refrigeration and air conditioning can be classified into:

# CLASSIFICATION OF COMPRESSORS

I. Reciprocating type

II. Rotary type with sliding vanes (rolling piston type or multiple vane type)

III. Rotary screw type (single screw or twin-screw type)

IV. Orbital compressors, and

V. Acoustic compressors

In roto-dynamic compressors, the pressure rise of refrigerant is achieved by imparting kinetic energy to a steadily flowing stream of refrigerant by a rotating mechanical element and then converting into pressure as the refrigerant flows through a diverging passage. Unlike positive displacement type, the roto-dynamic type compressors are steady flow devices, hence are subjected to less wear and vibration.

# CLASSIFICATION OF COMPRESSORS

Depending upon the construction, roto-dynamic type compressors can be classified into:

I. Radial flow type, or

II. Axial flow type

Centrifugal compressors (also known as turbo-compressors) are radial flow type, roto-dynamic compressors. These compressors are widely used in large capacity refrigeration and air conditioning systems. Axial flow compressors are normally used in gas liquefaction applications.

# CLASSIFICATION OF COMPRESSORS

b) Based on arrangement of compressor motor or external drive:

I. Open type

II. Hermetic (or sealed) type

iii. Semi-hermetic (or semi-sealed) type

In open type compressors the rotating shaft of the compressor extends through a seal in the crankcase for an external drive. The external drive may be an electrical motor or an engine (e.g. diesel engine). The compressor may be belt driven or gear driven. Open type compressors are normally used in medium to large capacity refrigeration system for all refrigerants and for ammonia (due to its incompatibility with hermetic motor materials). Open type compressors are characterized by high efficiency, flexibility, better compressor cooling and serviceability. However, since the shaft has to extend through the seal, refrigerant leakage from the system cannot be eliminated completely. Hence refrigeration systems using open type compressors require a refrigerant reservoir to take care of the refrigerant leakage for some time, and then regular maintenance for charging the system with refrigerant, changing of seals, gaskets etc.

# CLASSIFICATION OF COMPRESSORS

In hermetic compressors, the motor and the compressor are enclosed in the same housing to prevent refrigerant leakage. The housing has welded connections for refrigerant inlet and outlet and for power input socket. As a result of this, there is virtually no possibility of refrigerant leakage from the compressor. All motors reject a part of the power supplied to it due to eddy currents and friction, that is, inefficiencies. Similarly the compressor also gets heated-up due to friction and also due to temperature rise of the vapor during compression. In Open type, both the compressor and the motor normally reject heat to the surrounding air for efficient operation. In hermetic compressors heat cannot be rejected to the surrounding air since both are enclosed in a shell. Hence, the cold suction gas is made to flow over the motor and the compressor before entering the compressor. This keeps the motor cool. The motor winding is in direct contact with the refrigerant hence only those refrigerants, which have high dielectric strength, can be used in hermetic compressors. The cooling rate depends upon the flow rate of the refrigerant, its temperature and the thermal properties of the refrigerant. If flow rate is not sufficient and/or if the temperature is not low enough the insulation on the winding of the motor can burn out and short-circuiting may occur. Hence, hermetically sealed compressors give satisfactory and safe performance over a very narrow range of design temperature and should not be used for off-design conditions.

# CLASSIFICATION OF COMPRESSORS

The COP of the hermetic compressor based systems is lower than that of the open compressor based systems since a part of the refrigeration effect is lost in cooling the motor and the compressor. However, hermetic compressors are almost universally used in small systems such as domestic refrigerators, water coolers, air conditioners etc, where efficiency is not as important as customer convenience (due to absence of continuous maintenance). In addition to this, the use of hermetic compressors is ideal in systems, which use capillary tubes as expansion devices and are critically charged systems. Hermetic compressors are normally not serviceable. They are not very flexible as it is difficult to vary their speed to control the cooling capacity.

In some (usually larger) hermetic units, the cylinder head is usually removable so that the valves and the piston can be serviced. This type of unit is called a semi-hermetic (or semi-sealed) compressor.

# RECIPROCATING COMPRESSORS

Reciprocating compressor is the workhorse of the refrigeration and air conditioning industry. It is the most widely used compressor with cooling capacities ranging from a few Watts to hundreds of kilowatts. Modern day reciprocating compressors are high speed (≈ 3000 to 3600 rpm), single acting, single or multi-cylinder (upto 16 cylinders) type.

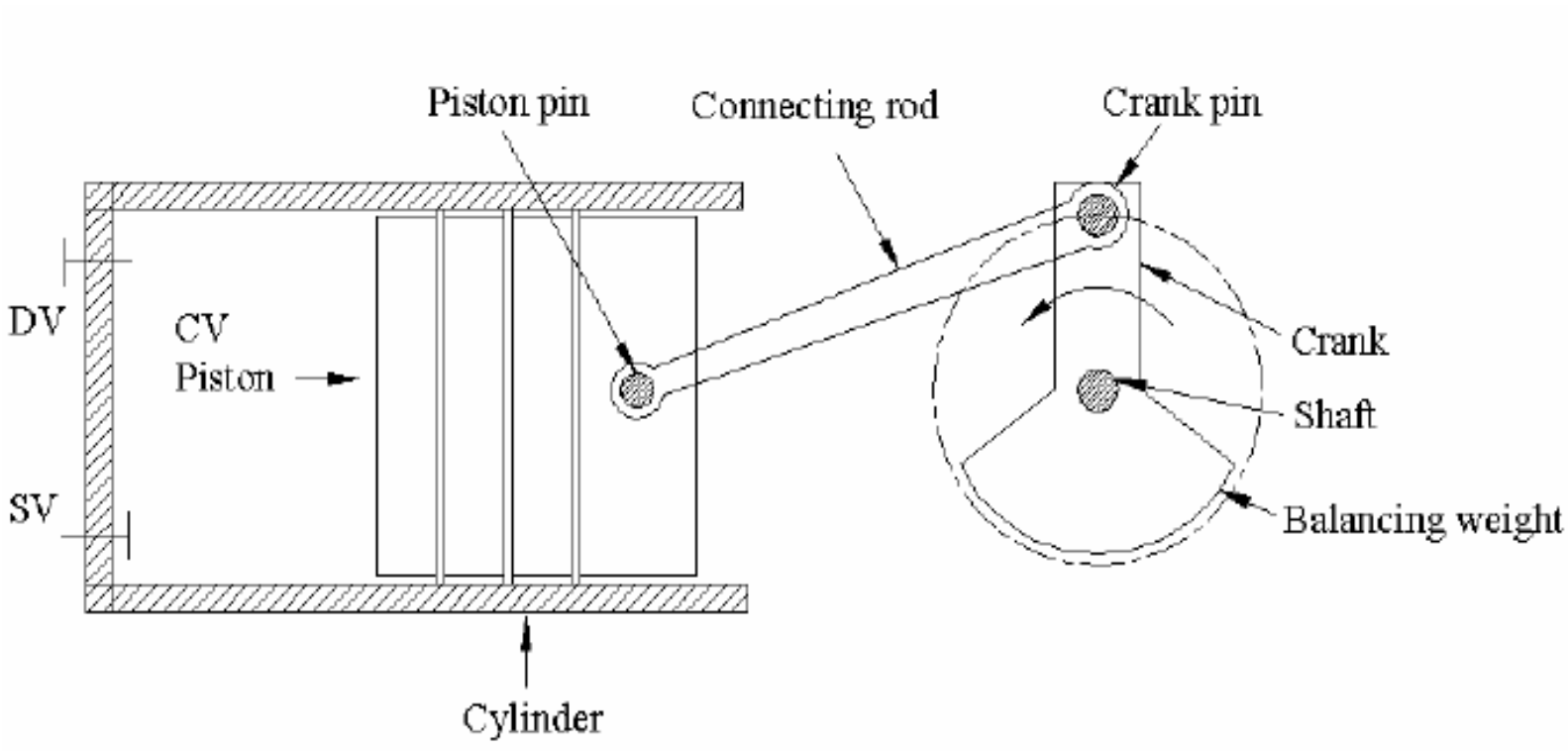
Figure shows the schematic of a reciprocating compressor. Reciprocating compressors consist of a piston moving back and forth in a cylinder, with suction and discharge valves to achieve suction and compression of the refrigerant vapor. Its construction and working are somewhat similar to a two-stroke engine, as suction and compression of the refrigerant vapor are completed in one revolution of the crank. The suction side of the compressor is connected to the exit of the evaporator, while the discharge side of the compressor is connected to

# RECIPROCATING COMPRESSORS

the condenser inlet. The suction (inlet) and the discharge (outlet) valves open and close due to pressure differences between the cylinder and inlet or outlet manifolds respectively. The pressure in the inlet manifold is equal to or slightly less than the evaporator pressure. Similarly the pressure in the outlet manifold is equal to or slightly greater than the condenser pressure. The purpose of the manifolds is to provide stable inlet and outlet pressures for the smooth operation of the valves and also provide a space for mounting the valves.

The valves used are of reed or plate type, which are either floating or clamped. Usually, backstops are provided to limit the valve displacement and springs may be provided for smooth return after opening or closing. The piston speed is decided by valve type. Too high a speed will give excessive vapor velocities that will decrease the volumetric efficiency and the throttling loss will decrease the compression efficiency.

# RECIPROCATING COMPRESSORS




*Schematic of a reciprocating compressor*

# PERFORMANCE OF RECIPROCATING COMPRESSORS

For a given evaporator and condenser pressures, the important performance parameters of a refrigerant compressor are:

a. The mass flow rate (m) of the compressor for a given displacement rate

b. Power consumption of the compressor ($W_c$)

c. Temperature of the refrigerant at compressor exit, $T_d$, and

d. Performance under part load conditions

# PERFORMANCE OF RECIPROCATING COMPRESSORS

The mass flow rate decides the refrigeration capacity of the system and for a given compressor inlet condition, it depends on the volumetric efficiency of the compressor. The volumetric efficiency, $\eta_V$ is defined as the ratio of volumetric flow rate of refrigerant to the maximum possible volumetric flow rate, which is equal to the compressor displacement rate, i.e.,

$$\eta_V = \frac{\text{Volumetric flow rate}}{\text{Compressor Displacement rate}} = \frac{\dot{m}.v_e}{\dot{V}_{SW}}$$

Where m and V SW are the mass flow rate of refrigerant (kg/s) and compressor displacement rate (.$m^3$/s) respectively, and $v_l$ is the specific volume ($m^3$/kg) of the refrigerant at compressor inlet.

# PERFORMANCE OF RECIPROCATING COMPRESSORS

- For a given evaporator and condenser temperatures, one can also use the volumetric refrigeration capacity (kW/m$^3$) to indicate the volumetric efficiency of the compressor. The actual volumetric efficiency (or volumetric capacity) of the compressor depends on the operating conditions and the design of the compressor.

- The power consumption (kW) or alternately the power input per unit refrigeration capacity (kW/kW) depends on the compressor efficiency ($\eta_C$), efficiency of the mechanical drive ($\eta_{mech}$) and the motor efficiency ($\eta_{motor}$). For a refrigerant compressor, the power input ($W_c$) is given by:

$$W_C = \frac{W_{ideal}}{\eta_C \eta_{mech} \eta_{motor}}$$

# PERFORMANCE OF RECIPROCATING COMPRESSORS

The performance of the compressor under part load conditions depends on the type and design of the compressor.

An ideal reciprocating compressor is one in which:

- The clearance volume is zero, i.e., at the end of discharge process, the volume of refrigerant inside the cylinder is zero.
- No pressure drops during suction and compression
- Suction, compression and discharge are reversible and adiabatic

# PERFORMANCE OF RECIPROCATING COMPRESSORS

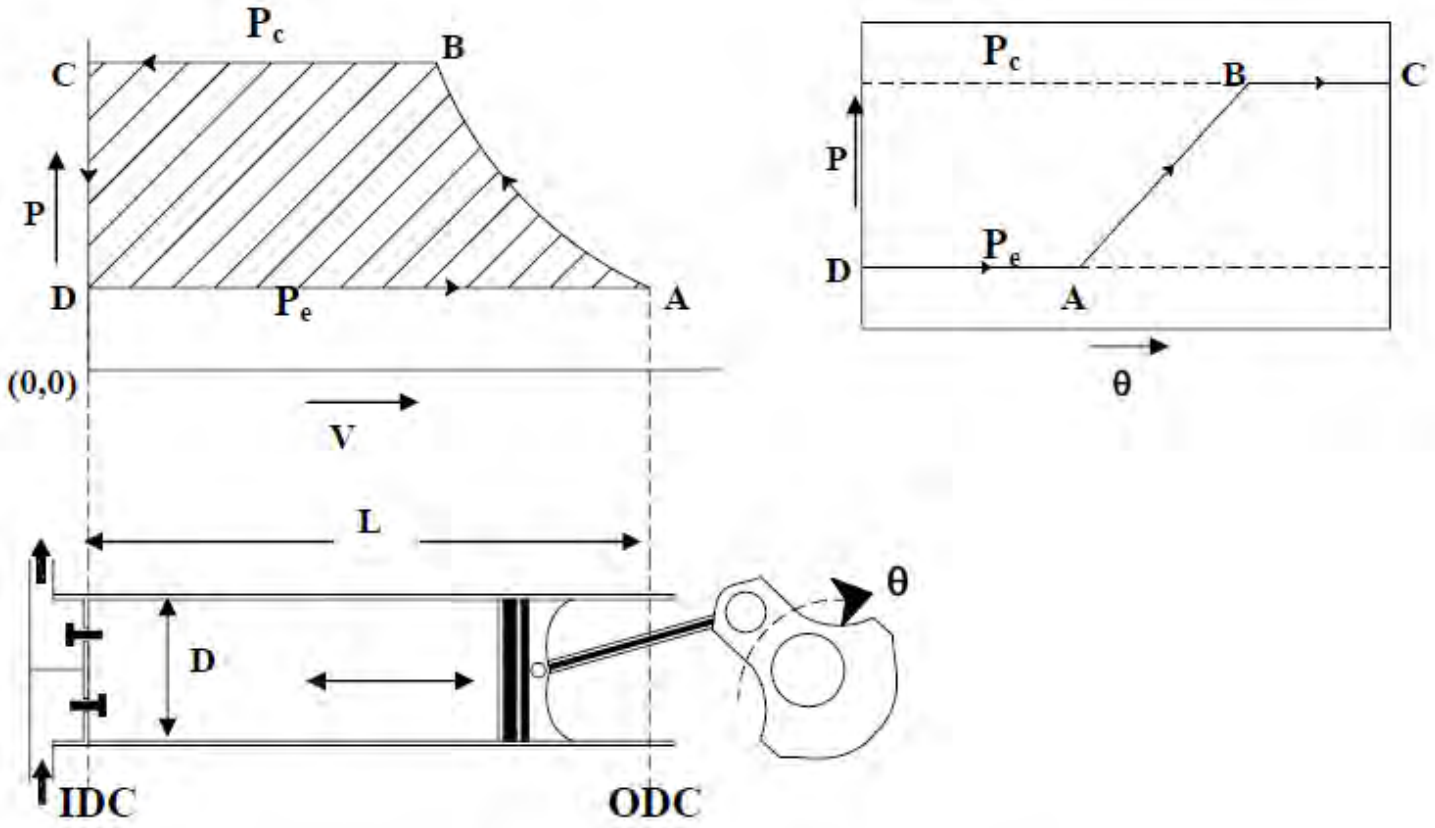
Figure shows the schematic of an ideal compression process on pressure-volume and pressure-crank angle (θ) diagrams. As shown in the figures, the cycle of operations consists of:

Process D-A: This is an isobaric suction process, during which the piston moves from the Inner Dead Centre (IDC) to the Outer Dead Centre (ODC). The suction valve remains open during this process and refrigerant at a constant pressure $P_e$ flows into the cylinder.

Process A-B: This is an isentropic compression process. During this process, the piston moves from ODC towards IDC. Both the suction and discharge valves remain closed during the process and the pressure of refrigerant increases from $P_e$ to $P_c$.

Process B-C: This is an isobaric discharge process. During this process, the suction valve remains closed and the discharge valve opens. Refrigerant at a constant P is expelled from the compressor as the piston moves to IDC.

# PERFORMANCE OF RECIPROCATING COMPRESSORS




*Ideal reciprocating compressor on P-V and P-θ diagrams*

# QUESTIONS

1. Which of the following is not positive displacement type compressor?
a. Rotary vane compressor
b. Rotary screw type compressor
c. Centrifugal compressor
d. Acoustic compressor

2. Compared to a hermetic compressor, an open type compressor:
a. Offers higher efficiency
b. Offers lower noise
c. Offers better compressor cooling
d. Offers serviceability and flexibility

3. Hermetic compressors are used mainly in smaller systems as they:
a. Yield higher COP
b. Do not require frequent servicing
c. Offer the flexibility of using any refrigerant
d. Can be used under different load conditions efficiently

# QUESTIONS

4. In reciprocating compressors, clearance is provided:
a. To improve the volumetric efficiency of the compressor
b. To accommodate valves
c. To account for thermal expansion due to temperature variation
d. To reduce power consumption of the compressor

5. The clearance volumetric efficiency of a reciprocating compressor depends on:
a. Properties of the refrigerant
b. Operating temperatures
c. Clearance volume
d. All of the above

# QUESTIONS

6. A spacer is used in reciprocating compressors to introduce clearance volume. A refrigerant manufacturer wishes to standardize the components of a reciprocating compressor for refrigeration systems of capacities of 2 kW and 2.5 kW by varying only the spacer. Both the systems use the same refrigerant, which has an isentropic index of compression of 1.116 and operate over a pressure ratio of 5. The operating temperatures are also same for both the systems. If the required clearance factor for the 2.5 kW system is 0.03, what should be the clearance factor for the 2.0 kW system?

7. Water is used in a Standard Single Stage (SSS) vapour compression refrigeration system. The system operates at an evaporator temperature of 4.5°C (pressure = 0.8424 kPa) and a condenser temperature of 38°C (pressure = 6.624 kPa). Assume that the water vapour behaves as an ideal gas with $c_p/c_v = 1.322$ and calculate the discharge temperature if compression is isentropic. Also calculate COP and volumic refrigeration effect if the refrigeration effect is 2355 kJ/kg. Molecular weight of water = 18 kg/kmol, Universal gas constant = 8.314 kJ/kmol.K

# STEAM TURBINE

This session is intended to discuss the following:

- Classification of steam turbines
- Compounding of steam turbines
- Forces, work done and efficiency of steam turbine
- Numerical examples

# STEAM

Steam is a vapour used as a working substance in the operation of steam turbine.

## Is steam a perfect gas?

Steam possess properties like those of gases: namely pressure, volume, temperature, internal energy, enthalpy and entropy. But the pressure volume and temperature of steam as a vapour are not connected by any simple relationship such as is expressed by the characteristic equation for a perfect gas.

Sensible heat – The heat absorbed by water in attaining its boiling point.

Latent heat – The heat absorbed to convert boiling water into steam.

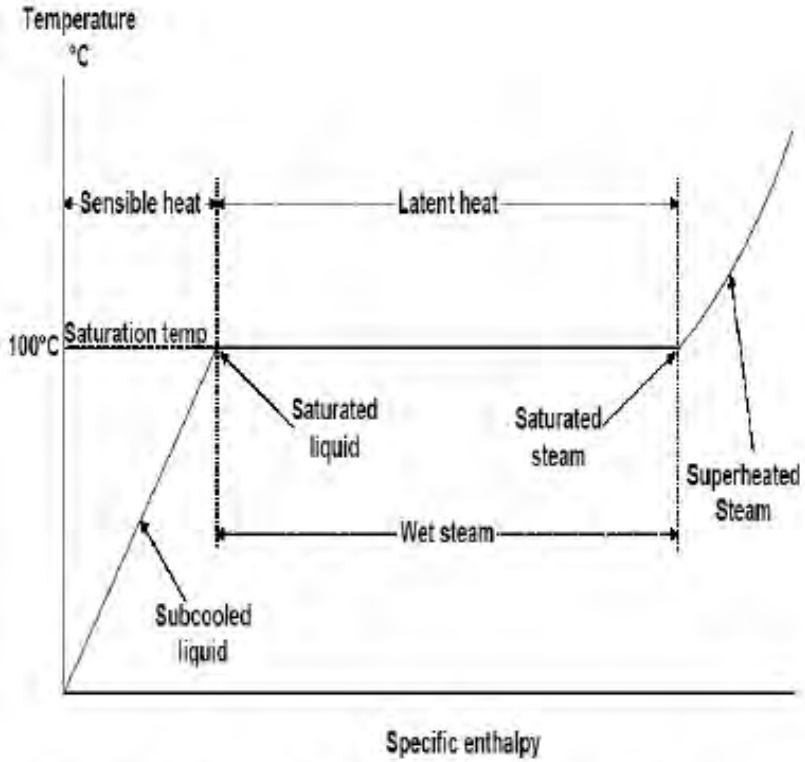
Wet steam – Steam containing some quantity of moisture.

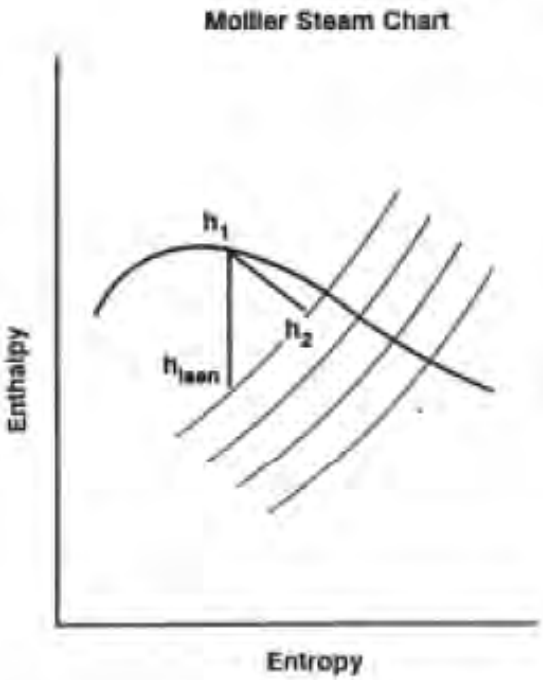
Dry steam – Steam that has no moisture content.

Superheated steam – Dry steam, when heated at constant pressure, attains superheat

The properties of steam are dependent on its pressure

# STEAM PROPERTIES

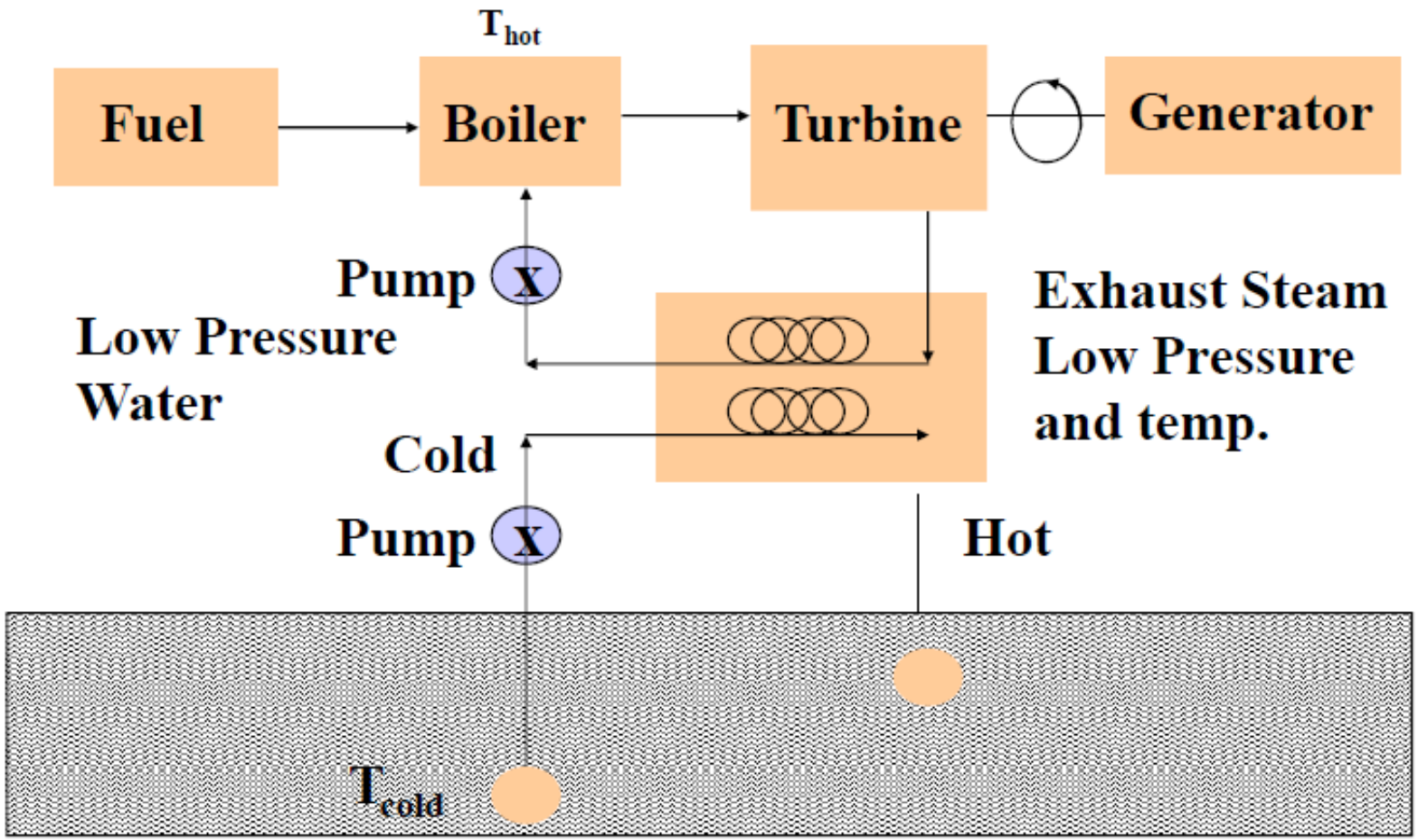



Enthalpy (H) kJ/kg
Entropy (S) kJ/kg-K
Density (ρ) kg/m$^3$

Internal energy (U) kJ/kg
Specific volume (v) m$^3$/kg
Isobaric heat capacity ($C_p$)

# STEAM POWER PLANT PROCESS



# STEAM TURBINE

- Steam turbine convert a part of the energy of the steam evidenced by high temperature and pressure into mechanical power-in turn electrical power
- The steam from the boiler is expanded in a nozzle, resulting in the emission of a high velocity jet. This jet of steam impinges on the moving vanes or blades, mounted on a shaft. Here it undergoes a change of direction of motion which gives rise to a change in momentum and therefore a force.
- The motive power in a steam turbine is obtained by the rate of change in momentum of a high velocity jet of steam impinging on a curved blade which is free to rotate.
- The conversion of energy in the blades takes place by impulse, reaction or impulse reaction principle.
- Steam turbines are available in a few kW(as prime mover) to 1500 MW

# STEAM, GAS AND HYDRAULIC TURBINES

- The working substance differs for different types of turbines.
- Steam turbines are axial flow machines (radial steam turbines are rarely used) whereas gas turbines and hydraulic turbines of both axial and radial flow type are used based on applications.
- The pressure of working medium used in steam turbines is very high, whereas the temperature of working medium used is gas turbine is high comparatively.
- The pressure and temperature of working medium in hydraulic turbines is lower than steam turbines.
- Steam turbines of 1300 MW single units are available whereas largest gas turbines unit is 530 MW and 815 MW

# MERITS AND DEMERITS OF STEAM TURBINE

**Merits:**

- Ability to utilize high pressure and high temperature steam.
- High component efficiency.
- High rotational speed.
- High capacity/weight ratio.
- Smooth, nearly vibration-free operation.
- No internal lubrication.
- Oil free exhaust steam.
- Can be built in small or very large units (up to 1200 MW).

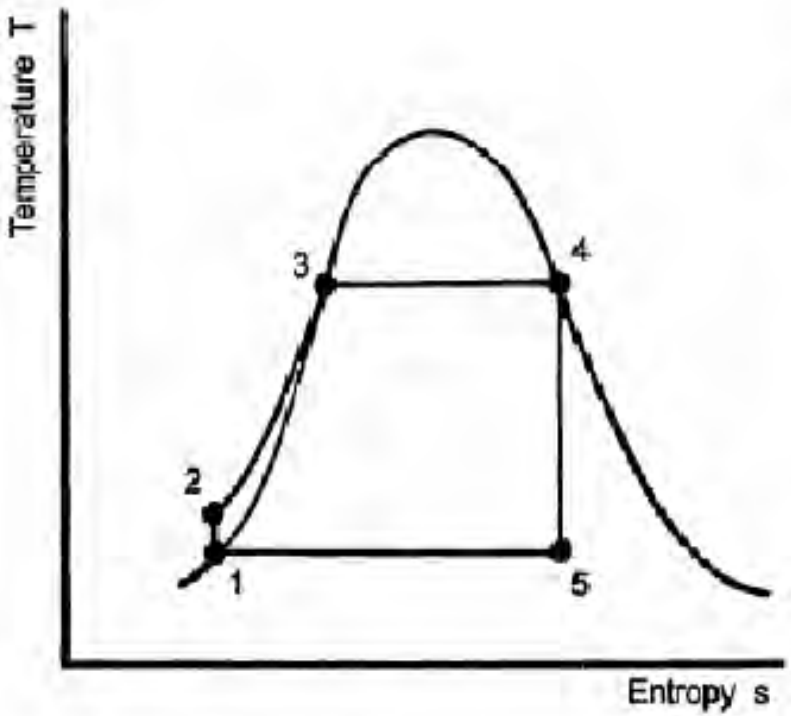
**Demerits:**

- For slow speed application reduction gears are required.
- The steam turbine cannot be made reversible.
- The efficiency of small simple steam turbines is poor.

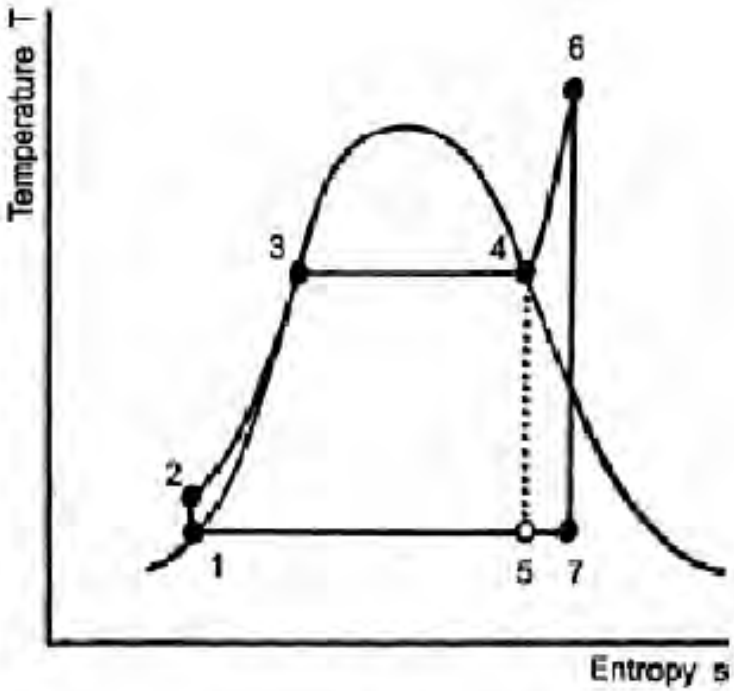
# APPLICATION

- Power generation
- Refinery, Petrochemical,
- Pharmaceuticals,
- Food processing,
- Petroleum/Gas processing,
- Pulp & Paper mills,
- Waste-to-energy

# RANKINE CYCLE




Saturated Rankine cycle

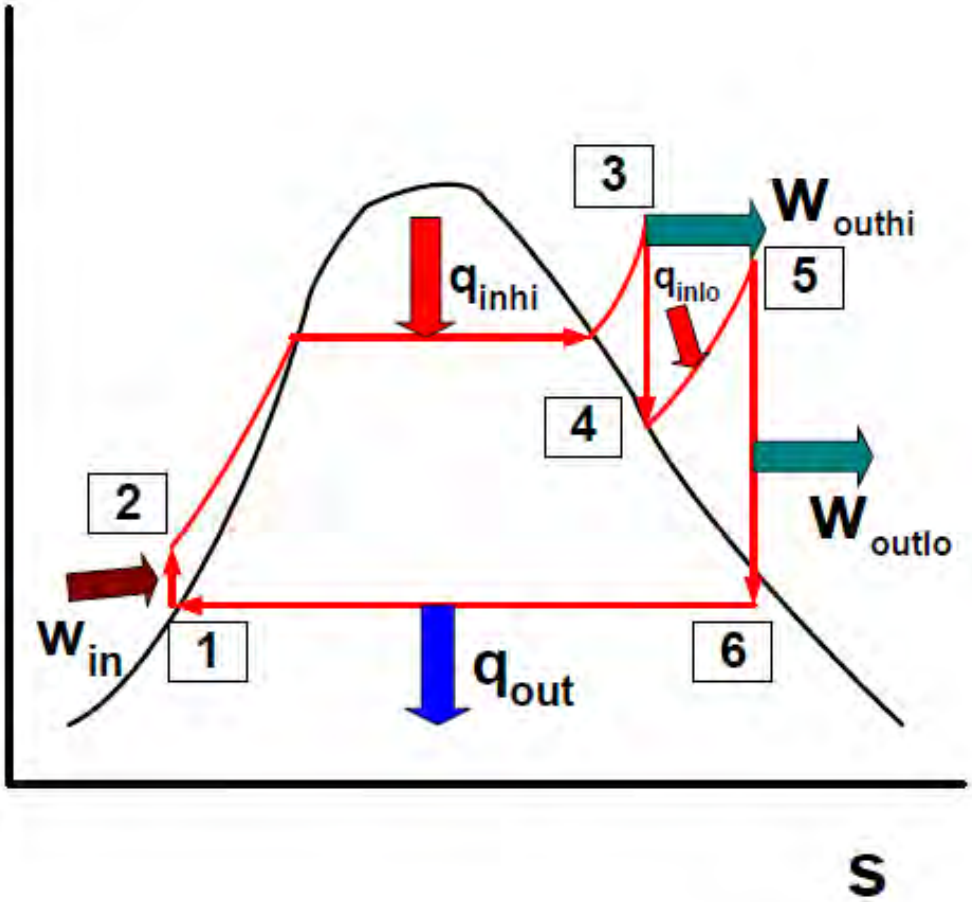



Superheated Rankine cycle

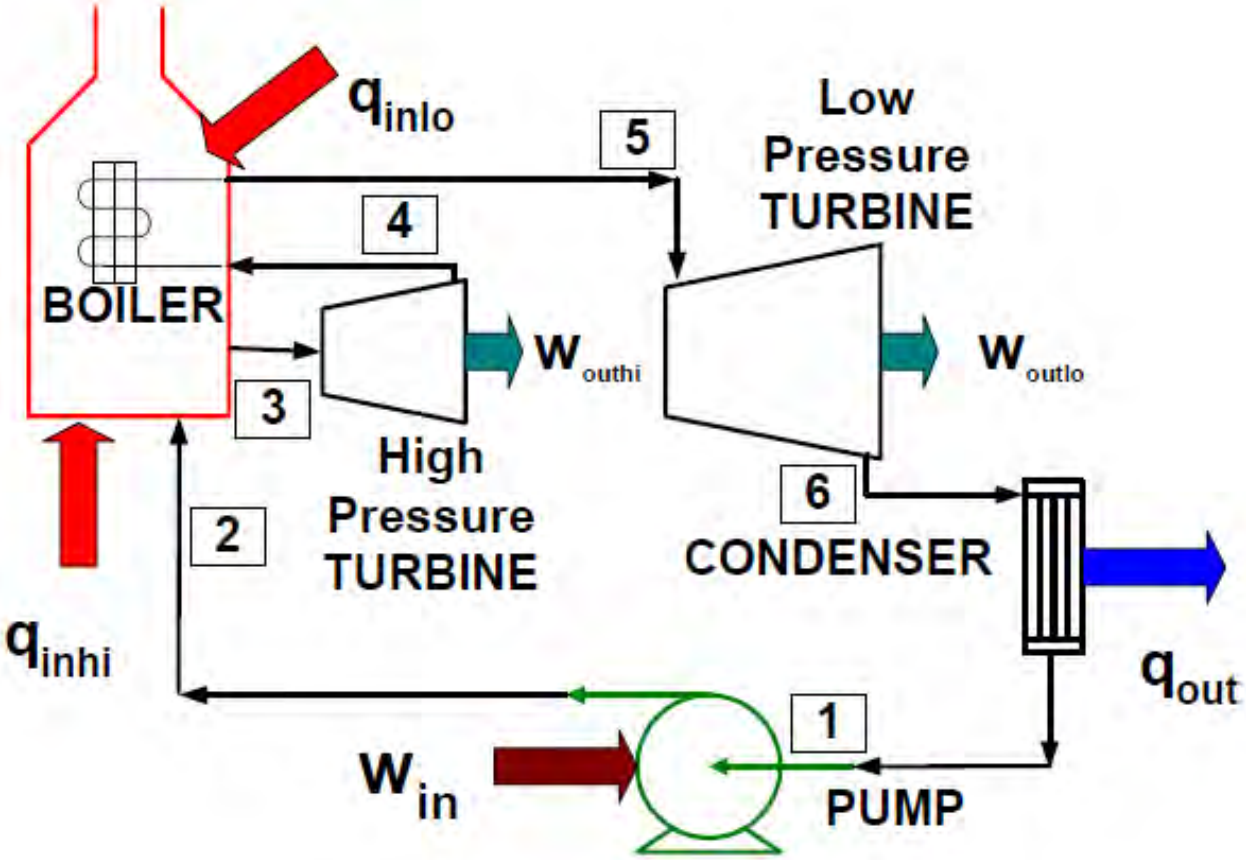
# REHEAT ON *T-S* DIAGRAM

Note that $T_5 < T_3$. Many systems reheat to the same temperature ($T_3 = T_5$).

Reheat is usually not offered for turbines less than 50 MW

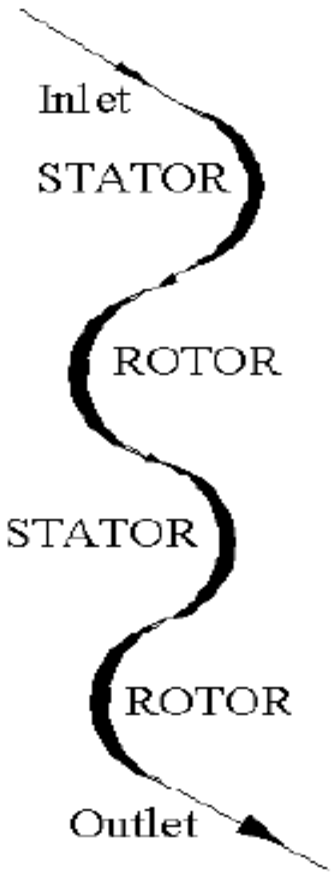


# SCHEMATIC OF RANKINE REHEAT CYCLE



# STEAM TURBINE STAGE

- A turbine stage consists of stationary stator row (guide vanes or nozzle ring) and rotating rotor row.
- In the guide vanes high pressure, high temperature steam is expanded resulting in high
- The guide vanes direct the flow to the rotor blades at an appropriate angle.
- In the rotor, the flow direction is changed and kinetic energy of the working fluid is absorbed by the rotor shaft producing mechanical energy



# TYPES OF STEAM TURBINE

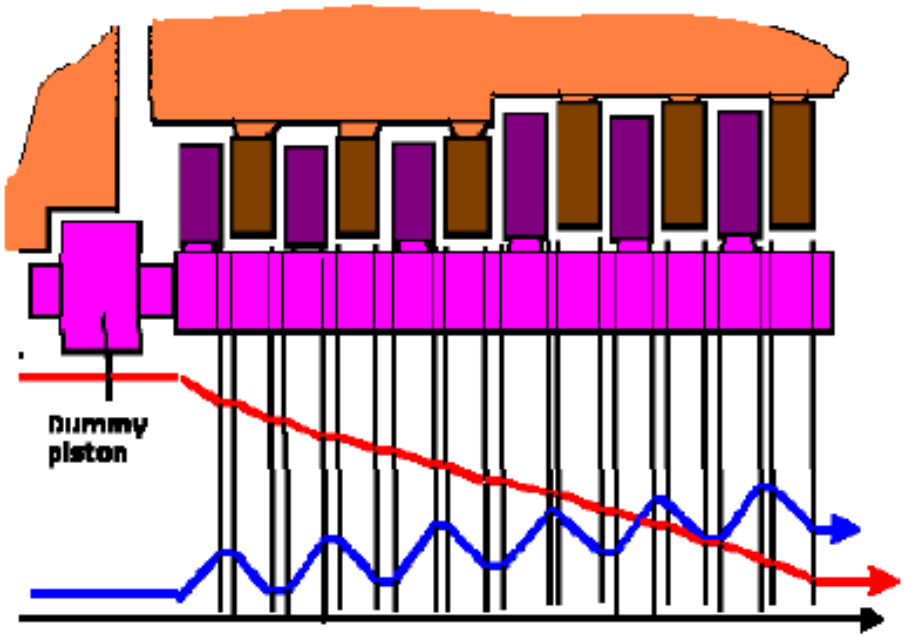
**Impulse Turbine**

Process of complete expansion of steam takes place in stationary nozzle and the velocity energy is converted into mechanical work on the turbine blades.

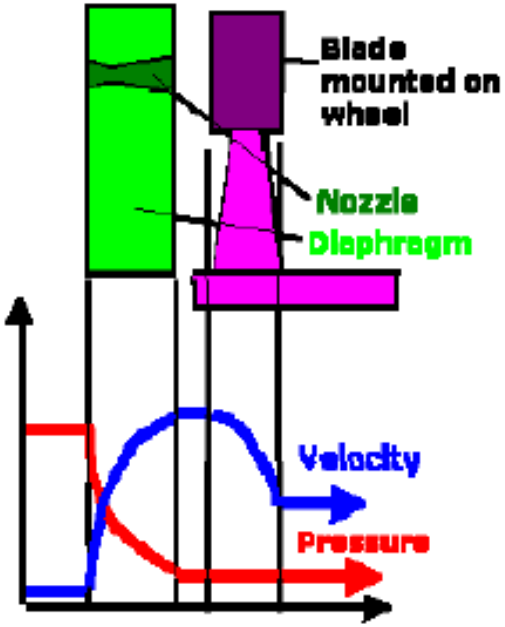
**Reaction Turbine**

Pressure drop with expansion and generation of kinetic energy takes place in the moving blades.

# TYPES OF STEAM TURBINE




Parsons Reaction Turbine



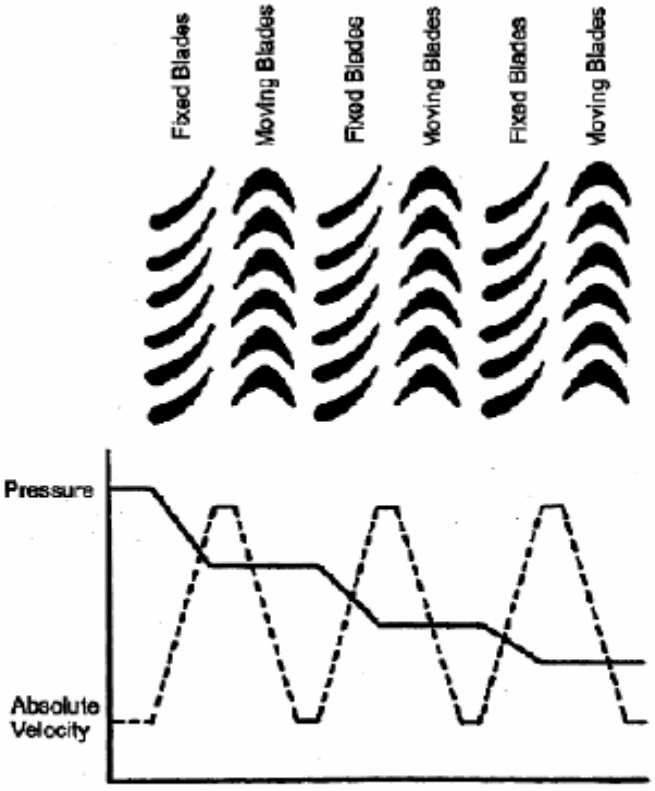

De Laval Impulse Turbine.

# IMPULSE REACTION TURBINE

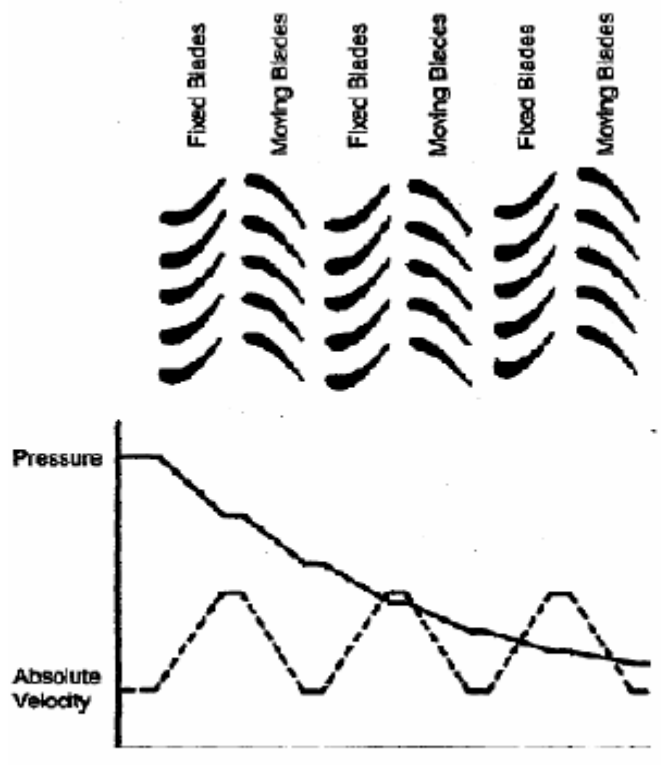
- Modern turbines are neither purely impulse or reaction but a combination of both.
- Pressure drop is effected partly in nozzles and partly in moving blades which are so designed that expansion of steam takes place in them.
- High velocity jet from nozzles produce an impulse on the moving blade and jet coming out from still higher velocity from moving blades produces a reaction.
- Impulse turbine began employing reaction of upto 20% at the root of the moving blades in order to counteract the poor efficiency incurred from zero or even negative reaction.
- Reaction at the root of reaction turbines has come down to as little as 30% to 40% resulting in the reduction of the number of stages required and the sustaining of 50% reaction at mid point.
- It may be more accurate to describe the two design as

✓ Disc and diaphragm turbine using low reaction blading
✓ Drum rotor turbine using high reaction blading

# FLOW THROUGH STEAM TURBINE STAGE




Impulse turbine blading and conditions




Reaction turbine blading and conditions

# COMPOUNDING OF STEAM TURBINES

- This is done to reduce the rotational speed of the impulse turbine to practical limits.

- Compounding is achieved by using more than one set of nozzles, Blades, rotors, in a series keyed to a common shaft; so that either the steam pressure or the jet velocity is absorbed by the turbine in stages.

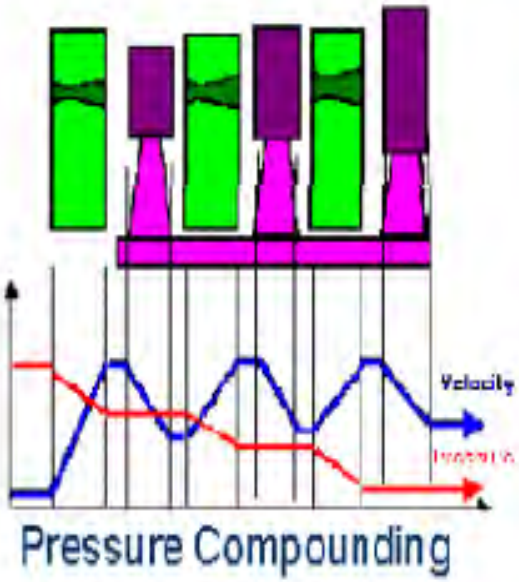
- Three main types of compounded impulse turbines are:

  a. Pressure compounded

  b. Velocity compounded

  c. Pressure and velocity compounded impulse turbines.

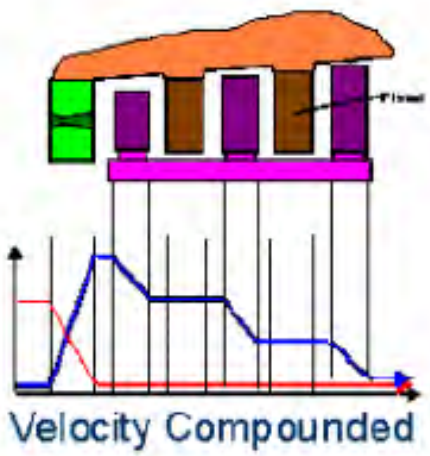
# COMPOUNDING OF STEAM TURBINES



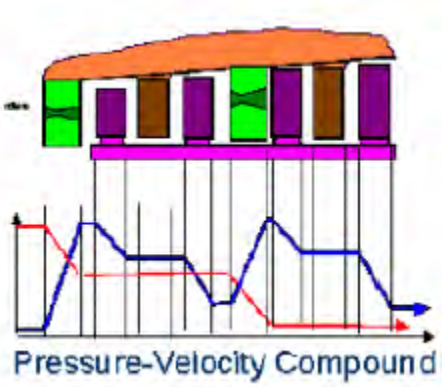

Involves splitting up of the whole pressure drop into a series of smaller pressure drops across several stages of impulse turbine.

The nozzles are fitted into a diaphragm locked in the casing that separates one wheel chamber from another. All rotors are mounted on the same shaft.

# COMPOUNDING OF STEAM TURBINES




Velocity drop is achieved through many moving rows of blades instead of a single row of moving blades. It consists of a nozzle or a set of nozzles and rows of moving blades attached to the rotor or the wheel and rows of fixed blades attached to the casing.




Pressure velocity compounding gives the advantage of producing a shortened rotor compared to pure velocity compounding.
In this design steam velocity at exit to the nozzles is kept reasonable and thus the blade speed (hence rotor rpm) reduced.

# COMPARISON BETWEEN IMPULSE & REACTION TURBINE

**Impulse Turbines**

- An impulse turbine has fixed nozzles that orient the steam flow into high speed jets.
- Blade profile is symmetrical as no pressure drop takes place in the rotor blades.
- Suitable for efficiently absorbing the high velocity and high pressure.
- Steam pressure is constant across the blades and therefore fine tip clearances are not necessary.
- Efficiency is not maintained in the lower pressure stages (high velocity cannot be achieved in steam for the lower pressure stages).

**Reaction Turbines**

- Reaction turbine makes use of the reaction force produced as the steam accelerates through the nozzles formed by the rotor
- Blades have aerofoil profile (convergent passage) since pressure drop occurs partly in the rotor blades.
- Efficient at the lower pressure stages
- Fine blade tip clearances are necessary due to the pressure leakages.
- Inefficient at the high pressure stages due to the pressure leakages around the blade tips.
- Fine tip clearances can cause damage to the tips of the blades.

# NOMENCLATURE

| | |
|---|---|
| $V$ | Absolute velocity of steam |
| $U$ | Blade velocity |
| $W$ | Relative velocity of steam |
| $V_a = V_f = V_m$ | Axial component or flow velocity |
| $V_w$ | Whirl or tangential component |
| $\alpha$ | Nozzle angle |
| $\beta$ | Blade angle |
| $h$ | enthalpy |

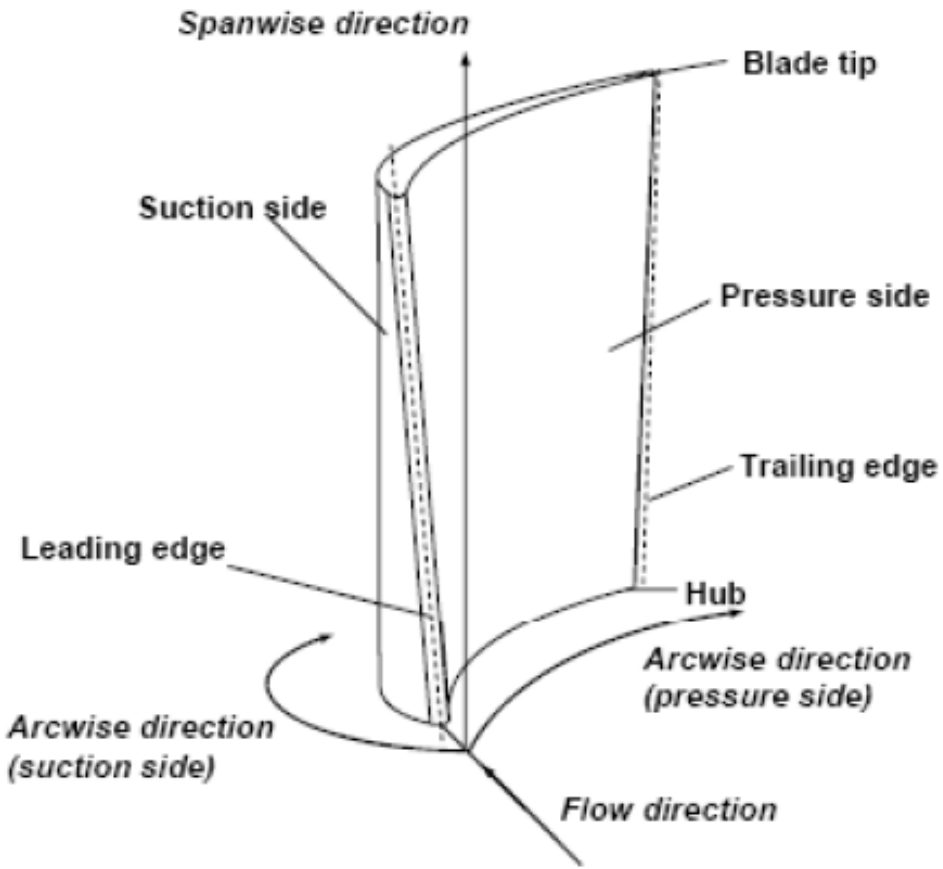
*Suffix*

| | |
|---|---|
| 1 | Inlet |
| 2 | Outlet |

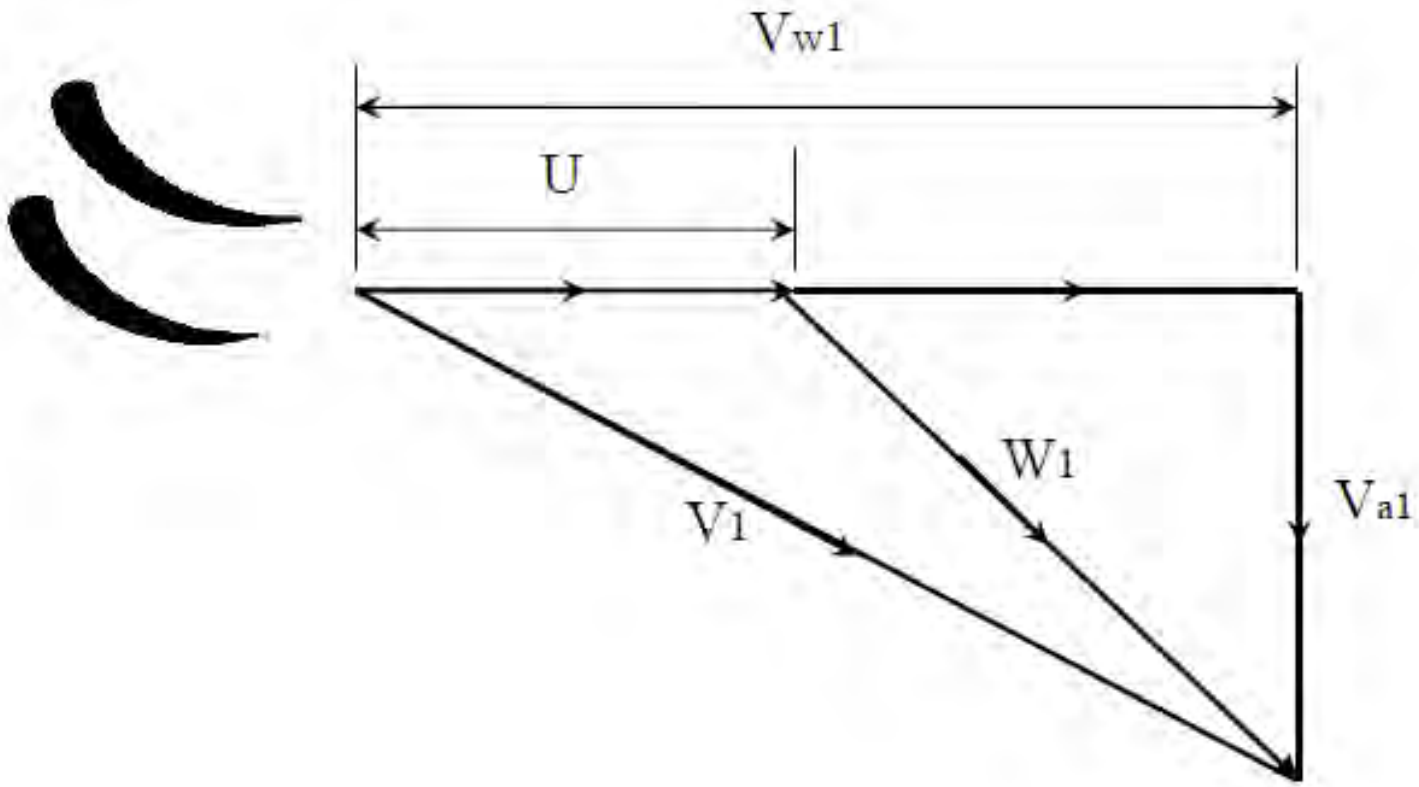
# VELOCITY TRIANGLES

- The three velocity vectors namely, blade speed, absolute velocity and relative velocity in relation to the rotor are used to form a triangle called velocity triangle.
- Velocity triangles are used to illustrate the flow in the bladings of turbomachinery.
- Changes in the flow direction and velocity are easy to understand with the help of the velocity triangles.
- Note that the velocity triangles are drawn for the inlet and outlet of the rotor at certain radii.

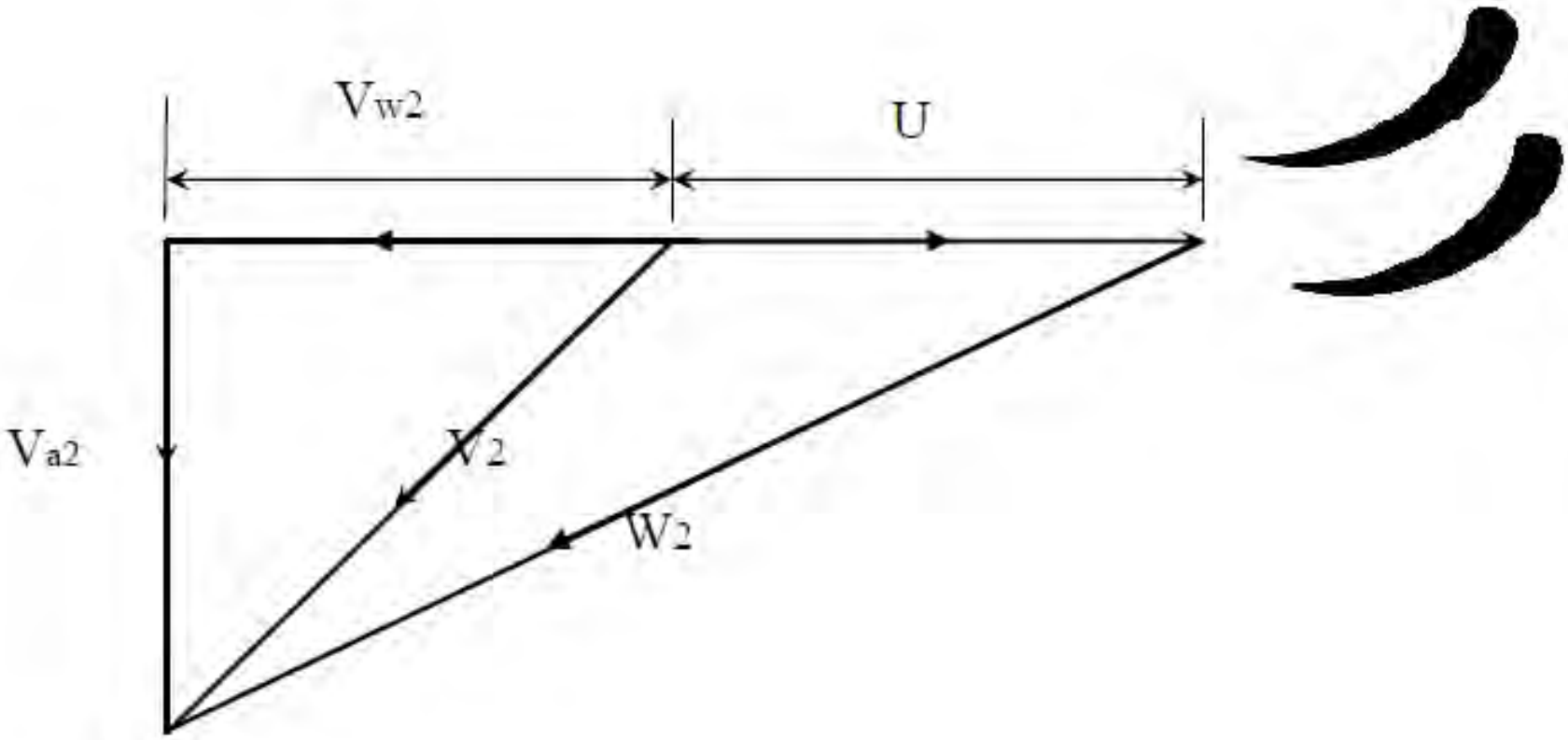
# STEAM TURBINE BLADE TERMINOLOGY



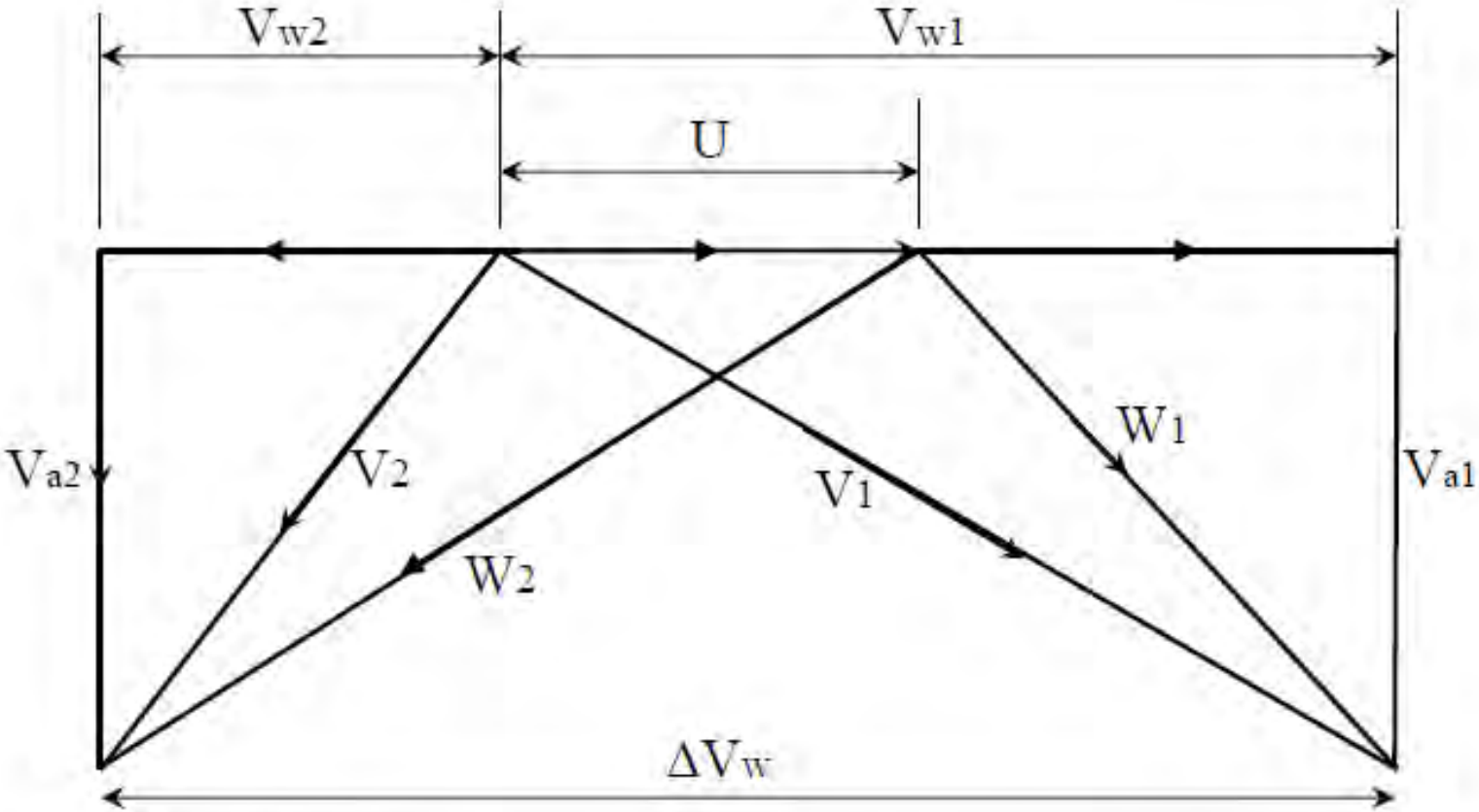
# INLET VELOCITY TRIANGLE



# OUTLET VELOCITY TRIANGLES



# COMBINED VELOCITY TRIANGLES




For 50% reaction design

# WORK DONE – IMPULSE STEAM TURBINE

If the blade is symmetrical then $\beta_1 = \beta_2$ and neglecting frictional effects of the blades on the steam, $W_1 = W_2$.

In actual case, the relative velocity is reduced by friction and expressed by a blade velocity coefficient k.

Thus $k = W_2/W^1$

From Euler's equation, work done by the steam is given by:

$$W_t = U(V_{w1} \pm V_{w2}) \quad (1)$$

Since $V_{w2}$ is in the negative r direction, the work done per unit mass flow is given by,

$$W_t = U(V_{w1}+V_{w2}) \quad (2)$$

If $V_{a1} \neq V_{a2}$, there will an axial thrust in the flow direction. Assume that Va is constant then,

$$W_t = UV_a (\tan\alpha_1 + \tan\alpha_2) \quad (3)$$

$$W_t = UV_a (\tan\beta_1 + \tan\beta_2) \quad (4)$$

Equation (4) is often referred to as the diagram work per unit mass flow and hence the diagram efficiency Is defined as

# WORK DONE – IMPULSE STEAM TURBINE

$$\eta_d = \frac{\text{Diagram work done per unit mass flow}}{\text{Work available per unit mass flow}} \quad (5)$$

Referring to the combined diagram $\Delta V_w$ is the change in the velocity of whirl. Therefore:

The driving force on the wheel = $mV_w$ (6)

The product of the driving force and the blade velocity gives the rate at which work is done on the wheel. From equation (6)

Power output = $mU\Delta V_w$ (7)

If $V_{a1}$-$V_{a2}$ = $\Delta V_a$, the axial thrust is given by;

Axial thrust = $m\Delta V_a$ (8)

The maximum velocity of the steam striking the blades

$$V_1 = \sqrt{\{2(h_0-h_1)\}} \quad (9)$$

Where $h_0$ is the enthalpy at the entry to the nozzle and $h_1$ is the enthalpy at the exit, neglecting the velocity at the inlet to the nozzle. The energy supplied to the blades is the kinetic energy of the jet $V_1^2/2$ and the blading or diagram efficiency;

# WORK DONE – IMPULSE STEAM TURBINE

$$\eta_d = \frac{\text{Rate of work performed per unit mass flow}}{\text{Energy supplied per unit mass of steam}}$$

$$\eta_d = (U\Delta V_w) x \frac{2}{V_1^2} = \frac{2U\Delta V_w}{V_1^2} \qquad (10)$$

Using the blade velocity coefficient ($k=W_2/W_1$) and symmetrical blades ($\beta_1 = \beta_2$),

then; $\Delta V_w = 2V_1 \cos\alpha_1 - U$

Hence $\Delta V_w = 2(V_1 \cos\alpha_1 - U)U$ (11)

And the rate of work performed per unit mass $= 2(V_1 \cos\alpha_1 - U)U$

Therefore; $\eta_d = 2(V_1 \cos\alpha_1 - U)U \times 2/V_1^2$

$$\eta_d = \frac{4(V_1 \cos\alpha_1 - U)U}{V_1^2} = \frac{4U}{V_1}\left(\cos\alpha_1 \frac{U}{V_1}\right)$$

where $\frac{U}{V_1}$ is called the blade speed ratio (12)

# WORK DONE – IMPULSE STEAM TURBINE

Differentiating equation (12) and equating it to zero provides the maximum diagram efficiency;

$$\frac{d(\eta d)}{d\left(U/V_1\right)} = 4\cos\alpha_1 - 8U/V_1 = 0$$

or $$\frac{U}{V_1} = \frac{\cos\alpha_1}{2} \qquad (13)$$

i.e., maximum diagram efficiency

$$= \frac{4\cos\alpha_1}{2}\left(\cos\alpha_1 - \frac{\cos\alpha_1}{2}\right)$$

or $$\eta_d = 4\cos^2\alpha_1 \qquad (14)$$

Substituting this value in equation (7), the power output per unit mass flow rate at the maximum diagram efficiency

$$P = 2U^2 \qquad (15)$$

# DEGREE OF REACTION

- **Degree of reaction** is a parameter that describes the relation between the energy transfer due to the static pressure change and the energy transfer due to dynamic pressure change.

- **Degree of reaction** is defined as the ratio of static pressure drop in the rotor to the static pressure drop in the stage. It is also defined as the ratio of static enthalpy drop in the rotor to the static enthalpy drop in the stage

# DEGREE OF REACTION

$$\Lambda = \text{Degree of reaction} = \frac{\text{Static enthalpy change in rotor}}{\text{Total enthalpy change in stage}} = \frac{h_1 - h_2}{h_0 - h_2} \qquad (16)$$

The static enthalpy at the inlet to the fixed blade in terms of stagnation enthalpy and velocity at the inlet to the fixed blades is given by

$$h_0 = h_{00} - \frac{V_0^2}{2C_p} \quad \text{similarly} \quad h_2 = h_{02} - \frac{V_2^2}{2C_p}$$

Substituting $$\Lambda = \frac{(h_1 - h_2)}{\left(h_{00} - \frac{V_0^2}{2C_p}\right) - \left(h_{02} - \frac{V_2^2}{2C_p}\right)}$$

# DEGREE OF REACTION

But for a normal stage, $V_0 = V_2$ and since $h_{00} = h_{01}$ in the nozzle, then;

$$\Lambda = \frac{(h_1 - h_2)}{(h_{01} - h_{02})} \tag{17}$$

We know that $(h_{01} - h_{02}) = \quad (h_1 - h_2) + \frac{(V_{w1}^2 - V_{w2}^2)}{2} = 0$

Substituting for $(h_1 - h_2)$ in equation (17),

$$\Lambda = \frac{(V_{w2}^2 - V_{w1}^2)}{[2(h_{01} - h_{02})]} \quad = \frac{(V_{w2}^2 - V_{w1}^2)}{[2U(V_{w1} - V_{w2})]} \tag{18}$$

Assuming the axial velocity is constant through out the stage, then

$$\Lambda = \frac{(V_{w2}^2 - V_{w1}^2)}{[2U(U + V_{w1} + V_{w2} - U)]}$$

$$\Lambda = \frac{(V_{w2} - V_{w1})(V_{w2} + V_{w1})}{[2U(V_{w1} + V_{w2})]} \tag{19}$$

# DEGREE OF REACTION

$$\Lambda = \frac{V_a(\tan\beta_2 + \tan\beta_1)}{2U} \qquad (20)$$

From the velocity triangles it is seen that

$$V_{w1} = U + V_{w1} \qquad V_{w2} = V_{w2} - U$$

Therefore equation (20) can be arranged into a second form:

$$\Lambda = \frac{1}{2} + \frac{V_a}{2U}(\tan\beta_2 + \tan\alpha_2) \qquad (21)$$

Putting $\Lambda = 0$ in equation (20), we get

$(\beta_2 = \beta_1)$ And $V_1 = V_2$ and for $\Lambda = 0.5$, $(\beta_2 = \alpha_1)$

**Zero reaction stage**

Let us first discuss the special case of zero reaction. According to the definition of reaction. When $\Lambda = 0$, equation (16) reveals that $h_1 = h_2$ and equation (20) that $\beta_1 = \beta_2$.

# BLADE HEIGHT IN AXIAL FLOW MACHINES

The continuity equation $\dot{m} = \rho A V$ may be used to find the blade height 'h'. The annular area of flow = πDh. Thus the mass flow rate through an axial flow turbine is

$$\dot{m} = \rho \pi D h V_a$$

$$h = \frac{\dot{m}}{\rho \pi D V_a}$$

Blade height will increase in the direction of flow in a turbine and decrease in the direction of flow in a compressor.

# MATERIALS OF STEAM TURBINE

| Part name | Material Code/Composition |
|---|---|
| Casing | IS:2063 |
| Inner casing | GS 22Mo4 Shaft |
| Shaft | 30CrMoV121 |
| Blade high pressure | X22CrMoV121 |
| Blade Low pressure | X20Cr3 |
| Casing joint bolt | 21CrMoV57 |
| Crossover pipe | ASTM 533 Gr.70 |
| Valve spindle | X22CrMoV121 |
| Valve body | GS17crmov511 |
| Valve seat | 21CrMo57 |